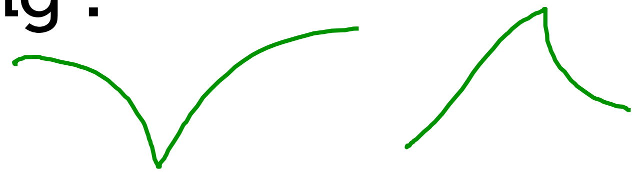
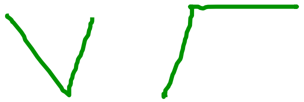
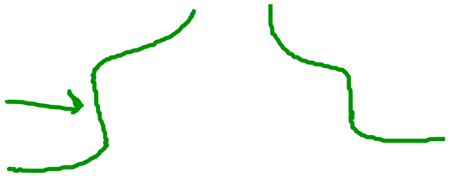
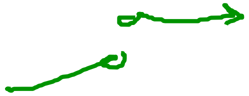
CURVE SKETCHING

Keeper 21

Honors Calculus

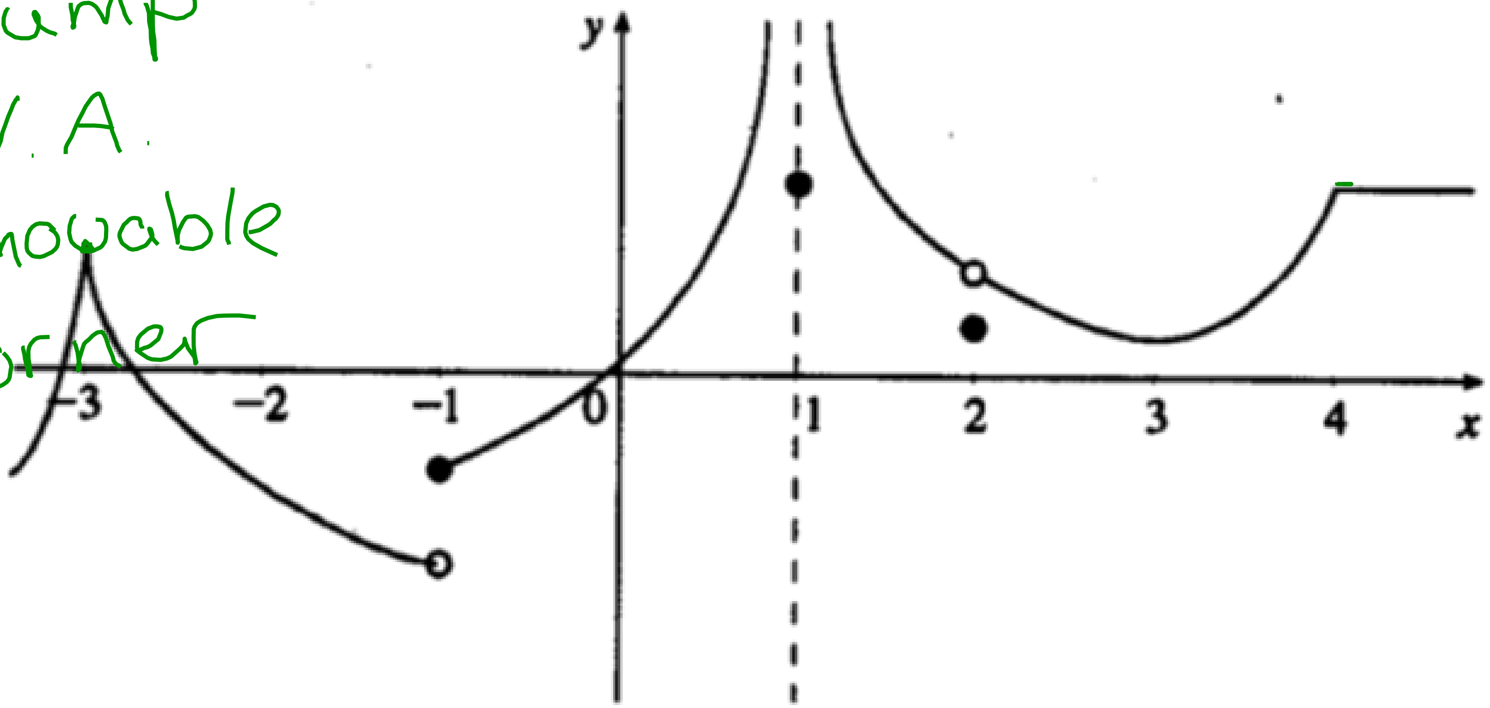
WHEN IS A GRAPH DIFFERENTIABLE

A graph is differentiable anywhere EXCEPT where there is the following :

- Cusp 
- Corner 
- Vertical Tangent 
- Discontinuity
  - Removable hole
  - Infinite V.A. 
  - Jump 

