RIEMANN SUMS

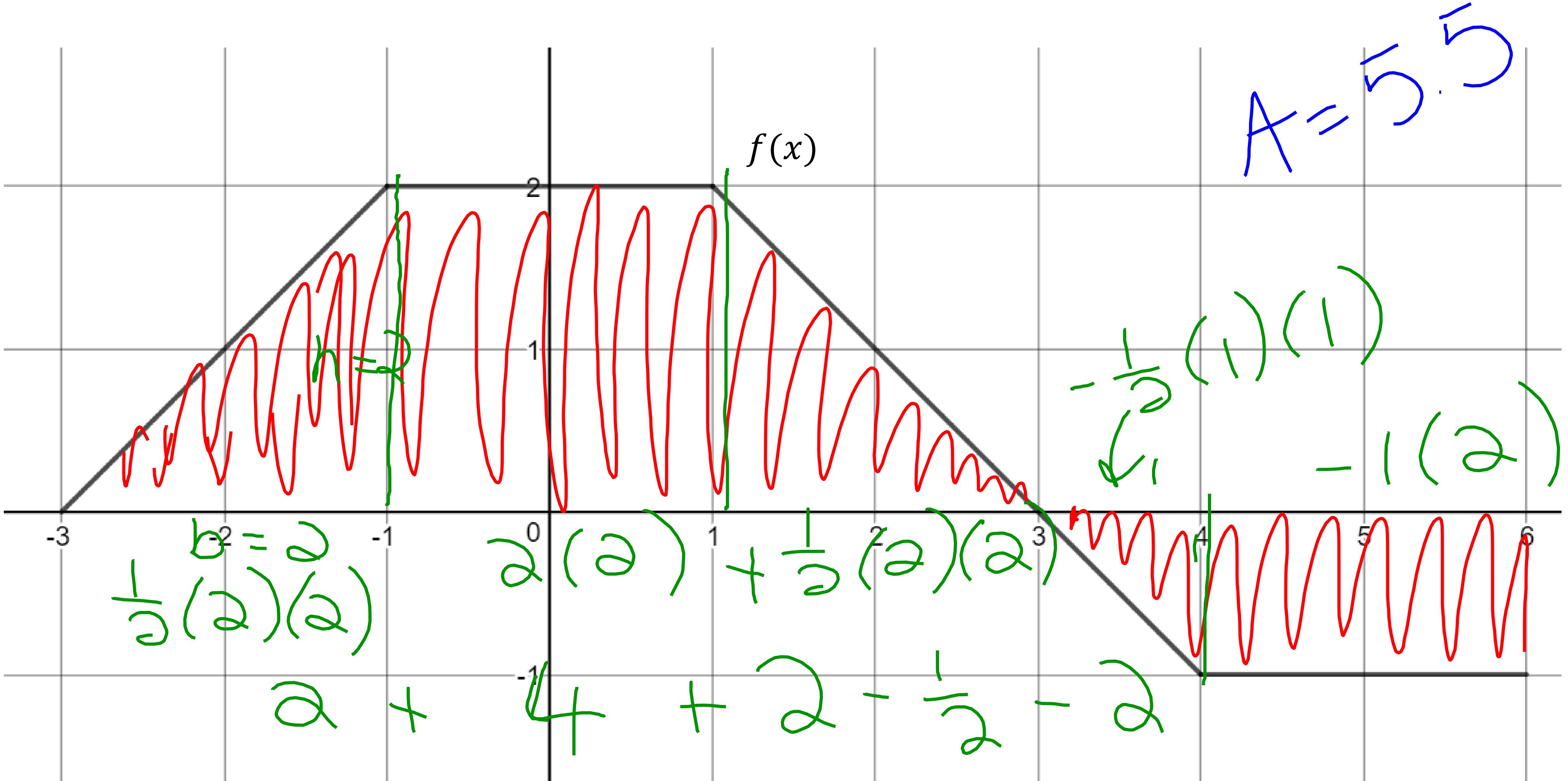
Keeper 27

Honors Calculus

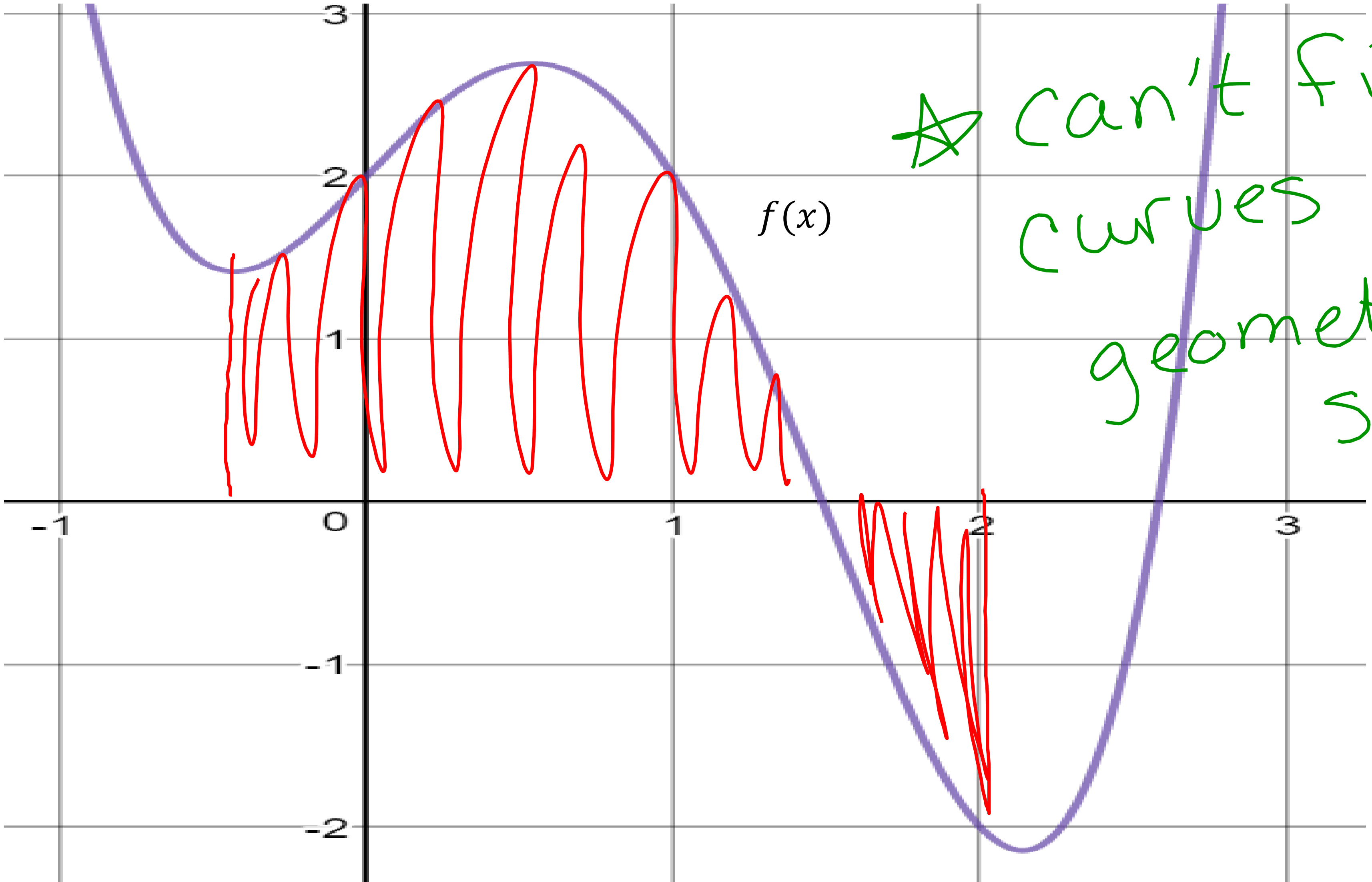
WHAT IS AN INTEGRAL???

The **AREA** under a curve!!!

EXAMPLE – FIND $\int_{-3}^{6} f(x)$



EXAMPLE – FIND $\int_{-.5}^{2} f(x)$



☆ can't fit all curves to a geometric shape

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The process of using rectangles to approximate area

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$$\int_a^b f(x)\,dx$$

Right Riemann Sum (RRS)

