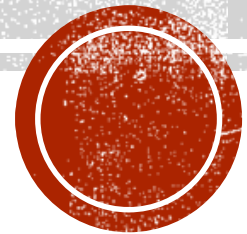


# L'HOPITAL'S RULE

Keeper 20

Honors Calculus



# L'HOPITAL'S RULE

Take deriv. of numerator AND denominator. Then find limit.  
Not Quotient Rule

Suppose that we have one of the following cases,

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{OR} \quad \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$$

where  $a$  can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$



# EVALUATE THE LIMIT

$$1. \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0} \text{ so use L'Hopital's}$$

$$\frac{\frac{d}{dx} \sin x}{\frac{d}{dx} x}$$

$$\lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1$$



# EVALUATE THE LIMIT

$$2. \lim_{t \rightarrow 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3} = \frac{5(1)^4 - 4(1)^2 - 1}{10 - 1 - 9(1)^3} = \frac{0}{0}$$

$$\lim_{t \rightarrow 1} \frac{\frac{d}{dx}(5t^4 - 4t^2 - 1)}{\frac{d}{dx}(10 - t - 9t^3)} = \lim_{t \rightarrow 1} \frac{20t^3 - 8t}{-1 - 27t^2} = \frac{20(1)^3 - 8(1)}{-1 - 27(1)^2} = \frac{12}{-28}$$

$$\left( -\frac{3}{7} \right)$$



# EVALUATE THE LIMIT

$$3. \lim_{x \rightarrow \infty} \frac{e^x}{x^2} = \frac{e^\infty}{\infty^2} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{d}{dx} e^x}{\frac{d}{dx} x^2} = \lim_{x \rightarrow \infty} \frac{e^x}{2x} = \frac{e^\infty}{2\infty} = \frac{\infty}{\infty}$$

use L'Hopital's again!

$$\lim_{x \rightarrow \infty} \frac{\frac{d}{dx} e^x}{\frac{d}{dx} 2x} = \lim_{x \rightarrow \infty} \frac{e^x}{2} = \frac{e^\infty}{2} = \frac{\infty}{2} = \infty$$



# EVALUATE THE LIMIT

4.  $\lim_{x \rightarrow 64} \frac{x^{1/3} - 4}{x - 64} = \frac{\sqrt[3]{64} - 4}{64 - 64} = \frac{0}{0}$

$$\lim_{x \rightarrow 64} \frac{\frac{d}{dx}(x^{1/3} - 4)}{\frac{d}{dx}(x - 4)}$$

$$= \lim_{x \rightarrow 64} \frac{\frac{1}{3}x^{-2/3}}{1}$$

$$= \frac{1}{3} (64)^{-2/3} = \frac{1}{3 \cdot 16}$$

$$\frac{1}{48}$$