STATE THE X VALUES WHERE f IS NOT DIFFERENTIABLE AND THE REASON

x = -3 cusp
x = -1 jump
x = 1 V.A.
x = 2 removable
x = 4 corner

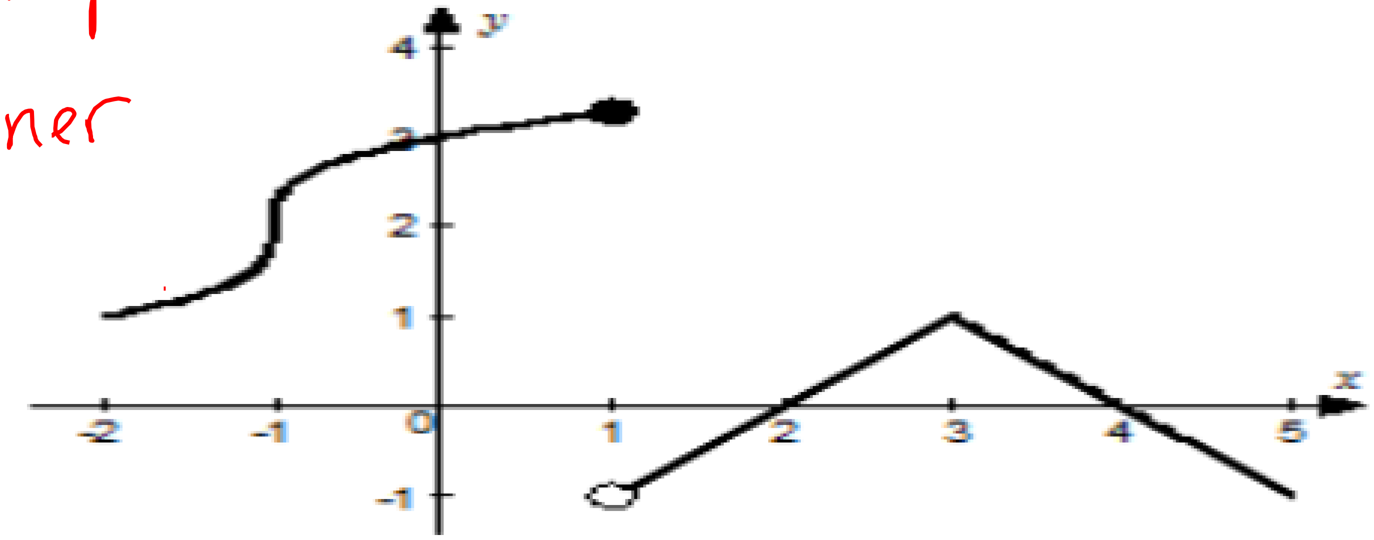


STATE THE X VALUES WHERE f IS NOT DIFFERENTIABLE AND THE REASON

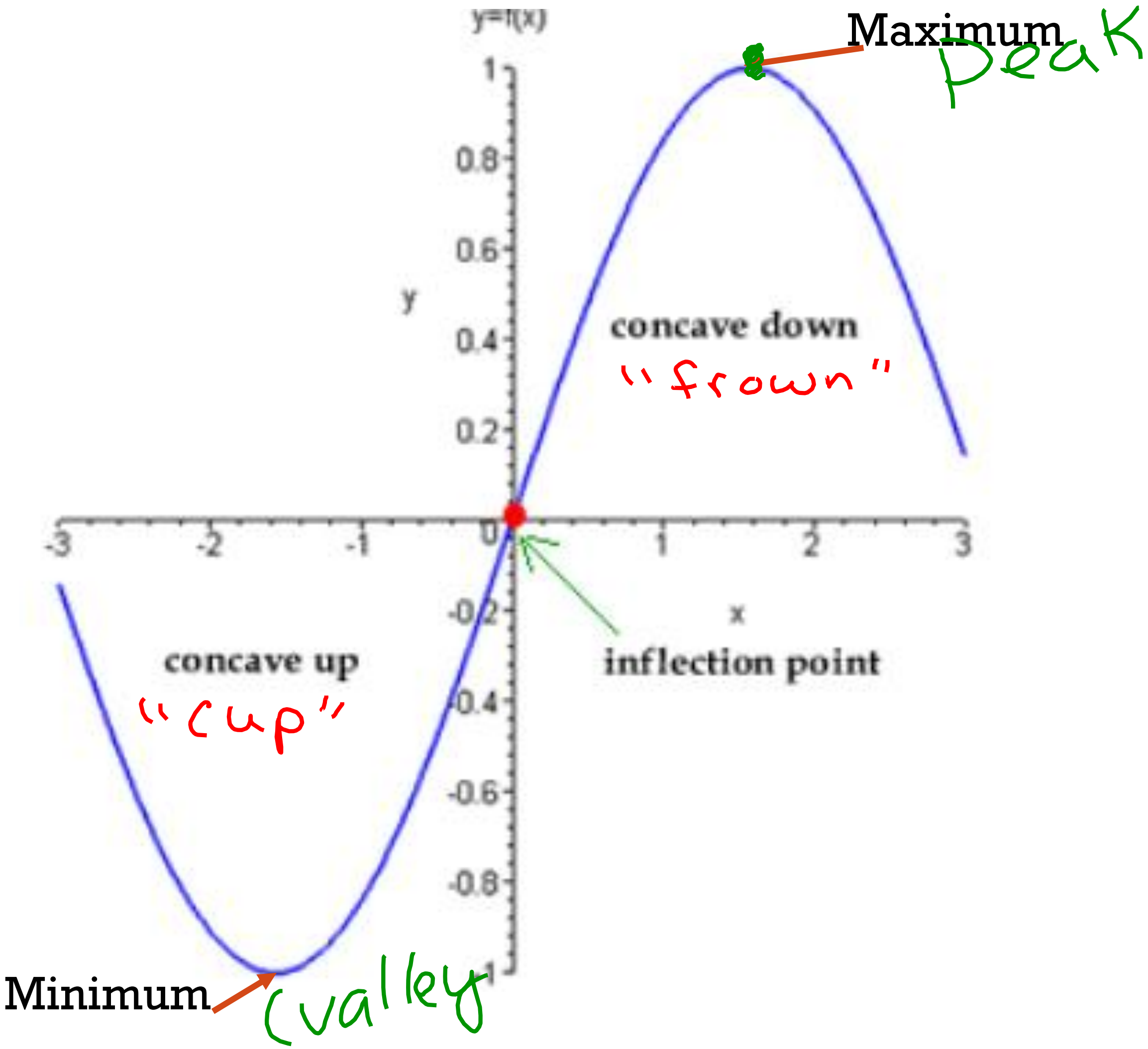
x = -1 vertical tangent

x = 1 jump

x = 3 corner

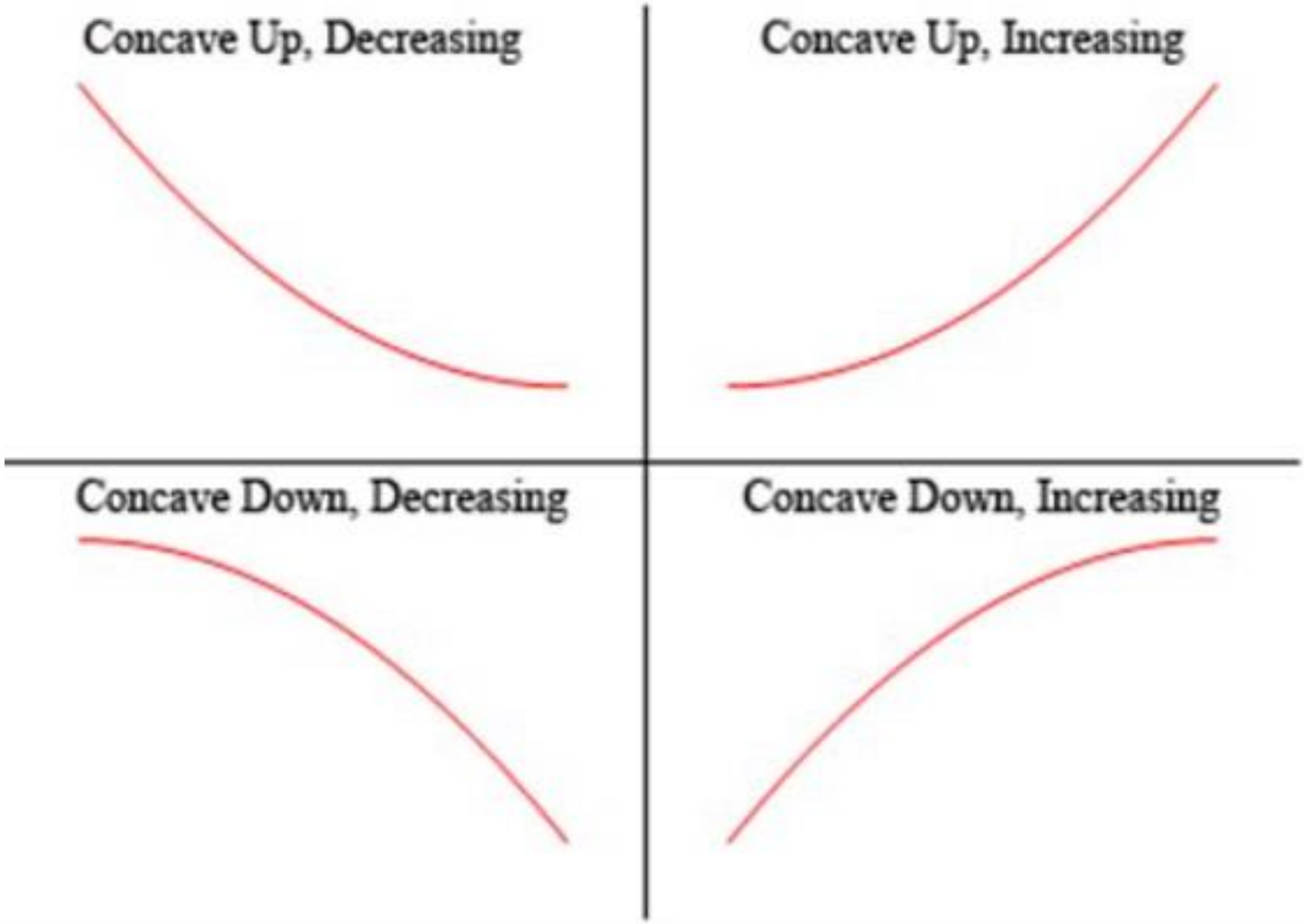


CRITICAL POINTS, CONCAVITY & INFLECTION POINTS



Critical Points – the graph's turning points or the Local Max (peaks) & Local Mins (valleys)

