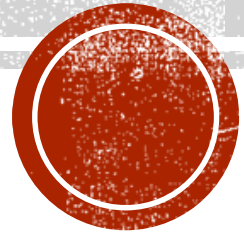
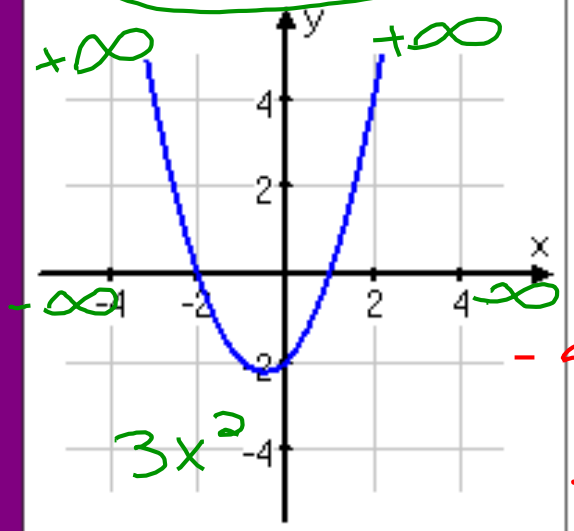
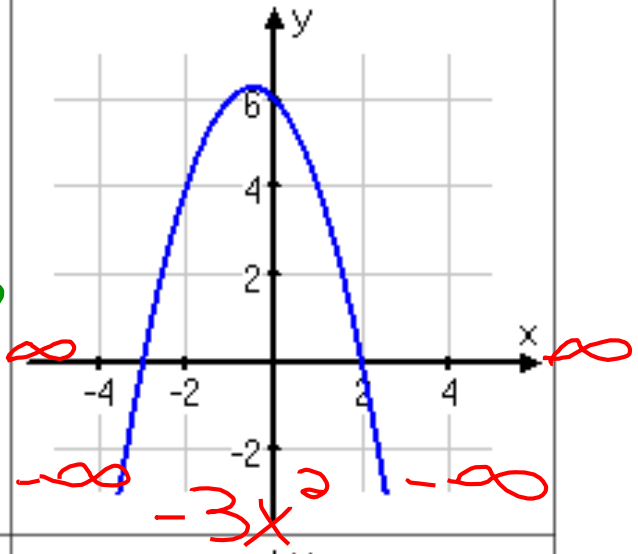
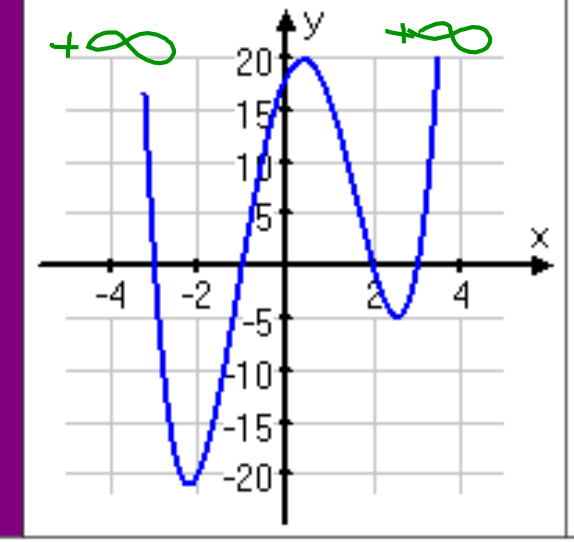
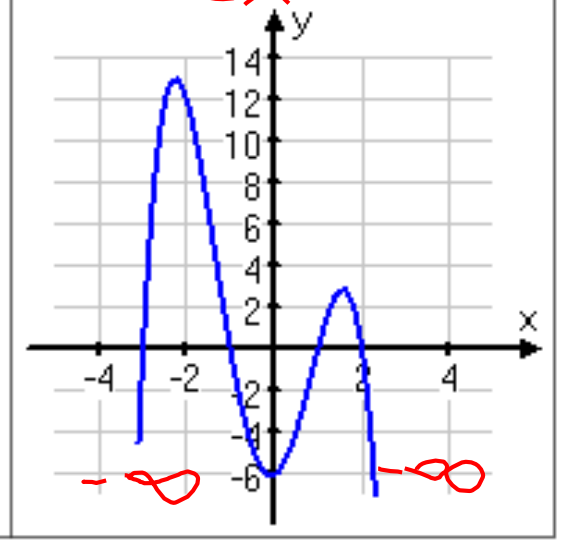
ASYMPTOTES, END BEHAVIOR, AND INFINITE LIMITS

