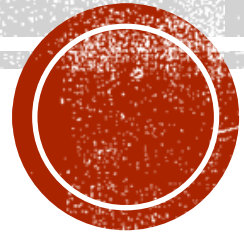
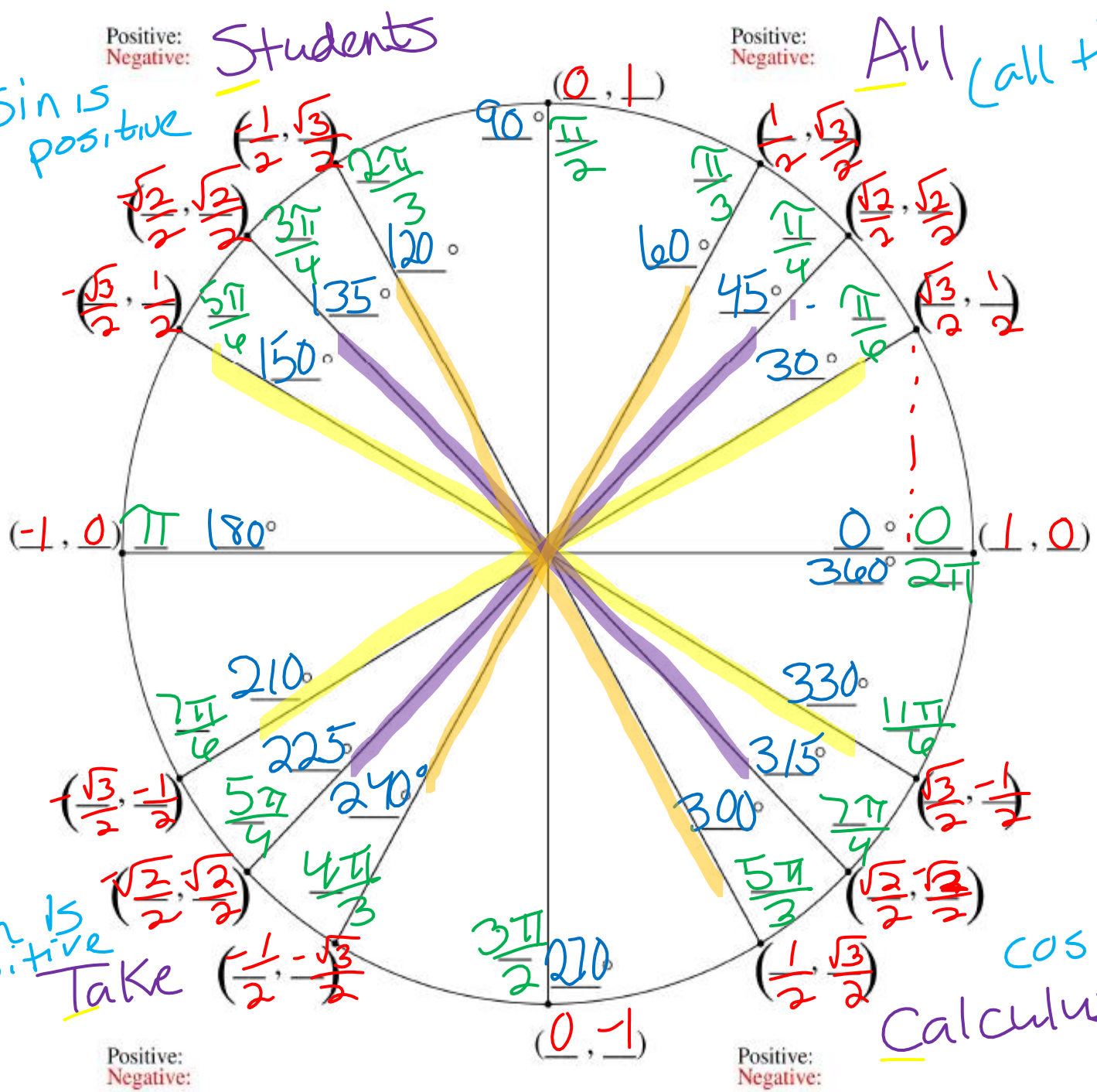
