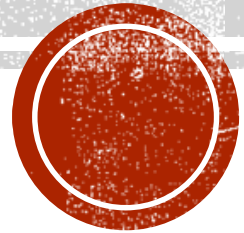
DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

Keeper 22

Honors Calculus



BASIC TRIG DERIVATIVES

sec → sec x tan x

SST

sec²x ← tan

csc → −csc x cot x

−csc²x ← cot x

Also, know your trig identities on p. 16!

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = (\sec x)^2 \text{ or } \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$$

$$\frac{d}{dx}(\cot x) = -(\csc x)^2 \text{ or } -\csc^2 x$$

Deriv. of trig fxns beg. w/ c (cos, csc + cot) are negative

YOU WILL USE THESE FORMULAS WITH THE PRODUCT, QUOTIENT & CHAIN RULE.

IF A TRIG FUNCTION CONTAINS ANYTHING EXCEPT A SINGLE VARIABLE (LIKE X), YOU HAVE TO USE THE CHAIN RULE TO DIFFERENTIATE.

FIND THE DERIVATIVE

1. $f(x) = \sin x \cdot \cos x$ (1st, 2nd) Product Rule

$f'(x) = \sin x \cdot -\sin x + \cos x \cdot \cos x$ ($\frac{d}{dx}\cos x$)

$f'(x) = -\sin^2 x + \cos^2 x$

2. $f(x) = x^2 \cdot \cos x$ Product

$f'(x) = x^2 \cdot -\sin x + \cos x \cdot 2x$

$f'(x) = -x^2 \sin(x) + 2x \cos(x)$

3. $f(x) = \dfrac{\cos x}{x \sin x}$

Simplify w/ trig identities 1st

$$f(x) = \frac{1}{x} \cdot \frac{\cos x}{\sin x}$$

$$f(x) = \frac{1}{x} \cdot \cot x$$

$f(x) = \dfrac{\cot x}{x}$ Quotient Rule

$f(x) = \dfrac{\cot x}{x}$ or $x^{-1} \cdot \cot x$

Quotient Product

$$f'(x) = \frac{x(-\csc^2 x) - \cot x (1)}{x^2}$$

$x^{-1} \cdot -\csc^2 x + \cot x \cdot \frac{-1}{x^2}$

$$f'(x) = -\frac{x \csc^2 x - \cot x}{x^2}$$

4. $f(x) = (x^2 + 1)\cdot\sec x$ Product

$f'(x) = (x^2+1)(\sec x \tan x) + \sec x\ (2x)$

or

$(x^2+1)(\sec x \tan x) + 2x \sec x$

5. $f(x) = x' - 4\csc x + 2\cot x$

sum/dif Rule

$f'(x) = 1 - 4(-\csc x \cot x) + 2(-\csc^2 x)$

$f'(x) = 1 + 4\csc x \cot x - 2\csc^2 x$

6. $f(x) = \csc x \tan x$

Simplify 1st (trig id.)

$f(x) = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x}$

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

7. $\frac{d}{dx}(\sin 3x) =$ means $\sin(3x)$

Chain Rule

out: $\sin x$

in: $3x$

$\sin(\ )$

$\cos(3x) \cdot 3$

$3\cos(3x)$

8. $\frac{d}{dx}(\tan x^2) =$

means $\tan(x^2)$

Chain
out: $\tan(\ )$ $\tan x$ in: x^2

$\sec^2(x^2) \cdot 2x$

$2x\sec^2(x^2)$

9. $\frac{d}{dx}(tan^2 x) =$

means $(\tan x)^2$

chain
out: $(\ )^2$ in: $\tan x$

$2(\tan x)' \sec^2 x$

$2 \tan x \sec^2 x$

10. $f(x) = \dfrac{\csc x}{1 + \cot^2 x}$ ← Pythagorean identities

$f(x) = \dfrac{\csc x}{\csc^2 x} \rightarrow f(x) = \dfrac{1}{\csc x} = \sin x$

$\dfrac{x}{x^2}$

$f(x) = \sin x$

$f'(x) = \cos x$

$$11.\ f(x) = \csc\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)$$

$$= \frac{1}{\sin\left(\frac{x}{3}\right)} \cdot \frac{\cos\left(\frac{x}{3}\right)}{1}$$

$$= \frac{\cos\left(\frac{x}{3}\right)}{\sin\left(\frac{x}{3}\right)}$$

$f(x) = \cot\left(\frac{1}{3}x\right)$ chain

$= -\csc^2\left(\frac{1}{3}x\right) \cdot \frac{1}{3}$

$= -\frac{1}{3}\csc^2\left(\frac{1}{3}x\right)$