TANGENT LINES

Keeper 14

Honors Calculus

SLOPE OF THE TANGENT LINE

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

This formula represents the **INSTANTANEOUS** rate of change

TANGENT EQUATIONS

To find the **equation of the tangent line**:

-find the slope of the tangent line (find the derivative)

-input the slope and the point into the point-slope form of a line $y - y_1 = m(x - x_1)$

FIND THE EQUATION OF THE TANGENT LINE

1. $f(x) = x^2 + 2x$ $\quad$ *find the* $f'(3)$

$f(3) = 3^2 + 2(3)$

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$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

$$\frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + \overset{0}{h} + 2)}{\cancel{h}}$$

$$f'(x) = 2x + 2$$

$$f'(3) = 2(3) + 2 = 8$$

$m = 8 \quad pt\,(\underset{x_1}{3}, \underset{y_1}{15})$

$y - y_1 = m(x - x_1)$

$y - 15 = 8(x - 3)$

$y - 15 = 8x - 24$

$\boxed{y = 8x - 9}$

FIND THE EQUATION OF THE TANGENT LINE

2. $f(x) = \sqrt{x}$ $find\ the\ f'(3)$

$$(3, \sqrt{3})$$
$$x_1 \quad y_1$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(3) = \frac{1}{2\sqrt{3}}$$

$$m = \frac{1}{2\sqrt{3}}$$

$$\boxed{y - \sqrt{3} = \frac{1}{2\sqrt{3}}(x - 3)}$$

FIND THE EQUATION OF THE TANGENT LINE

3. $f(x) = 4x - 3x^2$ at $(2, -4)$

$(x^2 + 2xh + h^2)$

$m = -8$

$(2, -4)$
$x_1 \; y_1$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h) - 3(x+h)^2 - (4x - 3x^2)}{h}$$

$$\frac{\cancel{4x} + 4h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{4x} + \cancel{3x^2}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(4 - 6x - 3\cancel{h})}{\cancel{h}}$$

$$f'(x) = 4 - 6x \qquad f'(2) = -8$$

$y + 4 = -8(x - 2)$

$y + 4 = -8x + 16$

$y = -8x + 12$