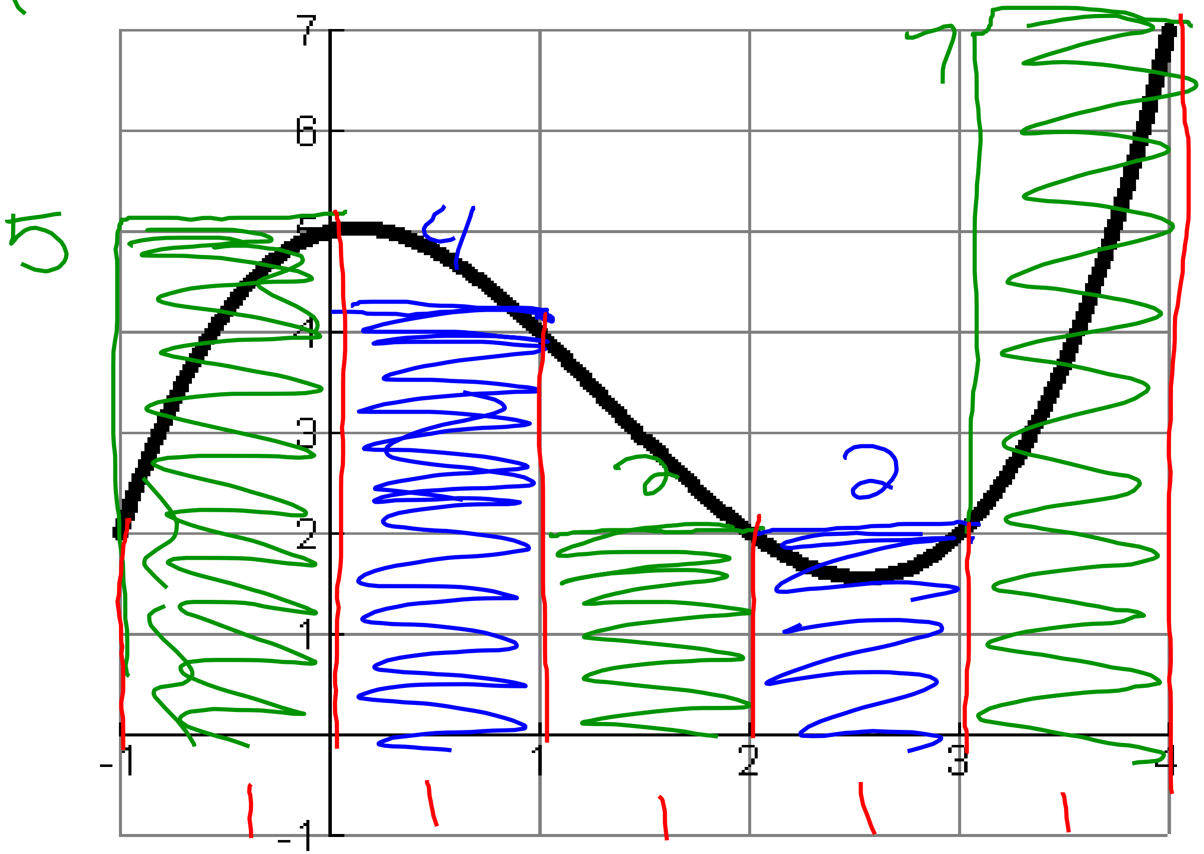
$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[f(x_1) + f(x_2) + \cdots + f(x_n)\right]$$

Left Riemann Sum (LRS)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[f(x_0) + f(x_1) + \cdots + f(x_n)\right]$$