Keeper 12

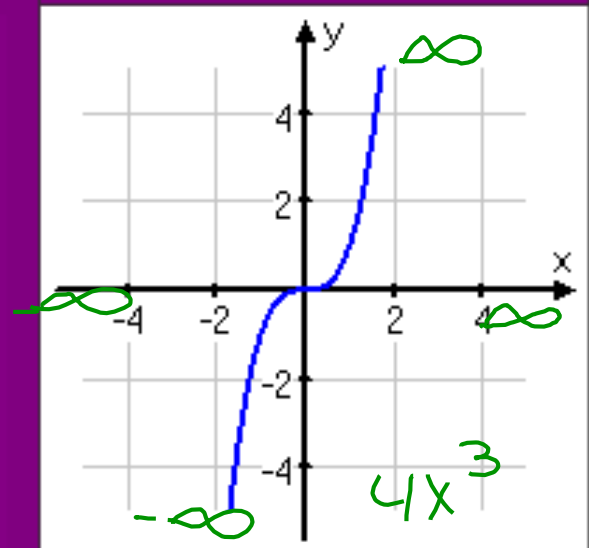
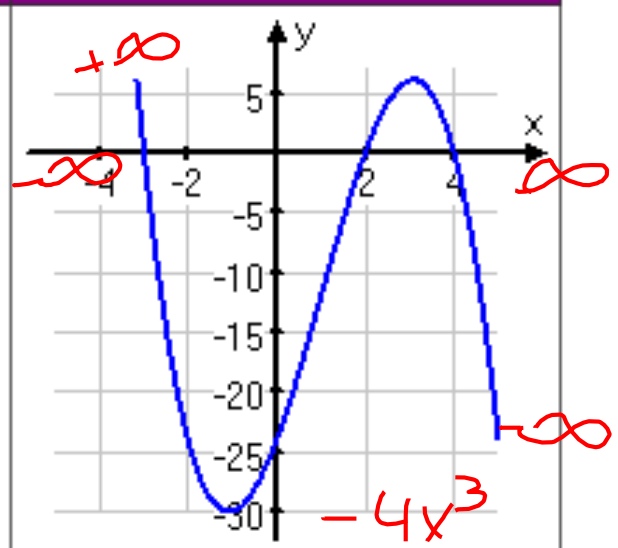
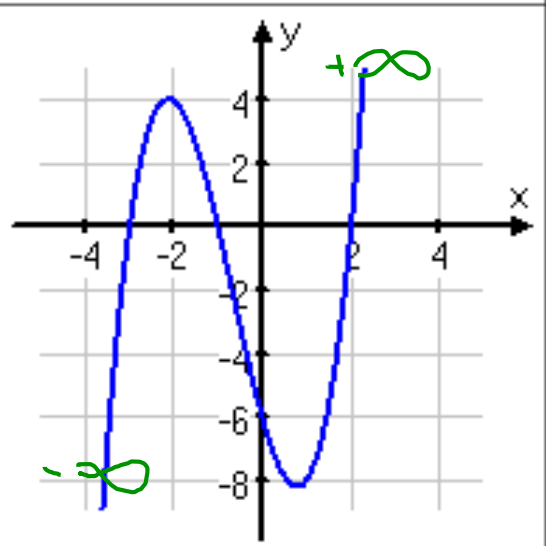
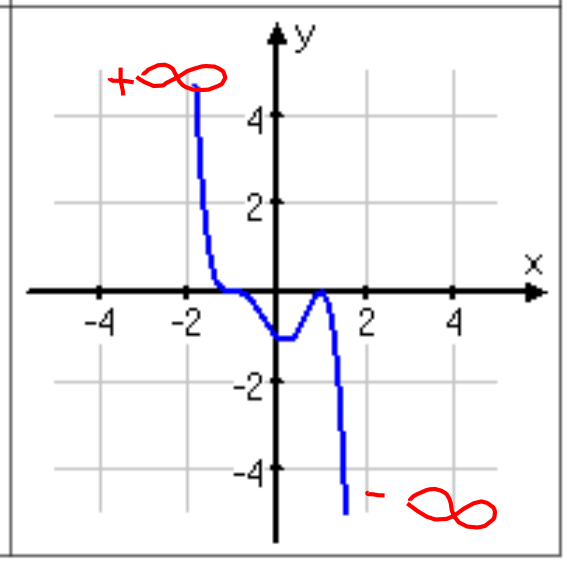
Honors Calculus



