ALGEBRAIC LIMITS

Keeper 10

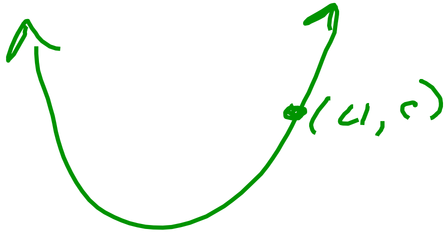
Honors Calculus

RULES

- Substitute First
- $\frac{0}{0}$ Factor, reduce, then substitute

  (OR PLAY TIME)
- $\frac{\#}{0}$ Use a t-chart

EXAMPLE 1



$$\lim_{x \to 4} (x^2 - 6x + 3)$$

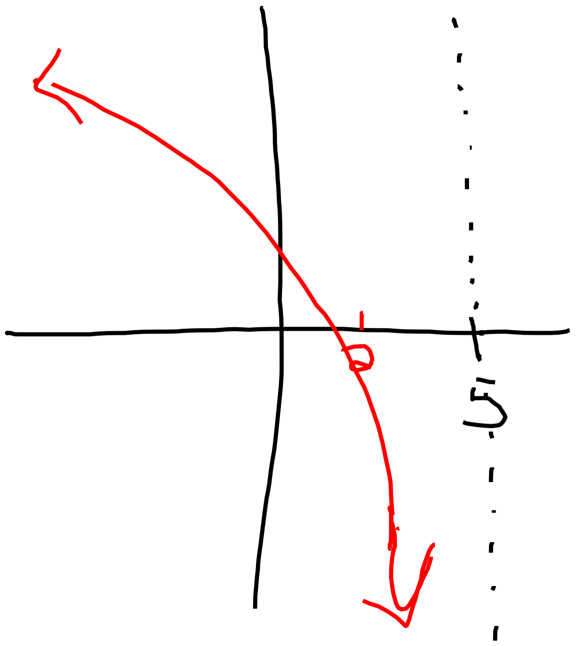
$$(4)^2 - 6(4) + 3$$

$$16 - 24 + 3$$

$$-5$$

EXAMPLE 2

$$\lim_{x\to 2}\frac{4x^3+1}{x-5}$$



$$\frac{4(2)^3+1}{2-5} = \frac{33}{-3} = -11$$

EXAMPLE 3

$$\lim_{x \to 2} \frac{x-3}{2x^2 - x - 15}$$

$$\frac{2-3}{2(2)^2-(2)-15} = \frac{-1}{-9} = \frac{1}{9}$$

$$\frac{\cancel{x-3}}{(2x+5)\cancel{(x-3)}}$$

VA at $x = -5/2$

hole at $x = 3$

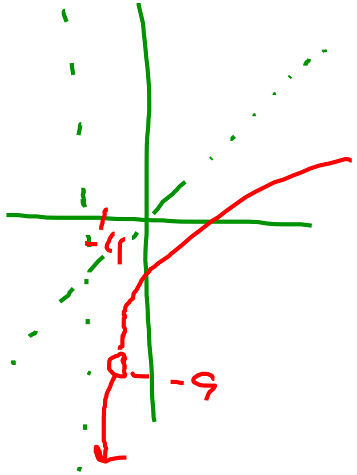
EXAMPLE 4

$$\lim_{x \to -4} \frac{x^2 - x - 20}{x + 4}$$

① Substitute $\frac{(-4)^2 - (-4) - 20}{-4+4} = \frac{0}{0}$ "play time"

② Factor + simplify $\frac{(x-5)\cancel{(x+4)}}{\cancel{(x+4)}} = x - 5$

③ Now substitute $-4 - 5 = -9$



EXAMPLE 5

$$\lim_{x\to 3} \frac{x-3}{(x^3-3x^2)(-7x+21)}$$

① Substitute

$$\frac{3-3}{(3)^3-3(3)^2-7(3)+21} = \frac{0}{0}$$

② Factor +simplify

$$\frac{x-3}{x^2(x-3)-7(x-3)} = \frac{\cancel{x-3}}{(x^2-7)\cancel{(x-3)}} = \frac{1}{x^2-7}$$

hole at $x=3$

Removable discontinuity

③ Substitute into simplified problem

$$\frac{1}{x^2-7} = \frac{1}{(3)^2-7} = \frac{1}{2}$$

EXAMPLE 6

$$\lim_{x\to 6} \frac{x^2 - 7x + 6}{3x^2 - 11x - 42}$$

① Subst.