Inflection Points – a point on a graph where it changes concavity. 1 side is concave down & 1 side is concave up


Concave Up, Decreasing
Concave Up, Increasing
Concave Down, Decreasing
Concave Down, Increasing

Relationship between f, f', f''

f	f'	f''
-Cusp -Corner -Discontinuity -Removable -Infinite -jump -Vertical Tangent	DNE	DNE
Local max, local min (local extrema), horizontal tangent	0 On the x-axis x-intercepts	
f increasing	Positive (Above the x-axis)	
f decreasing	Negative (Below the x-axis)	
f concave up	Increasing	Positive (Above the x-axis)
f concave down	Decreasing	Negative (Below the x-axis)
Points of Inflections	Local Extrema	Change Signs

WHAT CAN WE SAY ABOUT g, g', g'' FOR EACH SEGMENT OF THE GRAPH $y = g(x)$

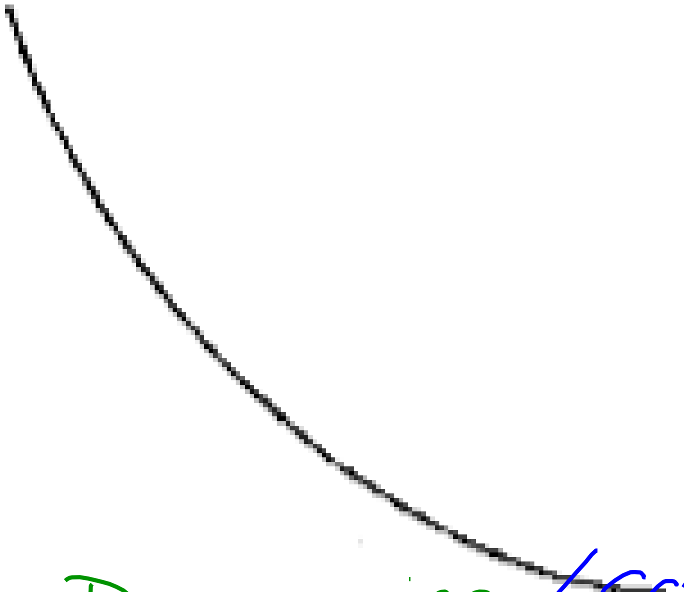
1.

g: Increasing / concave up

g': Positive (above x-axis) / increasing

g'': positive (above x-axis)

2.



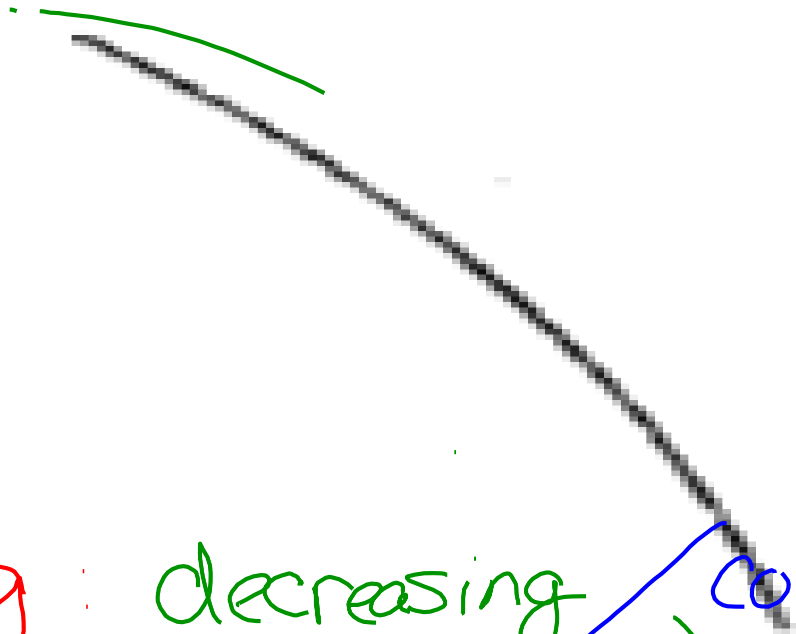
g: Decreasing / conc. up

g': negative (below x-axis) / Incr.

g'': positive (above)

WHAT CAN WE SAY ABOUT g, g', g'' FOR EACH SEGMENT OF THE GRAPH $y = g(x)$

3.



g : decreasing / conc. ↓

g' : negative (below x-axis) / decreasing

g'' : negative (below x-axis)

4.

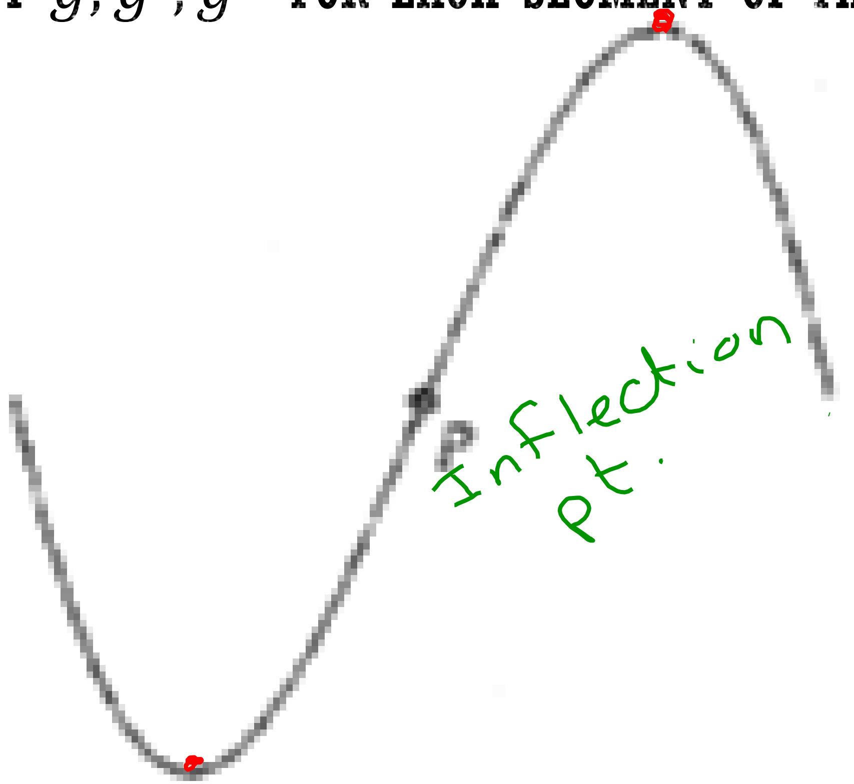
g : increasing / conc ↓

g' : positive (above x-axis) / decr.

g'' : negative (below)

WHAT CAN WE SAY ABOUT g, g', g'' FOR EACH SEGMENT OF THE GRAPH $y = g(x)$

5.