END BEHAVIOR OF POLYNOMIAL FUNCTIONS

	with a positive leading coefficient	with a negative leading coefficient
Even degree polynomials		
		

END BEHAVIOR OF POLYNOMIAL FUNCTIONS

Odd degree polynomials	with a positive leading coefficient	with a negative leading coefficient
		
		

Describe the End Behavior

1. $-x^7 + 2x^5 - 4x^2 + 2x - 4$

odd degree
– lead. coeff.

right
$x \to +\infty$, $f(x) \to -\infty$

$x \to -\infty$, $f(x) \to +\infty$
left

2. $x^8 + 2x^5 - 4x^{10} + 2x - 4$

even degree
– L.C.

$x \to \infty$, $f(x) \to -\infty$

$x \to -\infty$, $f(x) \to -\infty$

3. $-x^7 + 2x^5 - 4x^2 + 2x^9 - 4$

odd
+

$x \to \infty$, $f(x) \to \infty$

$x \to -\infty$, $f(x) \to -\infty$

ASYMPTOTES OF RATIONAL FUNCTIONS

Vertical Asymptotes – Set the denominator equal to 0 and solve

Horizontal Asymptotes – Focus on the degree of the numerator and denominator

- degree of numerator = degree of denominator: $y = \frac{lead\ coefficient\ of\ numerator}{lead\ coefficient\ of\ denominator}$

$y = \frac{3x^2+1}{5x^2-2}$ HA: $y = 3/5$

- degree of numerator < degree of denominator: $y = 0$ HA.

$y = \frac{5}{x+2}$

- degree of numerator > degree of denominator: No horizontal asymptote
  but there could be a slant asymptote

$y = \frac{x^2+3x-4}{x-5}$ no HA.

Oblique/Slant Asymptote – degree of numerator is 1 degree higher than that of the denominator - use long division to find equation of oblique asymptote

sometimes synth. ÷

$x-5=0$
$x=5$

$5 \mid 1 \quad 3 \quad -4$
$\quad\quad\ \downarrow \quad 5 \quad 40$
$\quad\quad 1 \quad 8 \quad 36$

