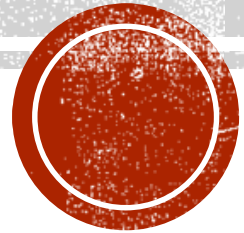
IMPLICIT DIFFERENTIATION

Keeper 19

Honors Calculus



IMPLICIT DIFFERENTIATION

- Explicit equations are solved for y, like $y = 3x^4 - 2x + 5$
- Implicit equations have y mixed throughout it, like

$$2xy = x^4y - 2x + 5y$$

- Use to find the derivative when there are 2 variables and you can't easily solve for y
- Goal is to find $\frac{dy}{dx}$ (to derive with respect to x)

STEPS:

- Differentiate x terms as usual (apply the Power Rule, etc.)
- Differentiate y terms & ALWAYS put $\frac{dy}{dx}$ behind it
- Collect $\frac{dy}{dx}$ terms on left side & everything else on the right side.
- Factor out $\frac{dy}{dx}$
- Solve for $\frac{dy}{dx}$

EX 1: FIND THE DERIVATIVE

$$x^2 + y^2 = 1$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{2y\frac{dy}{dx}}{2y} = -\frac{2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

EX 2: FIND THE DERIVATIVE

$$y^2 + x^2 = 2x$$

$$2y\frac{dy}{dx} + 2x = 2$$

$$\frac{2y\frac{dy}{dx}}{2y} = \frac{2-2x}{2y}$$

$$\frac{dy}{dx} = \frac{1-x}{y}$$

EX 3: FIND THE DERIVATIVE

$$xy + y^2 = 1$$

product rule

$$x \cdot 1 \frac{dy}{dx} + y \cdot 1 + 2y \frac{dy}{dx} = 0$$

Factor out $\frac{dy}{dx}$

$$x \frac{dy}{dx} + 2y \frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(x + 2y) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x + 2y}$$

EX 4: FIND THE DERIVATIVE

$$x^3 - xy + y^3 = 1$$

product

$$3x^2 - \left(x \cdot 1\frac{dy}{dx} + y \cdot 1\right) + 3y^2\frac{dy}{dx} = 0$$

$$3x^2 - x\frac{dy}{dx} - y + 3y^2\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2 + y$$

$$\frac{dy}{dx}(-x + 3y^2) = -3x^2 + y$$

$$\frac{dy}{dx} = \frac{-3x^2 + y}{-x + 3y^2}$$

EX 5: FIND THE DERIVATIVE

$$x = \tan y$$

$$1 = \sec^2 y \frac{dy}{dx}$$

$$\frac{1}{\sec^2 y} \quad \frac{\sec^2 y}{\sec^2 y}$$

or $$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \cos^2 y$$

EX 6: FIND THE DERIVATIVE

chain rule

$$x + \sin(y^2) = xy$$

$$1 + \cos(y^2)\, 2y \frac{dy}{dx} = x\frac{dy}{dx} + y(1)$$

$$2y\cos(y^2)\frac{dy}{dx} - x\frac{dy}{dx} = y - 1$$

$$\frac{dy}{dx}(2y\cos y^2 - x) = y - 1$$

$$\frac{dy}{dx} = \frac{y-1}{2y\cos y^2 - x}$$

EX 7: FIND THE TANGENT LINE

$x^2y^2 = 9 \quad at \ (-1,3)$

$x_1, y_1 \quad m = 3$

$x^2 \cdot 2y\frac{dy}{dx} + y^2 \cdot 2x = 0$

$\frac{x^2 2y \frac{dy}{dx}}{x^2 2y} = \frac{-2x^1y^2}{2x^2y^1}$

$\frac{dy}{dx} = \frac{-y}{x}$ ← slope $= \frac{-3}{1} = -3$

$y - y_1 = m(x - x_1)$

$y - 3 = 3(x + 1)$

$y - 3 = 3x + 3$

$+3 \qquad +3$

$y = 3x + 6$

HW: p. 7-9 odds

Box Project + Take Home Quiz

$\frac{dy}{dx} = \frac{-\pi}{2+\pi \cdot 0}$

EX 8: FIND DERIVATIVE AT A POINT

$\frac{dy}{dx} = \frac{-\pi}{2}$

$$2xy + \pi \sin y = 2\pi \quad at \quad \left(1, \frac{\pi}{2}\right)$$ (x, y)

$2x(1)\frac{dy}{dx} + y(2) + \pi \cos(y)\frac{dy}{dx} = 0$

$$2x\frac{dy}{dx} + \pi \cos y \frac{dy}{dx} = -2y$$

$$\frac{dy}{dx}(2x + \pi \cos y) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{2x + \pi \cos y}$$

$$\frac{-2\left(\frac{\pi}{2}\right)}{2(1) + \pi \cos\frac{\pi}{2}}$$