+ means above x-axis
0 means on x-axis
− means below x-axis

g': − − − 0 + + + + 0 − − −

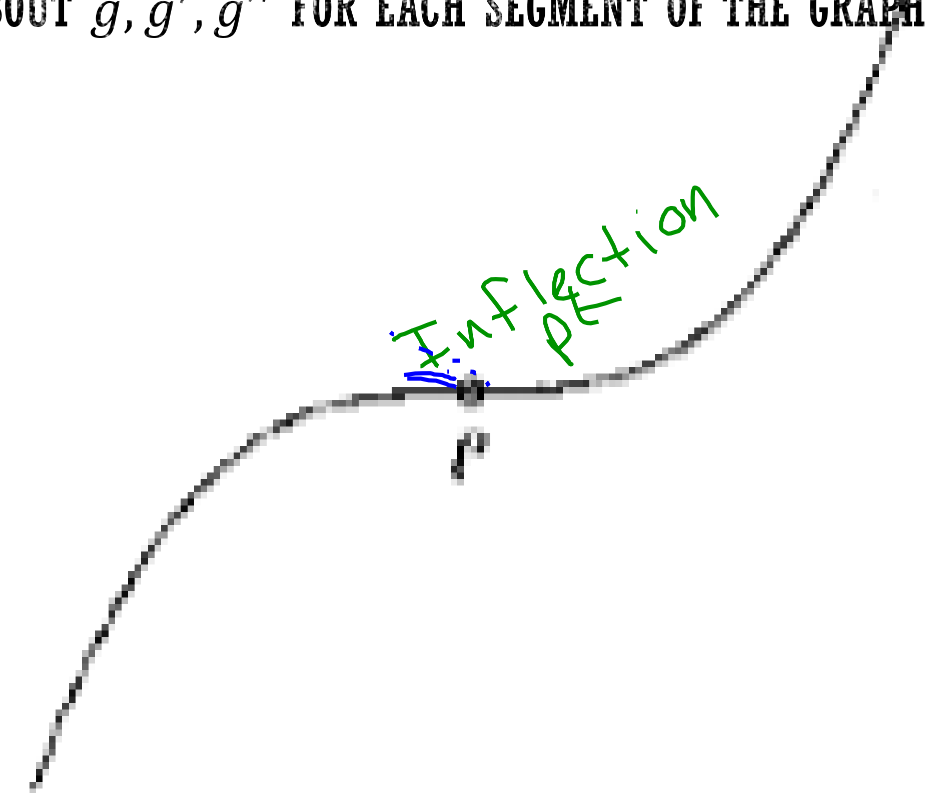
g'': + + + + 0 − − − − −

concave up   concave down

☆ concavity tells you if 2nd deriv. is + or −

WHAT CAN WE SAY ABOUT g, g', g'' FOR EACH SEGMENT OF THE GRAPH $y = g(x)$

6.



g' + + + + + + +

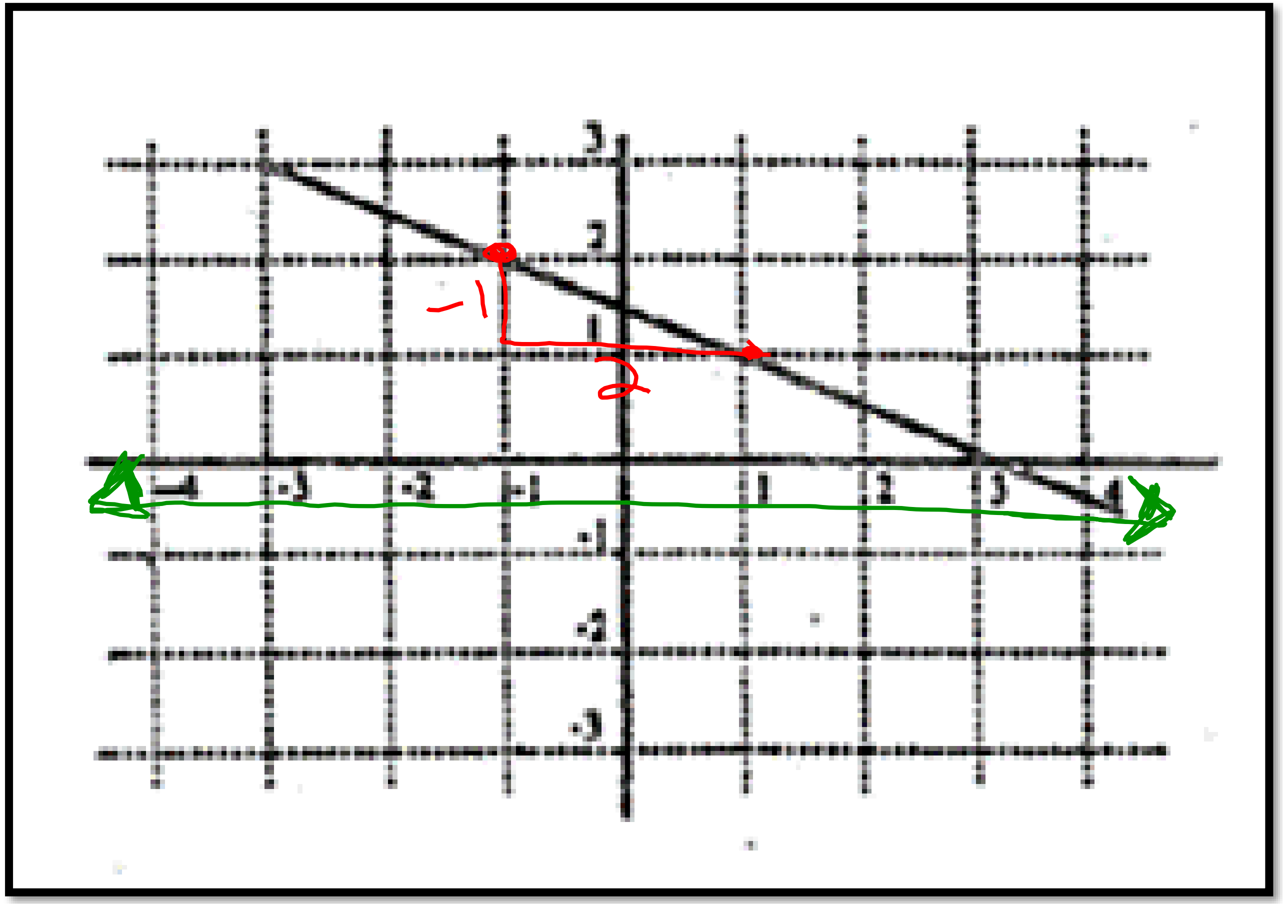
increasing everywhere

g' is all above x-axis

g'' - below - - 0 above + + + +

conc ↓ IP conc ↑

1. GRAPH THE FIRST DERIVATIVE



f'