S.A.
$y = x+8$

1x+8 : 36

***Watch out for holes!!!

- factor in the numerator = factor in denominator

FIND THE ASYMPTOTES, SKETCH THE GRAPH & DISCUSS THE END BEHAVIOR

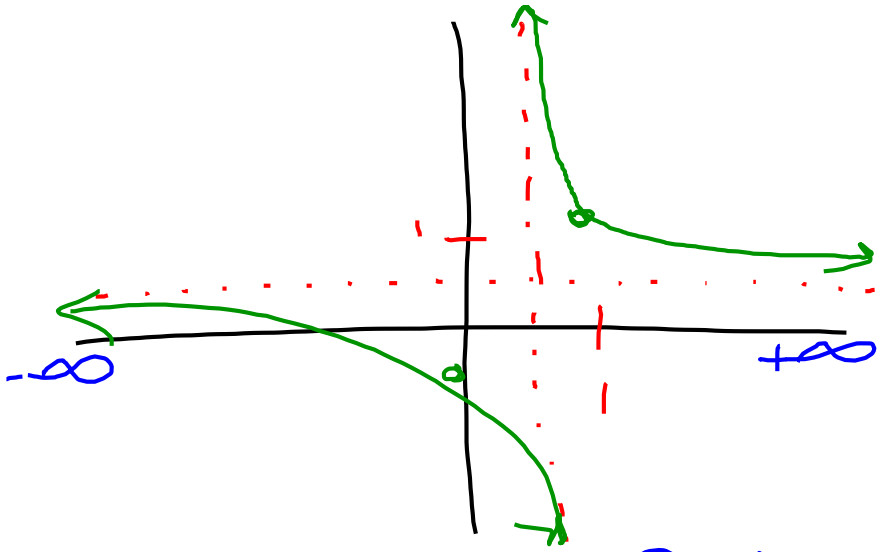
1. $f(x) = \dfrac{4x^1 + 5}{8x^1 - 3}$

UA: $8x - 3 = 0$

$x = 3/8$

HA: $y = 4/8$

$y = 1/2$

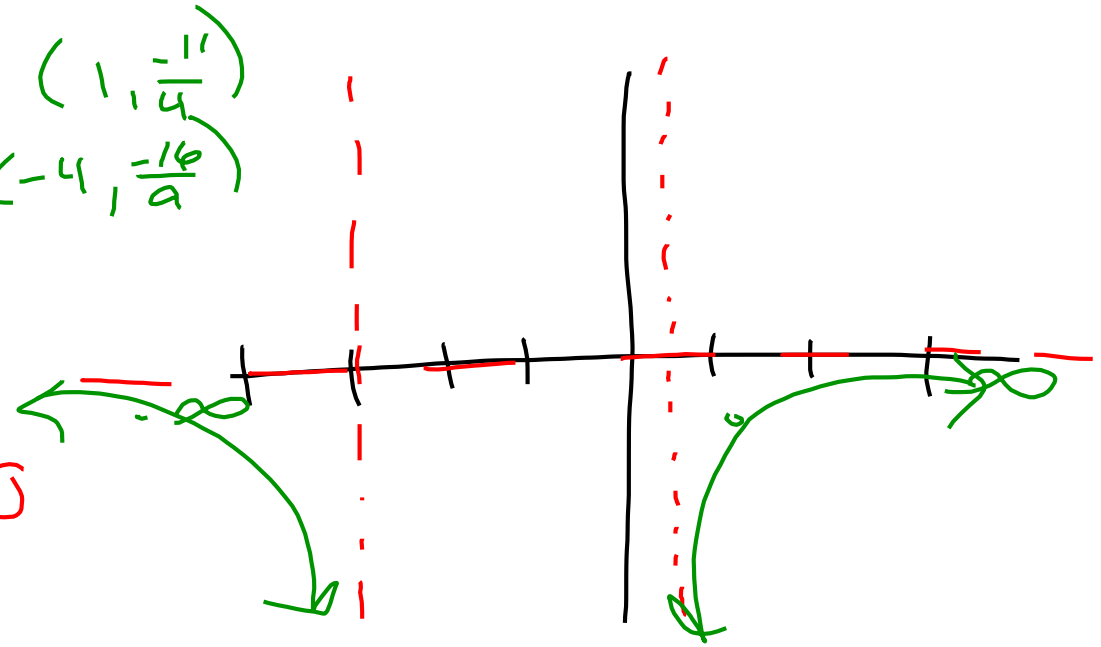


as $x \to \infty$, $f(x) \to 1/2$

$x \to -\infty$, $f(x) \to 1/2$

FIND THE ASYMPTOTES, SKETCH THE GRAPH & DISCUSS THE END BEHAVIOR

2. $f(x) = \dfrac{x - 12}{2x^2 + 5x - 3}$



VA: $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$x = 1/2 \quad x = -3$

HA: $y = 0$

$x \to \infty, f(x) \to 0$

$x \to -\infty, f(x) \to 0$

FIND THE ASYMPTOTES, SKETCH THE GRAPH & DISCUSS THE END BEHAVIOR

$$3.\ f(x) = \frac{x^2 - 9}{x + 4}$$

∞

VA: $x = -4$

HA: none

SA: $y = x - 4$

$-4 \mid 1 \quad 0 \quad -9$

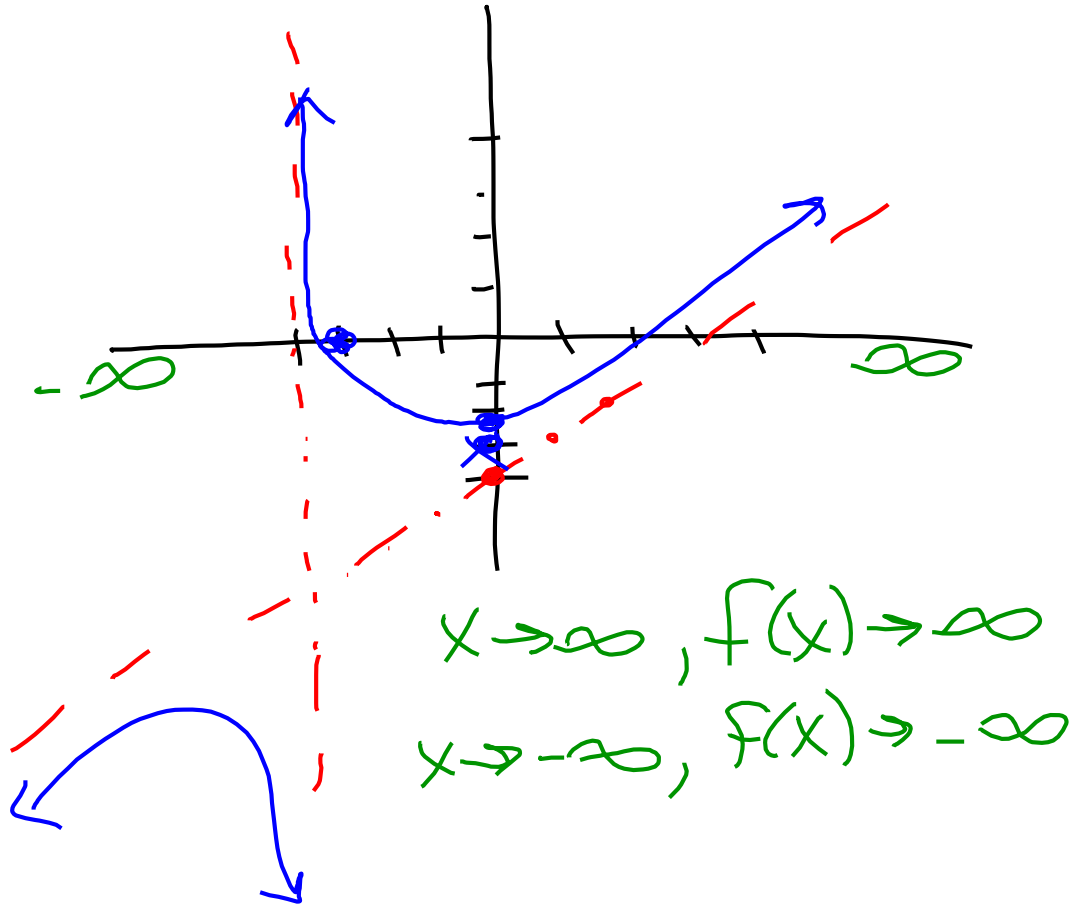
$\downarrow \quad -4 \quad 16$

$1 \quad -4 \quad \cancel{:} \quad \cancel{X}$

