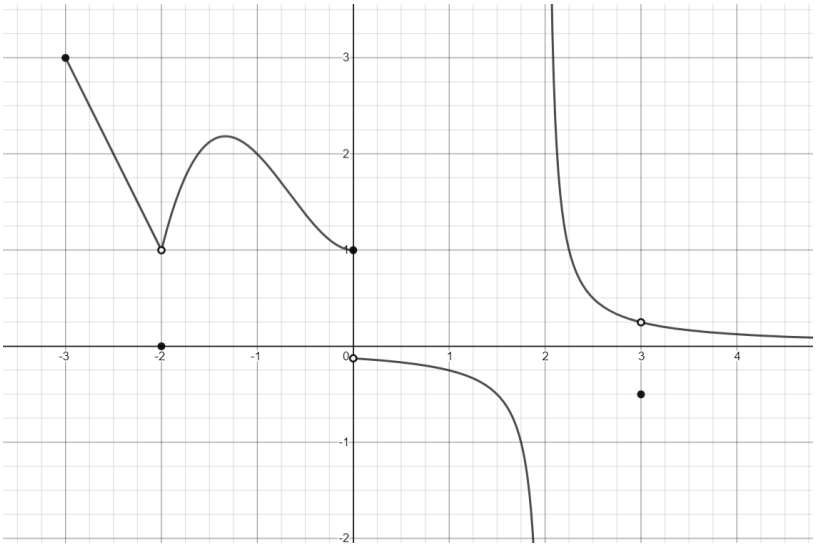
# Keeper 2.3 – Intermediate Value Theorem and Continuity

Example: Understanding Continuity




1. Does $f(3)$ exist?
2. Does $\lim_{x\to 3} f(x)$ exist?
3. Is $f(x)$ continuous at $x = 3$? Justify.
4. What new value should be assigned to $f(3)$ to remove the discontinuity?
5. Does $f(2)$ exist?
6. Does $\lim_{x\to 2} f(x)$ exist?
7. Does $f(0)$ exist?
8. Does $\lim_{x\to 0} f(x)$ exist?
9. Is $f(x)$ continuous at $x = 0$? Justify.
10. Is $f(x)$ right continuous, left continuous, or neither at $x = 0$?
11. Is $f(x)$ right continuous, left continuous, or neither at $x = -3$?
12. List all places where $f(x)$ is discontinuous and state the type of discontinuity.

Identify the type of discontinuity in the following equations.

13. $\mathrm{h}(x) = \frac{-3}{x-7}$

14. $p(x) = \begin{cases} 5x - 1, & if\ x \geq 1 \\ x^2 - 2, & if\ x < 1 \end{cases}$

15. $m(x) = \begin{cases} 2x - 5, & if\ x \geq -5 \\ 3x, & if\ x < -5 \end{cases}$

16. $k(x) = \frac{x-2}{5x-10}$

17. $j(x) = \frac{x^2-4}{x^2-2x}$

18. $b(x) = \sqrt[3]{x-1}$

19. Complete the Table

| Function | Discontinuous at? | Type of Discontinuity | If removable, what value would make it continuous |
|---|---|---|---|
| $f(x) = \frac{x^2-2x-3}{x-1}$ | | | |
| $f(x) = \begin{cases} 3x, & x < 3 \\ x + 2, & x \geq 3 \end{cases}$ | | | |
| $f(x) = \frac{\lvert x+1 \rvert}{x+1}$ | | | |
| $f(x) = \frac{x^2+6x+8}{x^2-4}$ | | | |

Find a value of the variable so that $f(x)$ is continuous

20. $f(x) = \begin{cases} 4x - 7k, & if\ x \geq -3 \\ 2k + x, & if\ x < -3 \end{cases}$

21. $f(x) = \begin{cases} 3x^2 - kx + m, & if\ x \geq 1 \\ mx - 2k, & if\ -1 < x < 1 \\ -3m + 4x^2k, & if\ x \leq -1 \end{cases}$

22. $f(x) = \begin{cases} cx + 1, & if\ x \leq 3 \\ cx^2 - 1, & if\ x > 3 \end{cases}$

IVT Examples:

23. Verify the conditions of the Intermediate Value Theorem and find the guaranteed $c$ value over $[-1,3]$ for $f(x) = 2x^2 + x - 4$ when $f(c) = 2$.

24. Use the Intermediate Value Theorem to show that $f(x) = x^3 - 3x^2 - 7x + 1$ has a root in the interval $(4,5)$.

25. Use the Intermediate Value Theorem to show that $f(x) = x^4 + 3x^2 - 6$ has a root in the interval $(1,2)$ and $(-2,-1)$.

26. Verify the conditions of the Intermediate Value Theorem and find the guaranteed $c$ value over $(-1,3)$ for $f(x) = x^3 + 5$ when $f(c) = 6$.