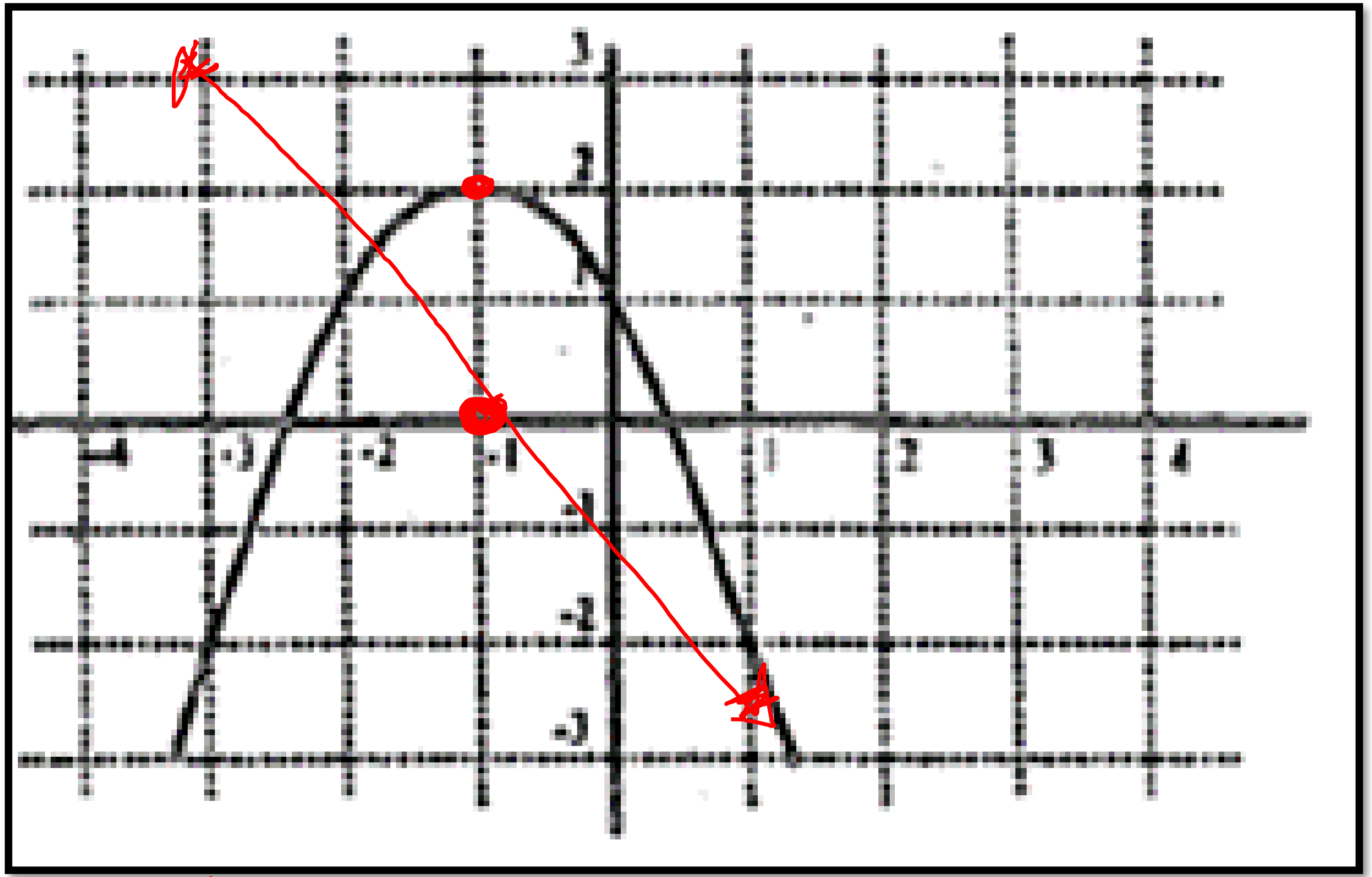
$m = -\frac{1}{2}$

$f(x) = -\frac{1}{2}x + \frac{3}{2}$

$f'(x) = -\frac{1}{2}$

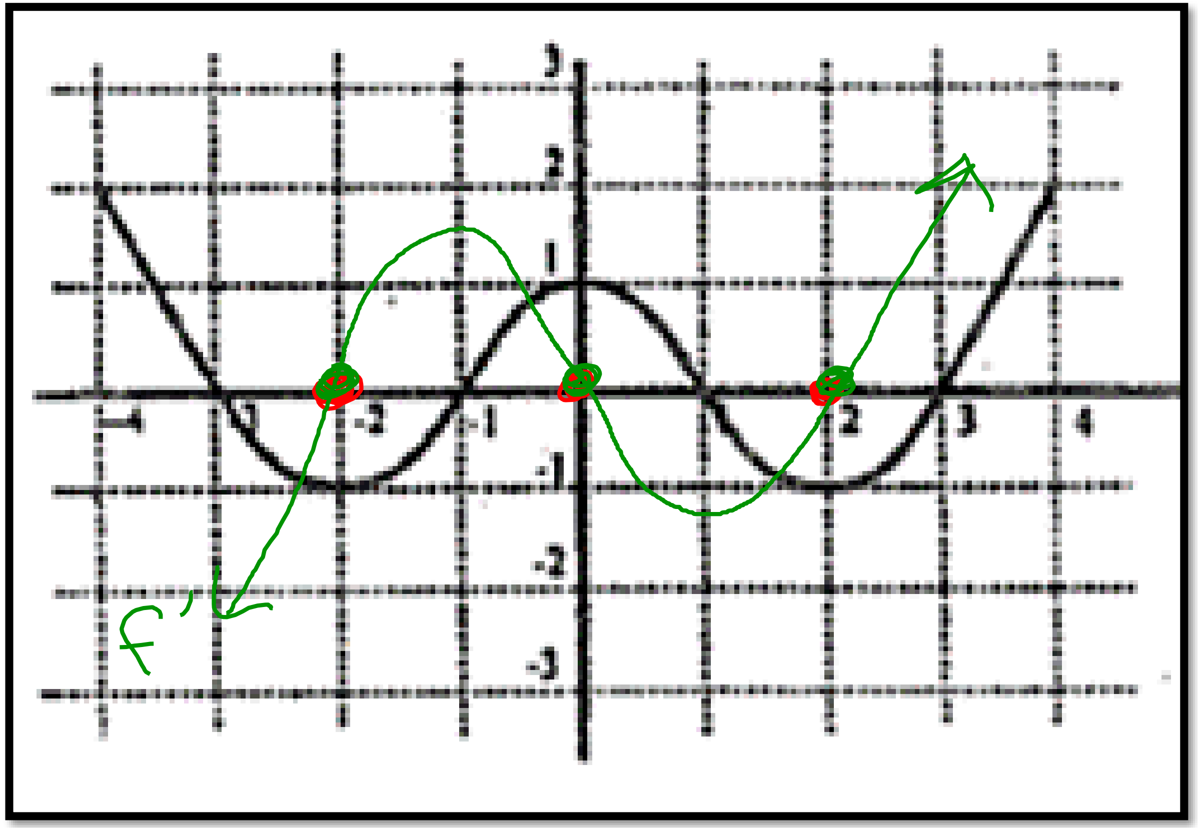
f' — — — — — — — below x-axis

2. GRAPH THE FIRST DERIVATIVE



f'   + + + above   0   below − − −

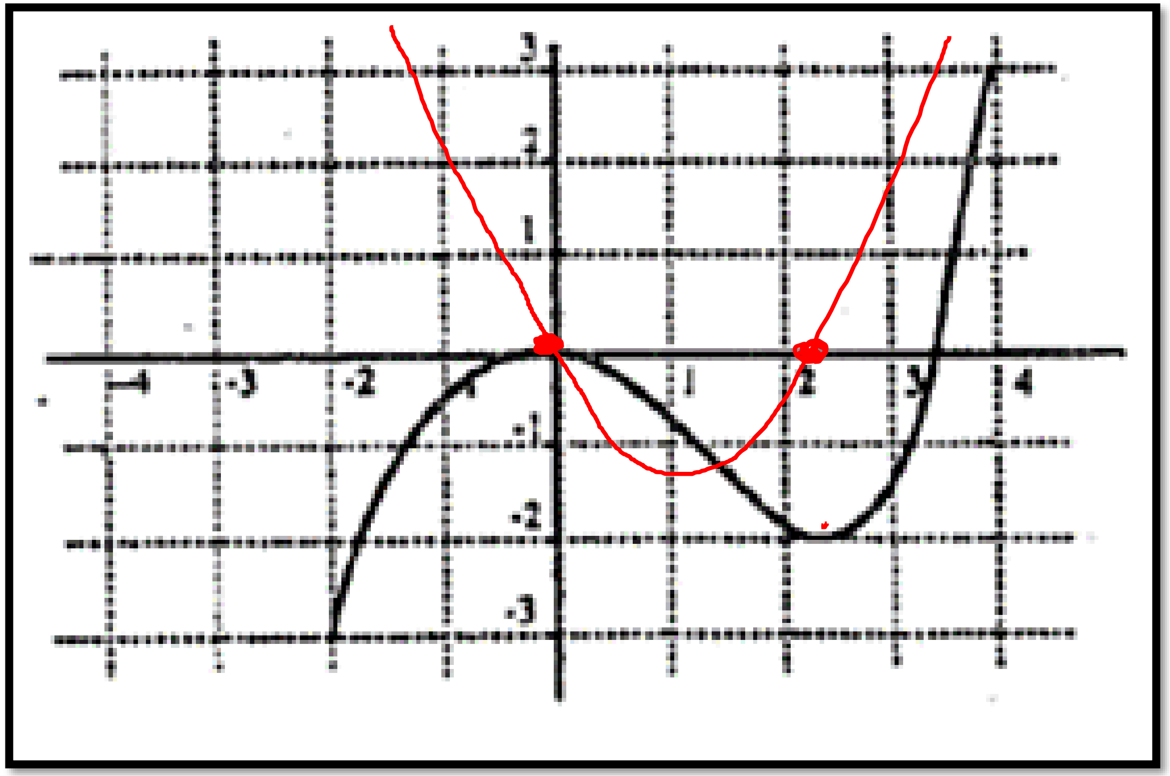