APPROXIMATE THE INTEGRAL USING THE RIGHT RIEMANN SUM METHOD – 5 SUBINTERVALS

$A = 1(5) + 1(4) + 1(2) + 1(2) + 1(7)$

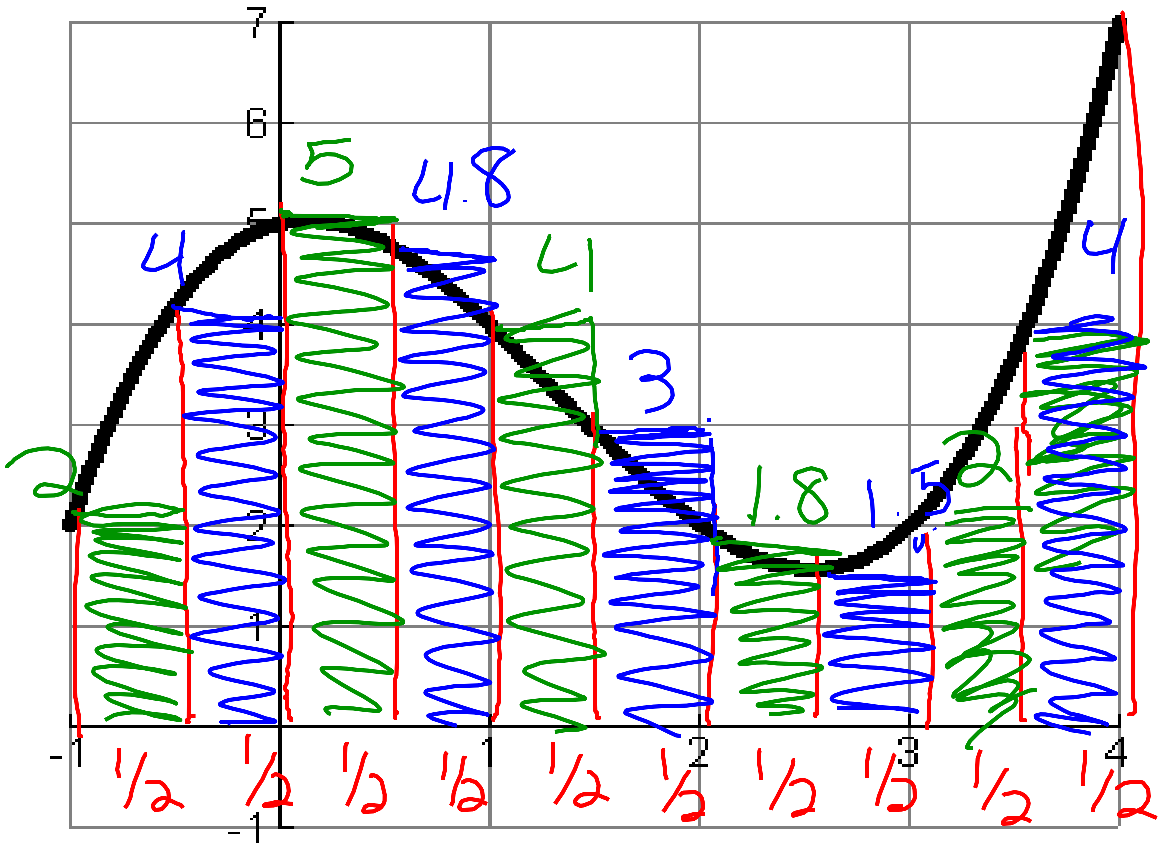


or

$A = 1(5+4+2+2+7)$

$A = 20$

APPROXIMATE THE INTEGRAL USING THE LEFT ENDPOINT METHOD – 10 SUBINTERVALS



$$\frac{1}{2}\left(2+4+5+4.8+4+3+1.8+1.5+2+4\right)$$

$$A = 16.05$$

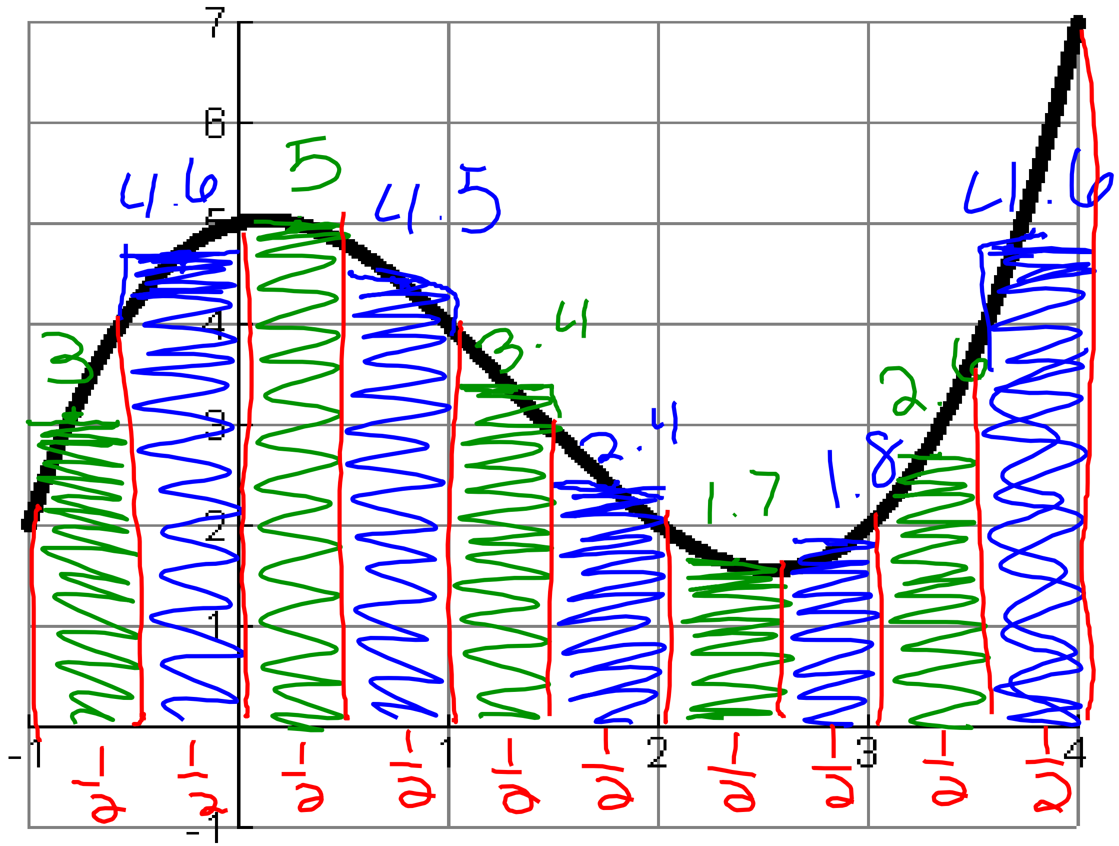
RIEMANN SUMS

$$\int_a^b f(x)\,dx$$