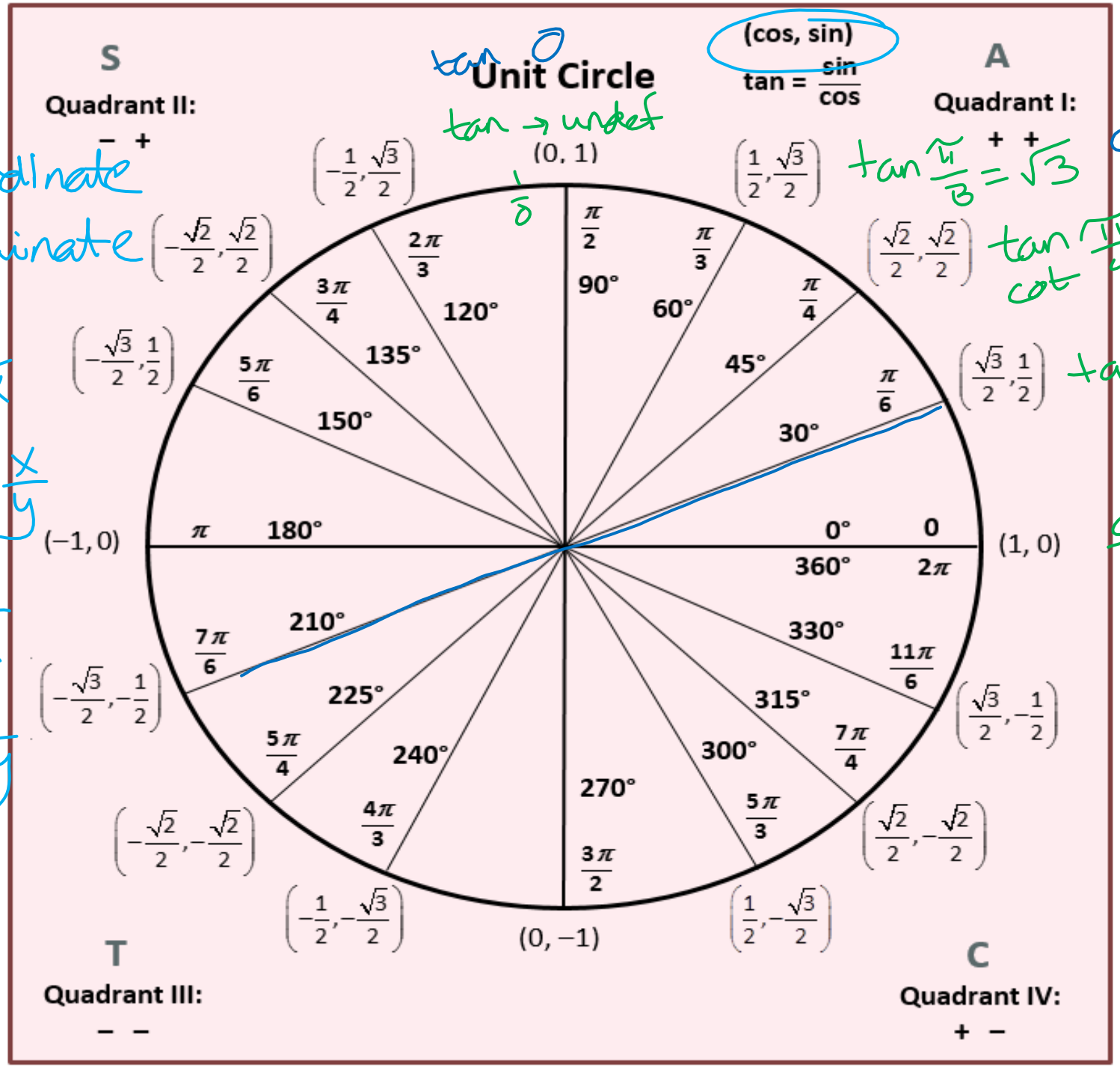
UNIT CIRCLE, TRIG EXACT VALUES, & SIMPLIFYING TRIG EXPRESSIONS