3. GRAPH THE FIRST DERIVATIVE



f' — — 0 + + 0 — — 0 + +

below on above on below on above x-axis

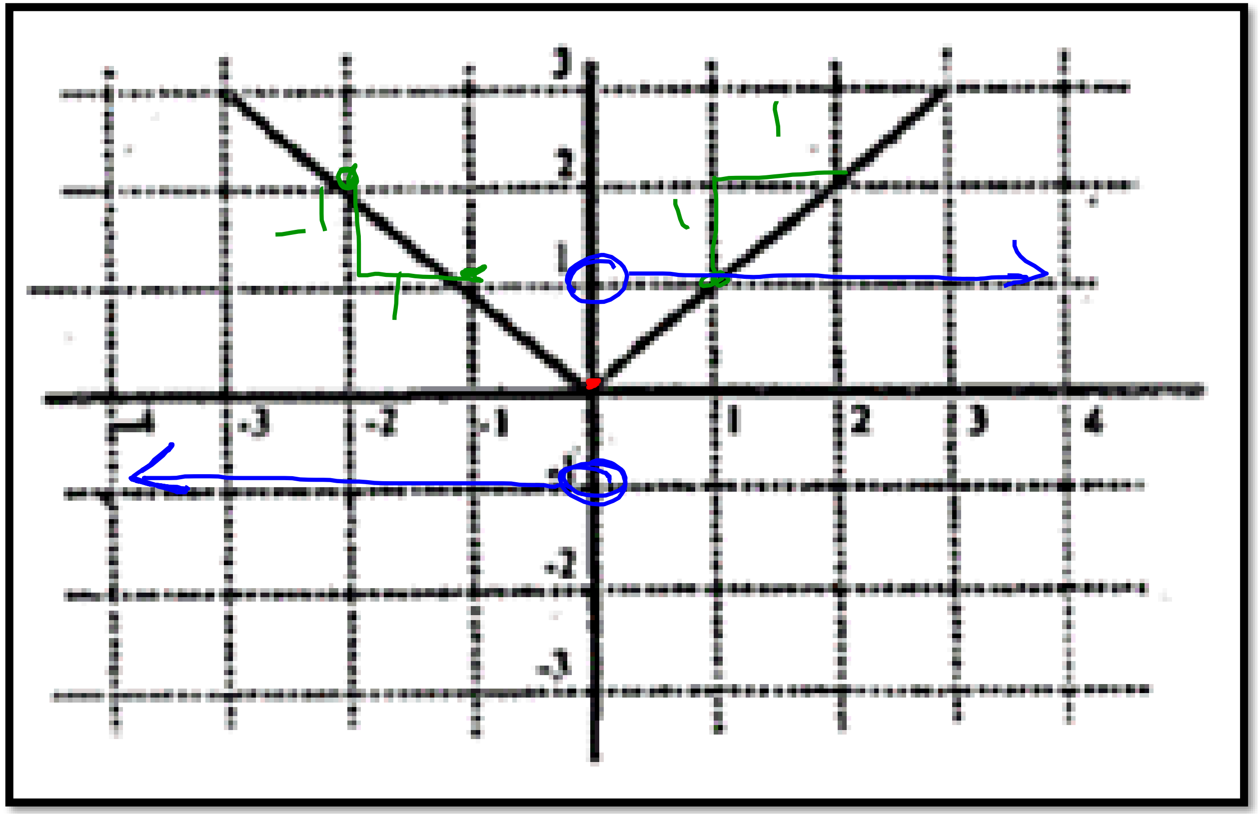
4. GRAPH THE FIRST DERIVATIVE



f′ + + + 0 - - 0 + + +

above on below on above

5. GRAPH THE FIRST DERIVATIVE



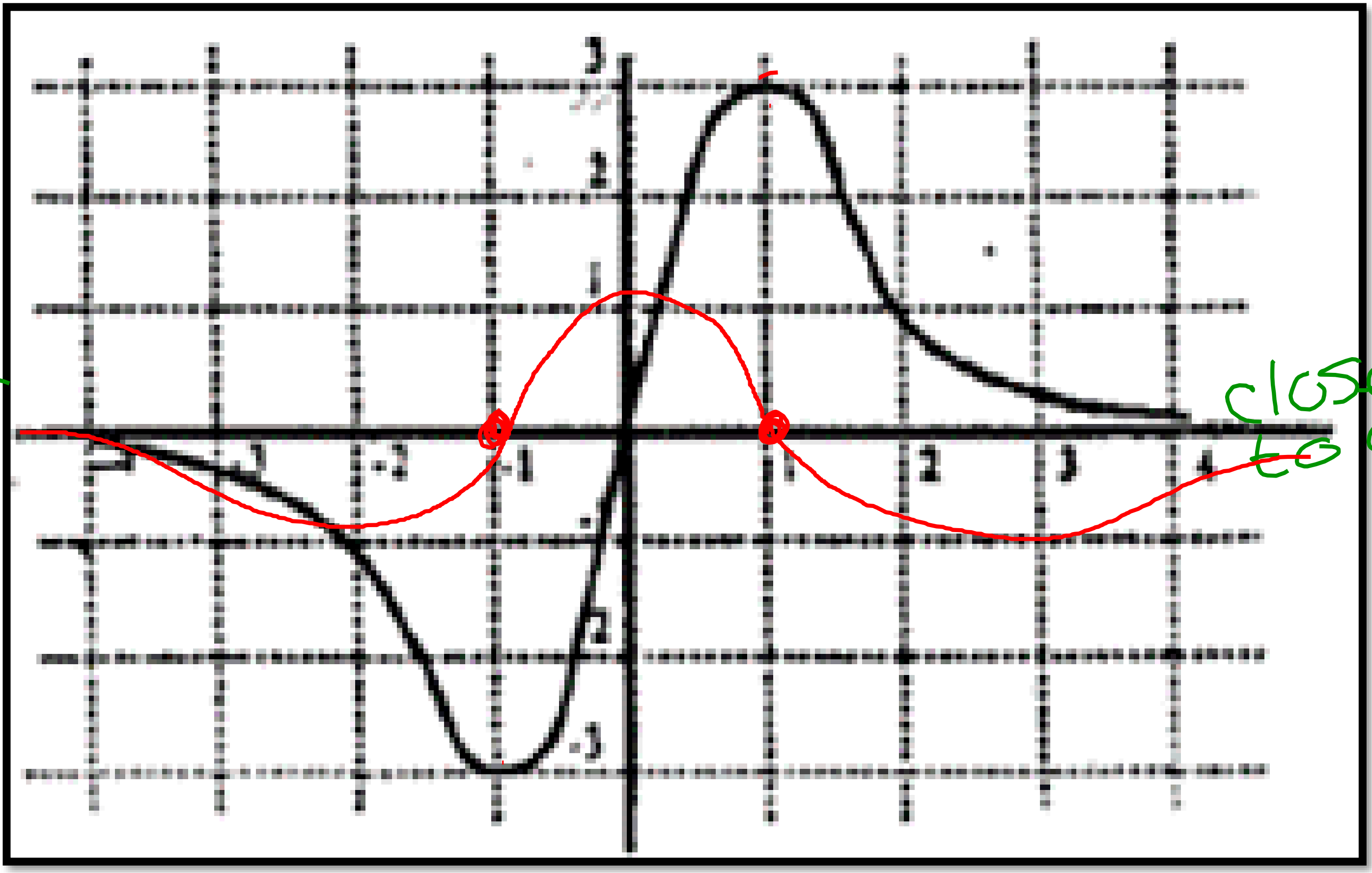