Midpoint Riemann Sum (MRS)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[f\left(\frac{x_0+x_1}{2}\right)+f\left(\frac{x_1+x_2}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$$

APPROXIMATE THE INTEGRAL USING THE MIDPOINT METHOD – 10 SUBINTERVALS



$$A = \frac{1}{2}(3 + 4.6 + 5 + 4.5 + 3.4 + 2.4 + 1.7 + 1.8 + 2.6 + 4.6)$$

$$A = 16.8$$

TRAPEZOID RULE

$$\int_a^b f(x)\,dx$$

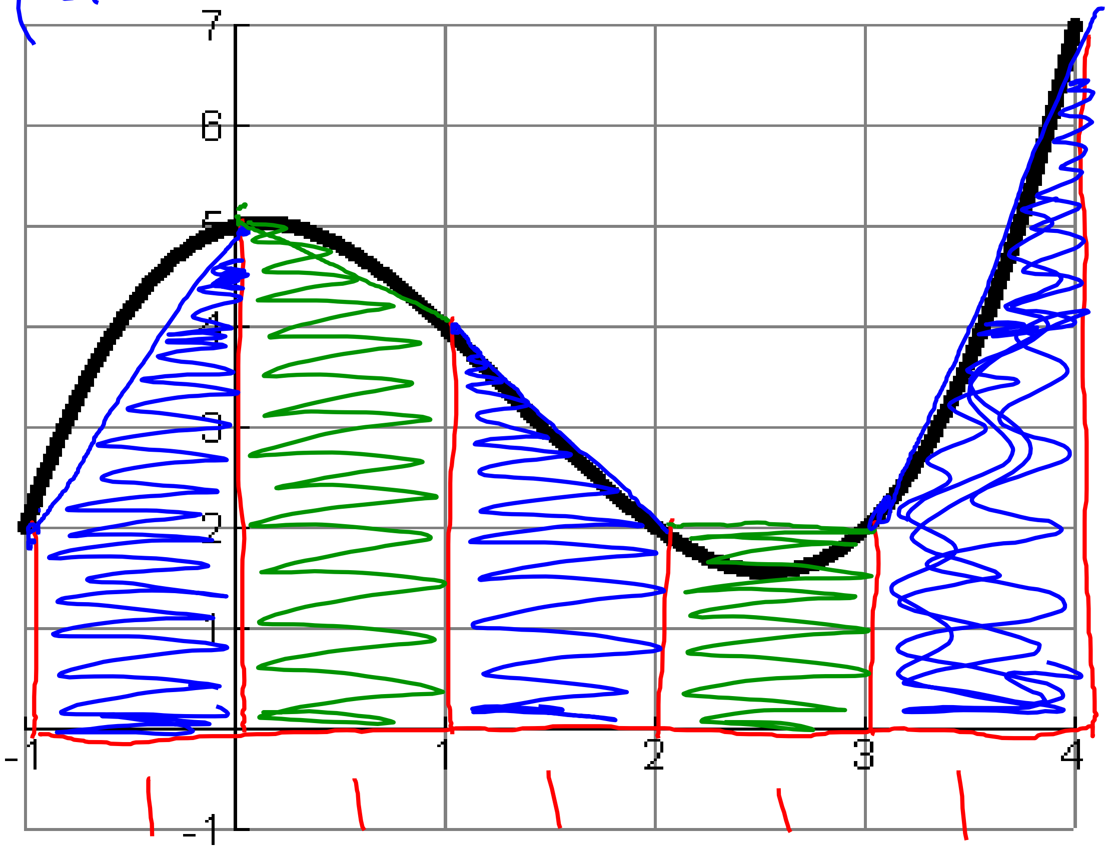
$$\int_a^b f(x)\,dx \approx \frac{1}{2} \cdot \frac{b-a}{n} [f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

