L'HOPITAL'S RULE

Keeper 20

Honors Calculus

L'HOPITAL'S RULE

Take deriv. of numerator AND denominator. Then find limit.

Not Quotient Rule

Suppose that we have one of the following cases,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \qquad \text{OR} \qquad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$$

where a can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

EVALUATE THE LIMIT

$$1.\ \lim_{x\to 0}\frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$$ so use L'Hopital's

$$\frac{\frac{d}{dx}\sin x}{\frac{d}{dx}x}$$

$$\lim_{x\to 0}\frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1$$

EVALUATE THE LIMIT

2. $$\lim_{t\to 1}\frac{5t^4-4t^2-1}{10-t-9t^3} = \frac{5(1)^4-4(1)^2-1}{10-1-9(1)^3} = \frac{0}{0}$$

$$\lim_{t\to 1}\frac{\frac{d}{dx}(5t^4-4t^2-1)}{\frac{d}{dx}(10-t-9t^3)} = \lim_{t\to 1}\frac{20t^3-8t}{-1-27t^2} = \frac{20(1)^3-8(1)}{-1-27(1)^2} = \frac{12}{-28}$$

$$-\frac{3}{7}$$

EVALUATE THE LIMIT

3. $\lim_{x\to\infty} \frac{e^x}{x^2} = \frac{e^{\infty}}{\infty^2} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{\frac{d}{dx} e^x}{\frac{d}{dx} x^2} = \lim_{x\to\infty} \frac{e^x}{2x} = \frac{e^{\infty}}{2\infty} = \frac{\infty}{\infty}$$

use L'Hopital's again!

$$\lim_{x\to\infty} \frac{\frac{d}{dx} e^x}{\frac{d}{dx} 2x} = \lim_{x\to\infty} \frac{e^x}{2} = \frac{e^{\infty}}{2} = \frac{\infty}{2} = \infty$$

EVALUATE THE LIMIT

$x^{1/3}$

4. $$\lim_{x\to 64} \frac{\sqrt[3]{x}-4}{x-64} = \frac{\sqrt[3]{64}-4}{64-64} = \frac{0}{0}$$

$$\lim_{x\to 64} \frac{\frac{d}{dx}\left(x^{1/3}-4\right)}{\frac{d}{dx}(x-4)} = \lim_{x\to 64} \frac{\frac{1}{3}x^{-2/3}}{1} = \frac{1}{3}(64)^{-2/3}$$

$$\frac{1}{3\sqrt[3]{64^2}} = \frac{1}{3\cdot 16}$$

$1/48$