Honors Calculus

Keeper 1.9




Positive:
Negative:
Students
Positive:
Negative:
All
(all +)
(−, +)
Sin is positive
(+, +)
(cos, sin)
(−, −)
(+, −)
tan is positive
Take
cos is positive
Calculus
Positive:
Negative:
Positive:
Negative:
(0, 1)
90°
π/2
(1/2, √3/2)
π/3
60°
(√2/2, √2/2)
π/4
45°
(√3/2, 1/2)
π/6
30°
2π/3
120°
3π/4
135°
(−√3/2, 1/2)
5π/6
150°
(−1, 0)
π
180°
0°
0
360°
2π
(1, 0)
210°
7π/6
225°
5π/4
240°
(−√3/2, −1/2)
4π/3
(−√2/2, −√2/2)
(−1/2, −√3/2)
3π/2
270°
(0, −1)
300°
5π/3
(1/2, √3/2)
315°
7π/4
(√2/2, −√2/2)
330°
11π/6
(√3/2, −1/2)



(cos, sin)

cos → x-coordinate

sin → y-coordinate

$\tan = \frac{\sin}{\cos}$ or $\frac{y}{x}$

$\cot = \frac{\cos}{\sin}$ or $\frac{x}{y}$

$\sec = \frac{1}{\cos}$ or $\frac{1}{x}$

$\csc = \frac{1}{\sin}$ or $\frac{1}{y}$

tan 0

tan → undef

$\tan\frac{\pi}{3} = \sqrt{3}$

$\cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

$\tan\frac{\pi}{4} = 1$

$\cot\frac{\pi}{4} = 1$

$\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\cot\frac{\pi}{6} = \sqrt{3}$

$\frac{0}{1}$ tan 0 cot und

FIND THE EXACT VALUE OF THE TRIG FUNCTION

1. $\cos\left(\frac{4\pi}{3}\right)$ $= -\frac{1}{2}$

2. $\tan 210°$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ (cos, sin)

$-\frac{1}{2} \div -\frac{\sqrt{3}}{2}$

$-\frac{1}{2} - \frac{2}{\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

3. $\csc \frac{11\pi}{6}$

reciprocal of $\sin\frac{11\pi}{6}$

$\sin\frac{11\pi}{6} = -\frac{1}{2}$

$\csc\frac{11\pi}{6} = -2$

FIND THE EXACT VALUE OF THE TRIG FUNCTION

4. $sec(270°)$ $\cos(270°) = \frac{0}{1}$ reciprocal is $\frac{1}{0}$ undefined DNE

5. $sin(-120°)$ $+360°$

$\sin(240°) = -\frac{\sqrt{3}}{2}$

6. $cot\left(\frac{17\pi}{6}\right)$ $\frac{17\pi}{6} - \frac{2\pi}{1} = \frac{17\pi}{6} - \frac{12\pi}{6}$

$5\pi/6$

$\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$

FIND THE EXACT VALUE OF THE TRIG FUNCTION

7. $\tan\left(-\frac{5\pi}{6}\right) = \tan\left(\frac{7\pi}{6}\right) = \frac{\sqrt{3}}{3}$

coterminal angle

III

S	A
T	C

~~8. $\csc\left(-\frac{5\pi}{3}\right)$~~

9. $\sec(480°) = \sec(120°) = -2$

-360

$\cos(120°) = -\frac{1}{2}$

reciprocal

Trigonometric Identities

Reciprocal Identities

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

- Know these basic identities
- Be able to rearrange the Pythagorean identities for various trig functions.

SIMPLIFY TO A SINGLE TRIG EXPRESSION OR VALUE

1. $\sec x \sin x$

$\frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$

2. $(1 - \cos^2 x) \csc x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x \quad \csc x$

$\sin^2 x \quad \frac{1}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$

SIMPLIFY TO A SINGLE TRIG EXPRESSION OR VALUE

or 4 $\frac{1}{\cot^2 x} + \frac{\cot^2 x}{\cot^2 x}$

$\tan^2 x + 1 = \sec^2 x$

3. $\frac{\tan^2 x}{\sin^2 x} = \tan^2 x \div \sin^2 x$

$\tan^2 x \cdot \frac{1}{\sin^2 x}$

$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

4. $\frac{1+\cot^2 x}{\cot^2 x} = \frac{\csc^2 x}{\cot^2 x} \rightarrow \csc^2 x \div \cot^2 x$

$\csc^2 x \cdot \frac{1}{\cot^2 x}$ or $\tan^2 x$

$\frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$