It is the average of the left and right sums and usually gives a better approximation than either does individually.

Area of Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

APPROXIMATE THE INTEGRAL USING THE TRAPEZOID METHOD – 5 SUBINTERVALS

$A=\frac{1}{2}(2+5)1+\frac{1}{2}(5+4)1+\frac{1}{2}(4+2)1+\frac{1}{2}(2+2)1+\frac{1}{2}(2+7)1$



or

$A=\frac{1}{2}(1)(2+5+5+4+4+2+2+2+2+7)$

$A=18$

RIEMANN SUMS – 2 MORE METHODS

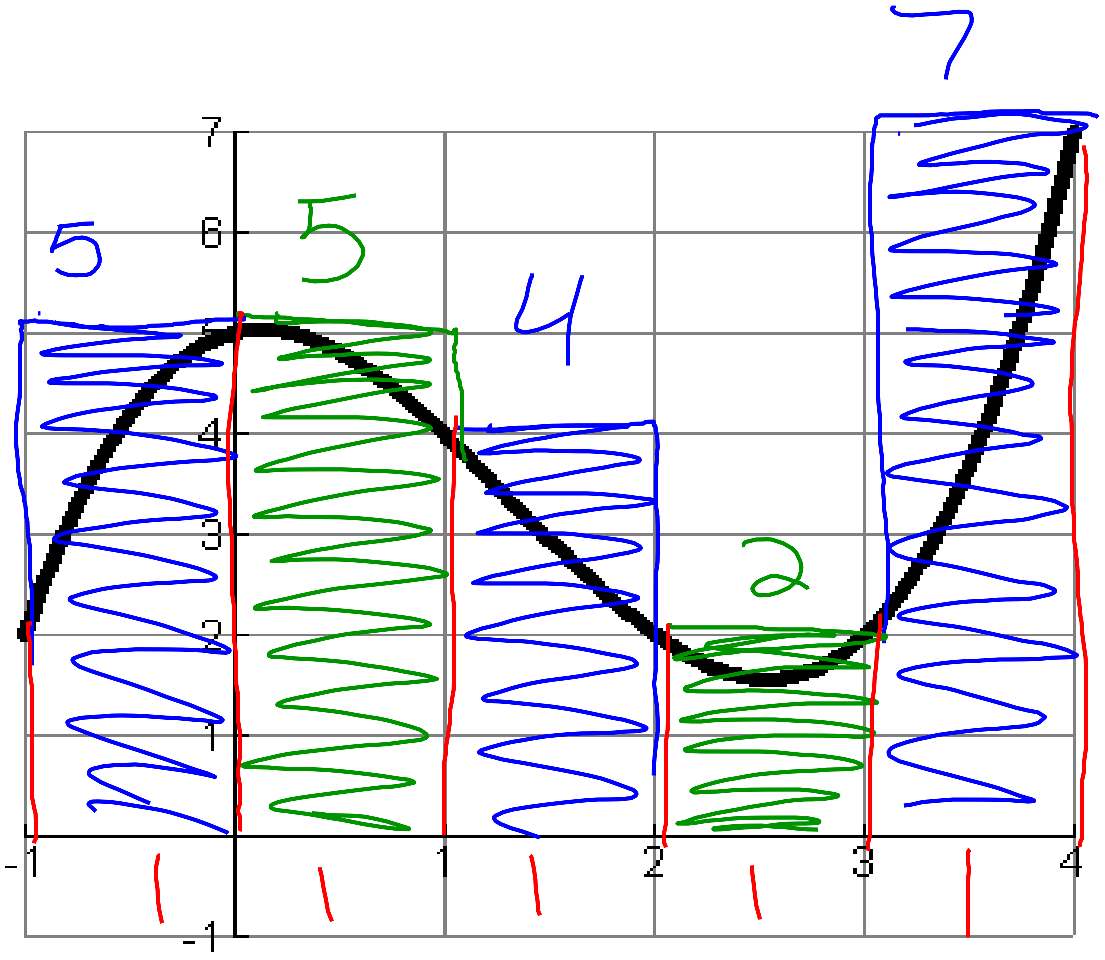
Circumscribed Method – Highest point in interval is used to create the rectangle.