absolute value is 2 linear functions

f' — — — DNE + + 1

$m = -1$ corner $m = 1$

$y = -1$ $y = 1$

6. GRAPH THE FIRST DERIVATIVE



f' close to 0 — — — 0 + + + 0 — — — close to 0

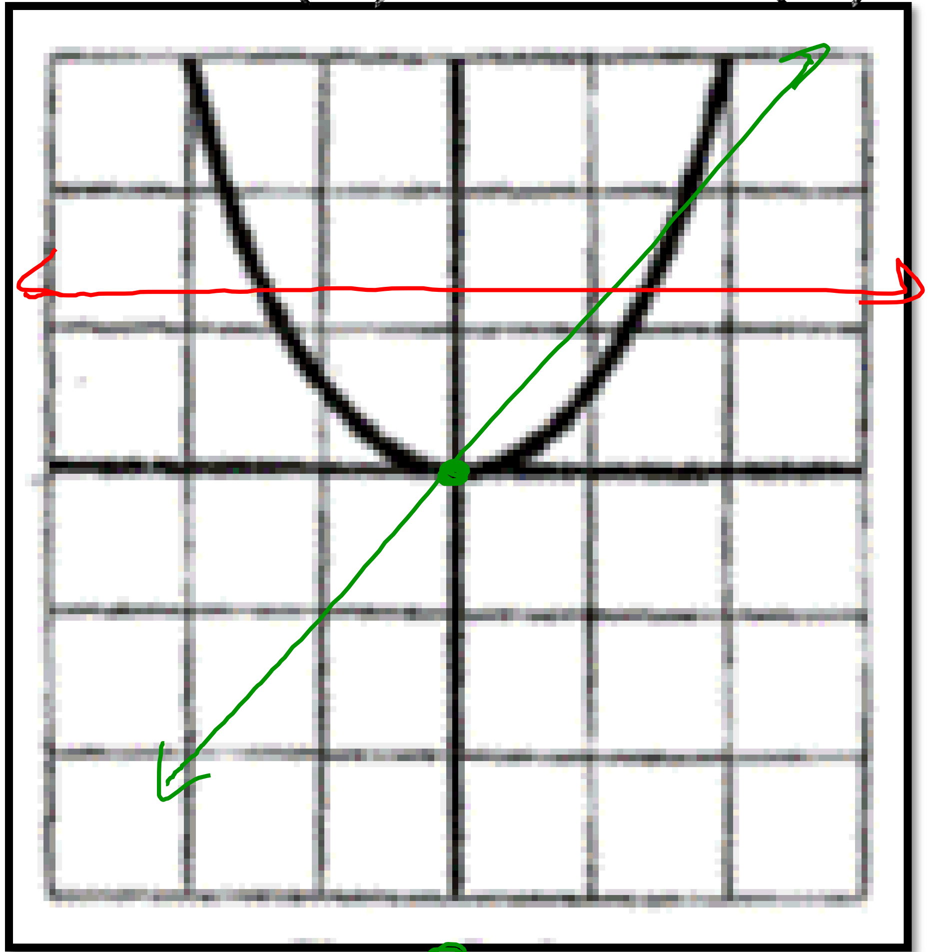
below above below

7. GRAPH THE F'(X) AND F''(X)

f → quad.

