# Exponentials and Logarithms

Honors Calculus

Keeper 1.8

| **Properties of Exponents** | **Let *a* and *b* be real #'s and let *m* and *n* be integers.** |
|---|---|
| Product of Powers | $a^m \cdot a^n = a^{m+n}$ |
| Power of a Power | $(a^m)^n = a^{mn}$ |
| Power of a Product | $(ab)^m = a^m b^m$ |
| Negative Exponent | $a^{-m} = \frac{1}{a^m}, a \neq 0$ |
| Zero Exponent | $a^0 = 1, a \neq 0$ |
| Quotient of Powers | $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ |

# Example 1: Simplify the expression completely.

$$3x^4 \cdot 6x^{\frac{1}{8}}$$

$4\frac{+1}{8}$

$\frac{33}{8}$

$18x^{4+\frac{1}{8}}$

$18x^{33/8}$

$8\ \frac{4}{8\cdot 1} + \frac{1}{8}$

$\frac{32}{8} + \frac{1}{8}$

# Example 2: Simplify the expression completely.

$$(16x^5y^{-1})^{\frac{1}{2}}$$

$b^{\frac{\text{exp}}{\text{root}}} = \sqrt[\text{root}]{b^{\text{exp}}}$

$16^{1/2} x^{5/2} y^{-1/2}$

$\frac{\sqrt{16}\, x^{5/2}}{y^{1/2}} = \frac{4x^{5/2}}{y^{1/2}}$

# Example 3: Simplify the expression completely.

$$\frac{4z^{\frac{5}{3}}}{2\sqrt[3]{z^2}} =$$

root

exp

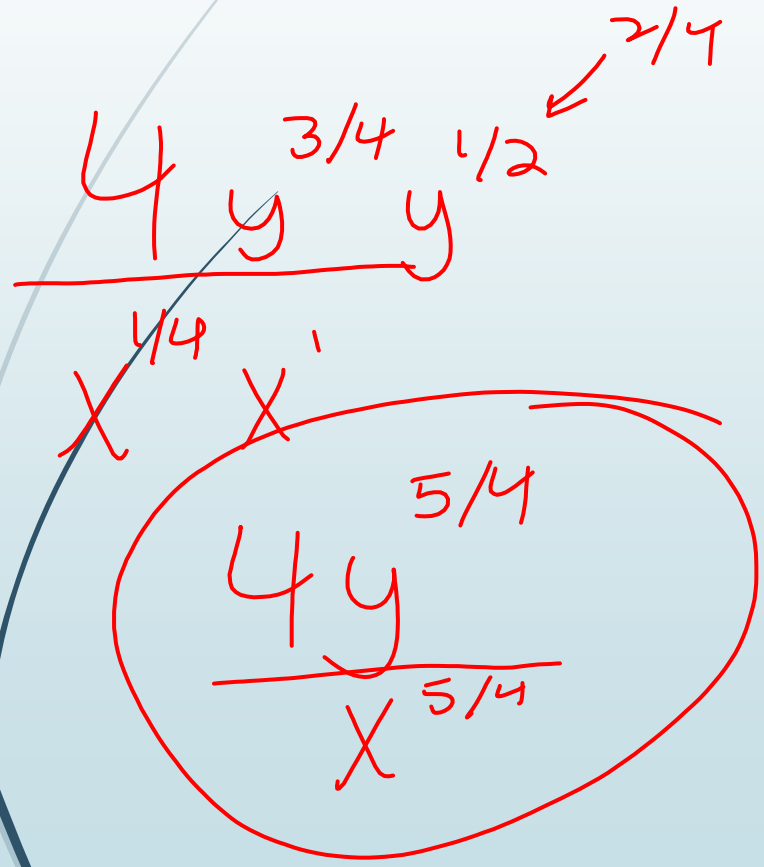
rewrite as rational exp

$$\frac{4z^{5/3}}{2z^{2/3}} = 2z^{5/3-2/3}$$

$$2z$$

# Example 4: Simplify the expression completely.

$$\frac{2x^{-\frac{1}{4}} \cdot 2y^{\frac{3}{4}}}{xy^{-\frac{1}{2}}}$$



# Example 5: Simplify the expression completely.

$$\frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{21x^5}$$

# Properties of Log & Exponentials

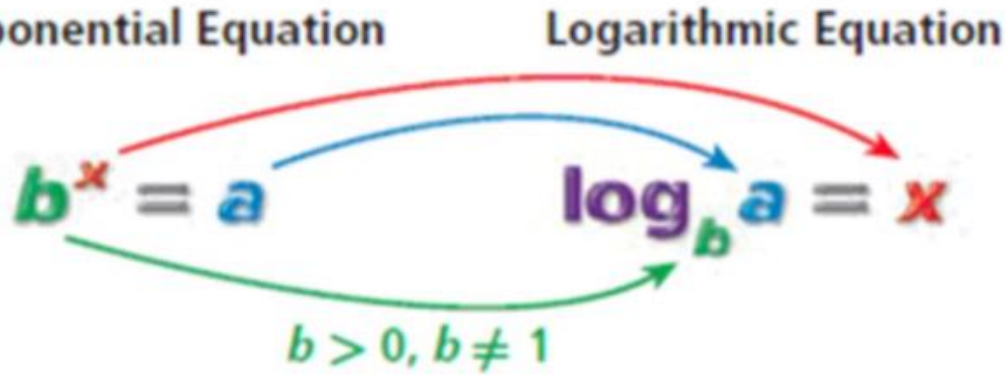
- $\log_e x = \ln x$
- $\ln e = 1$
- $\ln 1 = 0$
- $\ln e^b = b$

$y = b^x$ & $y = \log_b x$

- $y = e^x$ & $y = \ln x$ are inverses
- $\ln a^c = c \ln a$
- $\ln(ab) = \ln a + \ln b$
- $\ln \frac{a}{b} = \ln a - \ln b$

- $e^{-x} = \frac{1}{e^x}$
- $e^{\ln x} = x$
- $a^{x+y} = a^x \cdot a^y$
- $a^{x-y} = \frac{a^x}{a^y}$

# Writing Exponential Equations in Logarithmic Form




base$^{exp}$ = #