(outside the curve)

Inscribed Method – Lowest Point in the interval is used to create the rectangle.

(inside the curve)

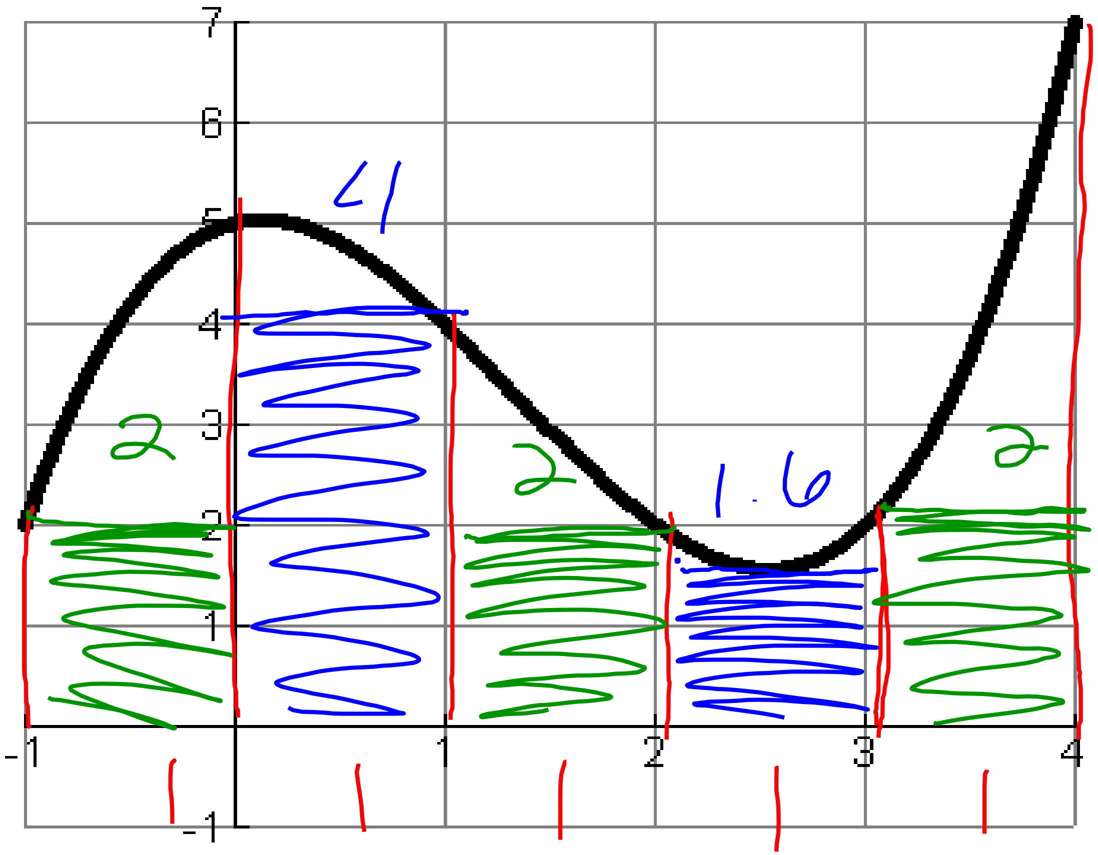
APPROXIMATE THE INTEGRAL USING THE CIRCUMSCRIBED METHOD – ~~10~~ 5 SUBINTERVALS



$A = 1(5+5+4+2+7)$

$A = 23$

APPROXIMATE THE INTEGRAL USING THE INSCRIBED METHOD – ~~10~~ 5 SUBINTERVALS



$A = 1(2 + 4 + 2 + 1.6 + 2)$

$A = 11.6$