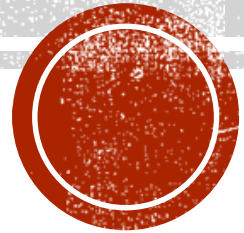
$y = 1x - 4$

$f(-3) = 0$

$f(0) = -2\frac{1}{4}$



INFINITE LIMITS



LIMITS OF POLYNOMIAL FUNCTIONS AT INFINITY

$\lim_{x \to \infty}$ or $\lim_{x \to -\infty}$

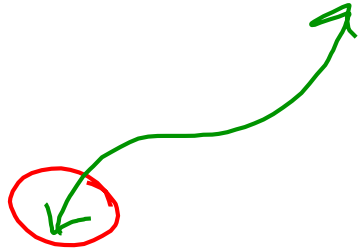
- follow your end behavior rules!

EXAMPLE 1

$$\lim_{x \to -\infty} (x^3 - 2x^2 + 5x - 1) = -\infty$$

left

odd +

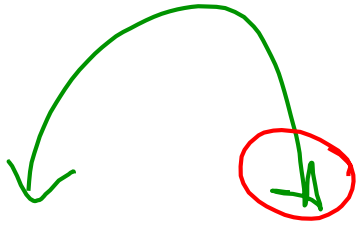


EXAMPLE 2

$$\lim_{x \to \infty} (4 + 3x - x^2) = -\infty$$

right

even −

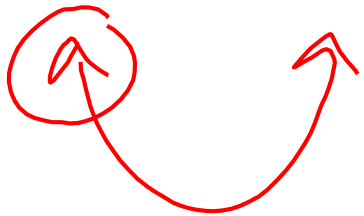


EXAMPLE 3

$$\lim_{x \to -\infty} (5x^4 - 3x) = \infty$$

left

even +



LIMITS OF RATIONAL FUNCTIONS:

FOLLOW HORIZONTAL ASYMPTOTE RULES

- Degree of numerator = Degree of denominator

$$\lim_{x\to\pm\infty} f(x) = \textit{ratio of leading coefficients}$$

- Degree of numerator < Degree of denominator

$$\lim_{x\to\pm\infty} f(x) = 0$$

- Degree of numerator > Degree of denominator

$$\lim_{x\to\pm\infty} f(x) = +\infty \textit{ or } -\infty$$

