

THE CHAIN RULE

Keeper 17

Honors Calculus



THE CHAIN RULE

Used to take the derivative of a
composition

$$f(x) = (\quad)^2$$
$$g(x) = x + 3$$
$$(x + 3)^2$$

$f(x)$ = outside function
 $g(x)$ = inside function

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

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FIND THE DERIVATIVE

1. $y = (2x^3 + 4)^5$

$f(x)$ outside = $()^5$
 $g(x)$ inside = $2x^3 + 4$

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$$y' = 5(2x^3 + 4)^4 \cdot 6x^2$$

$$y' = 30x^2(2x^3 + 4)^4$$



FIND THE DERIVATIVE

$$2. f(x) = (x^2 + 5x + 3)^4$$

out: $()^4$ or x^4
in: $x^2 + 5x + 3$

$$f'(x) = 4(x^2 + 5x + 3)^3 (2x + 5)$$

$$f'(x) = (8x + 20)(x^2 + 5x + 3)^3$$



FIND THE DERIVATIVE

$$3. f(x) = 4\sqrt[3]{x^2 + 3x}$$

$$f(x) = 4(x^2 + 3x)^{1/3}$$

out: $4(\)^{1/3}$

in: $x^2 + 3x$

$$f'(x) = \frac{4}{3}(x^2 + 3x)^{-2/3}(2x + 3)$$

$$f'(x) = \frac{4(2x + 3)}{3(x^2 + 3x)^{2/3}}$$

$$f'(x) = \frac{8x + 12}{3\sqrt[3]{(x^2 + 3x)^2}}$$



FIND THE DERIVATIVE

$$4. y = \sqrt[2]{(5x + 2)^3}$$

out: $(\quad)^{3/2}$

in: $5x + 2$

$$y = (5x + 2)^{3/2}$$

$$y' = \frac{3}{2} (5x + 2)^{1/2} (5)$$

$$y' = \frac{15}{2} \sqrt{5x + 2}$$

$$\text{or } \frac{15}{2} (5x + 2)^{1/2}$$

$$y' = \frac{15\sqrt{5x+2}}{2}$$



FIND THE DERIVATIVE

$$5. y = (\sqrt{x} - 1)^2$$

out: $(\quad)^2$
in: $\sqrt{x} - 1 = x^{1/2} - 1$

$$y' = 2(\sqrt{x} - 1)' \left(\frac{1}{2} x^{-1/2} \right)$$

$$y' = \frac{2}{1} (\frac{\sqrt{x} - 1}{1}) \left(\frac{1}{2\sqrt{x}} \right)$$

$$y' = \frac{\sqrt{x} - 1}{\sqrt{x}}$$

$$y' = \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$

$$y' = 1 - \frac{1}{\sqrt{x}}$$

FIND THE DERIVATIVE

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$$6. f(x) = \frac{1}{\sqrt{2x^3 - 7x^2}}$$

$$f(x) = (2x^3 - 7x^2)^{-1/2}$$

out: $\frac{1}{\sqrt{\quad}}$ or $(\quad)^{1/2}$ or $(\quad)^{-1/2}$

in: $2x^3 - 7x^2$

$$f'(x) = -\frac{1}{2} (2x^3 - 7x^2)^{-3/2} (6x^2 - 14x)$$

$$\frac{-\frac{1}{2}(6x^2 - 14x)}{(2x^3 - 7x^2)^{3/2}}$$

$$f'(x) = \frac{-3x^2 + 7x}{(2x^3 - 7x^2)^{3/2}}$$

$$f'(x) = \frac{-3x^2 + 7x}{\sqrt{(2x^3 - 7x^2)^3}}$$