$$\frac{(6)^2 - 7(6) + 6}{3(6)^2 - 11(6) - 42} = \frac{0}{0}$$

② Factor

$$\frac{\cancel{(x-6)}(x-1)}{(3x+7)\cancel{(x-6)}} = \frac{x-1}{3x+7}$$

$7x$
$-18x$
$-11x$

③ Subst
Find lim

$$\lim_{x\to 6} \frac{x-1}{3x+7} = \frac{6-1}{3(6)+7} = \frac{5}{25} = \frac{1}{5}$$

EXAMPLE 7

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$$

$a=\sqrt{x}$
$b=3$

① Subst. $\dfrac{\sqrt{9}-3}{9-9}=\dfrac{3-3}{9-0}=\dfrac{0}{0}$

② Factor with dif. of squares $\dfrac{\cancel{\sqrt{x}-3}}{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}=\dfrac{1}{\sqrt{x}+3}$

③ Find limit $\displaystyle\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}$

EXAMPLE 8

$$\lim_{x\to 0}\frac{2-\sqrt{x+4}}{x}\cdot\frac{(2+\sqrt{x+4})}{(2+\sqrt{x+4})}$$

① $\frac{2-\sqrt{0+4}}{0}=\frac{0}{0}$

② Rationalize the numerator by multiplying by the conjugate

$$\frac{4+\cancel{2\sqrt{x+4}}-\cancel{2\sqrt{x+4}}-(\sqrt{x+4})^2}{x(2+\sqrt{x+4})}$$

$$\frac{4-(x+4)}{x(2+\sqrt{x+4})}=\frac{\cancel{4}-x-\cancel{4}}{\longrightarrow}=\frac{-\cancel{x}1}{\cancel{x}(2+\sqrt{x+4})}$$

③ $\lim_{x\to 0}\frac{-1}{2+\sqrt{x+4}}$

$$\frac{-1}{2+\sqrt{0+4}}=\boxed{-\frac{1}{4}}$$

EXAMPLE 9

$$\lim_{x\to3}\frac{\left(3\cdot\frac{1}{x}+\frac{-1}{3}\cdot x\right)}{3\ \ x \quad 3\ \ x}{(x-3)} \rightarrow \frac{\frac{3}{3x}-\frac{x}{3x}}{x-3}$$

① Subst. $\frac{\frac{1}{3}-\frac{1}{3}}{3-3}=\frac{0}{0}$

② Simplify complex fraction

$$\frac{\left(\frac{3-x}{3x}\right)}{(x-3)}=\frac{3-x}{3x}\div\frac{x-3}{1}=\frac{(3-x)}{3x}\cdot\frac{1}{(x-3)}$$

$-1(x-3)$

$-1(-3+x)$

$$\frac{-1\cancel{(x-3)}}{3x}\cdot\frac{1}{\cancel{x-3}}$$

③ $\lim\limits_{x\to3}\frac{-1}{3x}=-\frac{1}{9}$

EXAMPLE 10

LCD is $2(2+x)$

$$\lim_{x\to 0} \frac{\frac{2 \cdot 1}{2(2+x)} + \frac{-1(2+x)}{2(2+x)}}{x}$$

$$\frac{\frac{1}{2} - \frac{1}{2}}{0} = \frac{0}{0}$$

Get common denom & simplify complex fraction

$$\frac{\frac{\cancel{2}}{2(2+x)} + \frac{\cancel{-2} - x}{2(2+x)}}{x} = \frac{\frac{-x}{2(2+x)}}{x} = \frac{-x}{2(2+x)} \div \frac{x}{1}$$

$$\frac{-\cancel{x}}{2(2+x)} \cdot \frac{1}{\cancel{x}}$$

$$\lim_{x\to 0} \frac{-1}{2(2+x)} = -\frac{1}{4}$$