EVALUATE

1. $\lim_{x \to \infty} \dfrac{4x^{1} + 5}{8x^{1} - 3} = \dfrac{1}{2}$

HA: $y = \frac{4}{8} = \frac{1}{2}$

$\dfrac{4x}{8x} = \dfrac{4}{8} = \dfrac{1}{2}$

EXAMPLE 2

$$\lim_{x \to -\infty} \frac{6x^2 - x}{3x^3 + 1} =$$

num deg < den deg.

HA: $y = 0$

EXAMPLE 3

num. deg > denom. deg = no HA.

So it's $+\infty$ or $-\infty$

$$\lim_{x\to\infty}\frac{5x^{4}}{9x^{3}+2x} = \frac{5(\infty)^4}{9(\infty)^3} = \text{or } \frac{5(+)^4}{9(+)^3} = +\infty$$

EXAMPLE 4

$$\lim_{x \to -\infty} \frac{5x^4}{9x^3 + 2x}$$

no HA
so
$+\infty$ or
$-\infty$

$$\frac{5(-\infty)^4}{9(-\infty)^3} = \frac{+}{-} = -$$

$-\infty$

EXAMPLE 5

$$\lim_{x\to\infty} \frac{4-3x^3}{2x^3+3x-1}$$

same deg

$$= \frac{-3}{2}$$

EXAMPLE 6

$$\lim_{x \to -\infty} \frac{\sqrt{4x^2 + 5}}{x - 3} = \frac{|2x|}{x} \ldots = -2$$

$\sqrt{x^2} = |x|$ so x or $-x$

EXAMPLE 7

$$\lim_{x\to\infty} \frac{10-3x}{(2x+1)^3}$$

$\frac{\deg 1}{\deg 3} = 0$

HW: p. 14-16

You don't have to graph p. 14 + 15