$\log_{base}$ # = exp

$\log$ # ↑ understood base 10

ln # understood base e

Rewrite into logarithmic form.

1. $5^3 = 125$ (exp, #)

$\log_5 125 = 3$

2. $e^2 = 7.39$

$\log_e 7.39 = 2$

$\ln 7.39 = 2$

Rewrite into exponential form.

3. $\log_6 \frac{1}{36} = -2$

$6^{-2} = \frac{1}{36}$

4. $3 = \ln 20.08$

$e^3 = 20.08$

# Evaluate without using a calculator

$\log_8 8 = ?$

$8^? = 8$

$8^? = 8^5$

1. $\log_8 8^5 = 5 \log_8 8 = 5 \cdot 1 = 5$

2. $\frac{3}{4} \ln e = \frac{3}{4} \cdot 1 = 3/4$

3. $6 \log 1000$ (understood: $\log_{10}$)

$6 \cdot 10^? = 1000$

$6 \cdot 3 = 18$

4. $2\left(\log_7 \frac{1}{49}\right) = 2 \cdot \left(7^x = \frac{1}{49}\right)$

$2 \cdot -2 = -4$

# Find the value without a calculator

1. $5(\ln e) + 2\ln\left(\frac{1}{e}\right)$

$5 \cdot 1 + 2\ln(e^{-1}) = 5 + 2(-1)$

$5 - 2 = 3$

2. $e^{-\ln e} - e^{\ln\sqrt{e}}$

$\frac{1}{e^{\ln e}} - e^{\ln(e^{1/2})} = \frac{1}{e} - e^{1/2}$ or $\frac{1}{e} - \sqrt{e}$

3. $3\ln(e^2 \ln(e \ln e))$

$3\ln(e^2) = 3 \cdot 2\ln e$ 6

# Remember your Logarithm Properties!!!!

The Product Rule: $log_a MN = log_a M + log_a N$

The Power Rule: $\log_a M^p = p \cdot \log_a M$

The Quotient Rule: $\log_a \frac{M}{N} = \log_a M - \log_a N$

# Express as a single logarithm (condense)

1. $2\ln x - 4\ln y - \ln 13$

$\ln x^2 - \ln y^4 - \ln 13 = \ln\left(\frac{x^2}{13y^4}\right)$

(pos) Adding → numerator
(neg) subtracting → denominator

2. $\log_3 7 + \frac{1}{2}\log_3 x - 5\log_3 y$

$\log_3\left(\frac{7x^{1/2}}{y^5}\right)$ or $\log_3 \frac{7\sqrt{x}}{y^5}$

# Expand the following Logarithms

7. $\log \frac{x^2 y^3}{w z^3}$ $= \log x^2 + \log y^3 - \log w - \log z^3$

$2\log x + 3\log y - \log w - 3\log z$

8. $\log \frac{b^3}{\sqrt{ac}}$ $= \log \frac{b^3}{a^{1/2}c^{1/2}} = \log b^3 - \log a^{1/2} - \log c^{1/2}$

rewrite as rat'l exp

$3\log b - \frac{1}{2}\log a - \frac{1}{2}\log c$