f' → linear

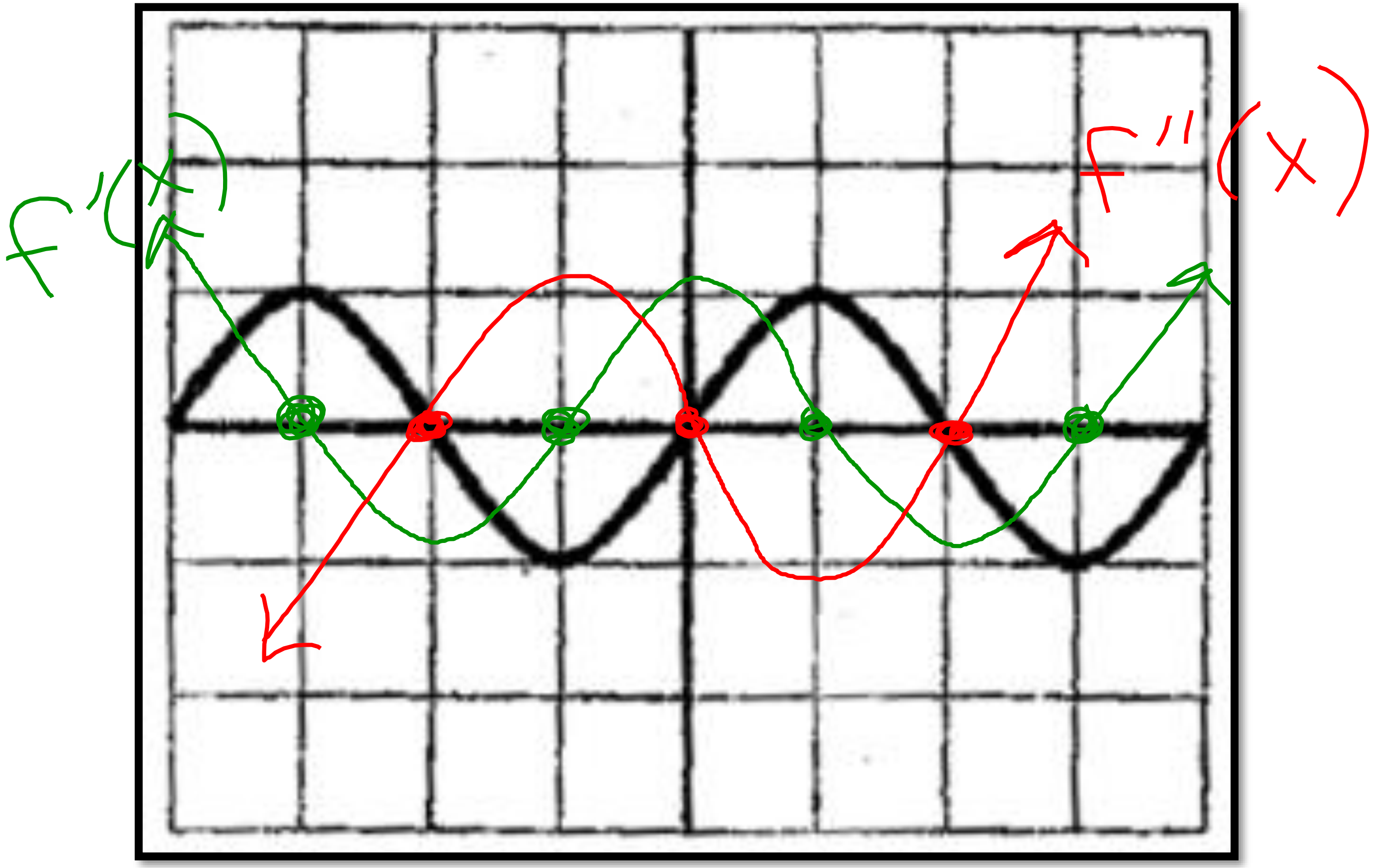
f'' → horiz.



f' — — 0 + +
below above

f'' + + + + +

8. GRAPH THE F'(X) AND F''(X)



$F(x) \to 4$ critical pts

$F'(x) \to 3$

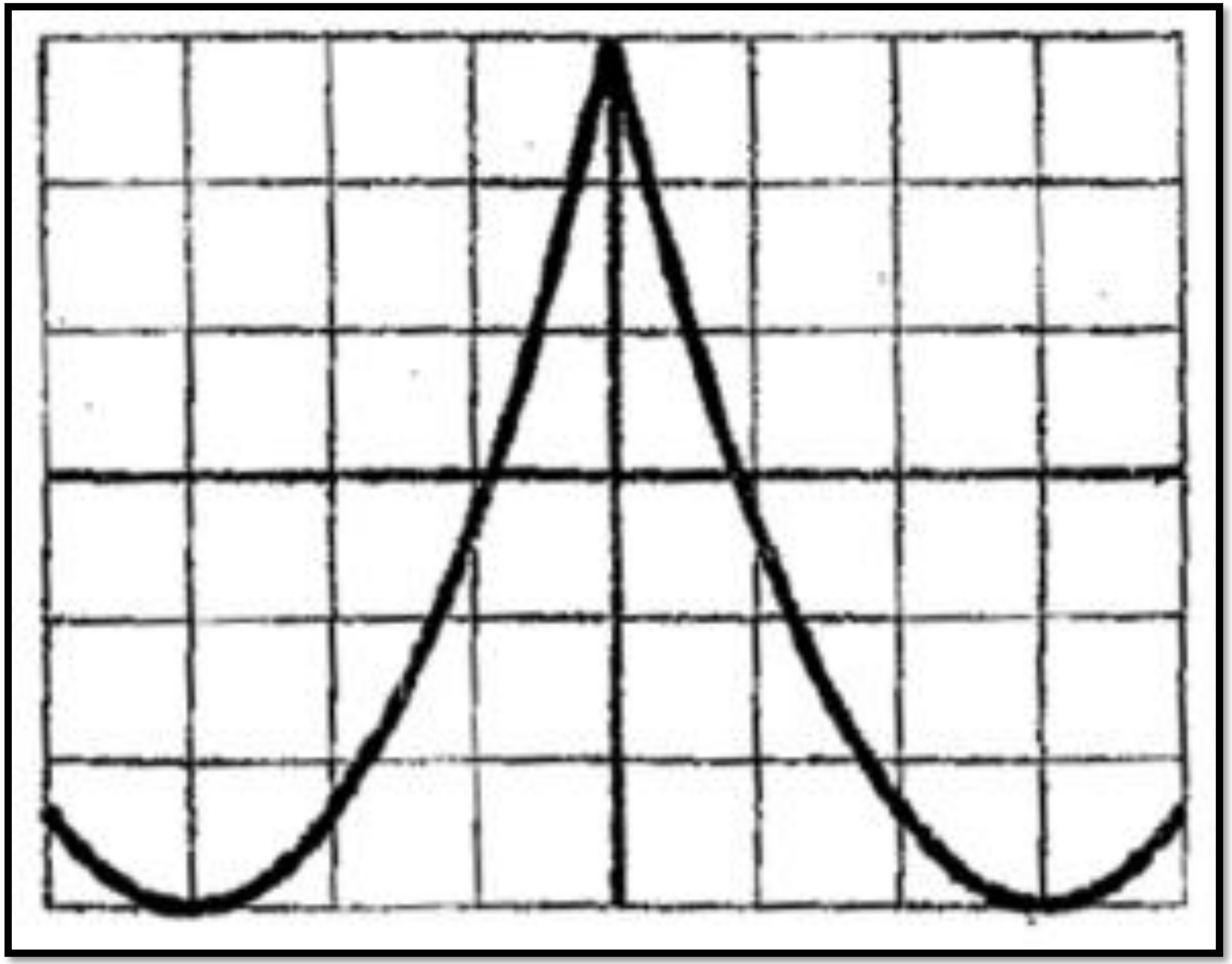
$F''(x) \to 2$

f' + 0 – – 0 + + 0 – – 0 +

f'' – – 0 + + 0 – – 0 + +

Below

GRAPH THE FIRST DERIVATIVE



GRAPH THE FIRST DERIVATIVE