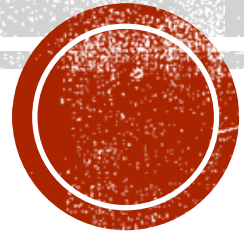
RATES OF CHANGE & WRITING LINEAR EQUATIONS

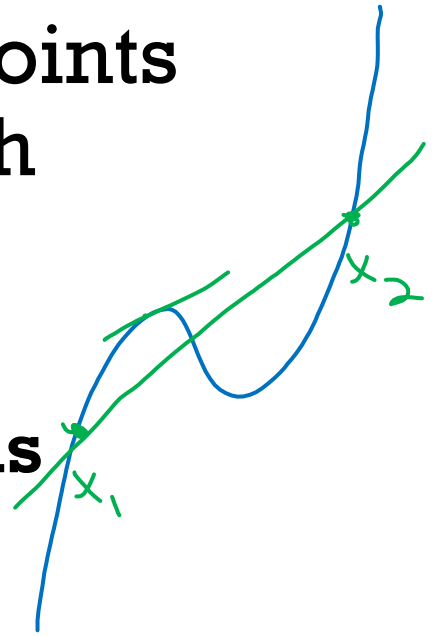
Honors Calculus

Keeper 1.2



AVERAGE RATE OF CHANGE

The average rate of change between any two points on the graph of f is the slope of the line through those points.



The average rate of change on the interval $[x_1, x_2]$ is

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

FIND AVERAGE RATE OF CHANGE

AROC

1. Find the average rate of change of $f(x) = -x^3 + 3x$ on each interval:

a. $[-2, -1]$

$x_1 \quad x_2$

$(-2, 2) \quad (-1, -2)$

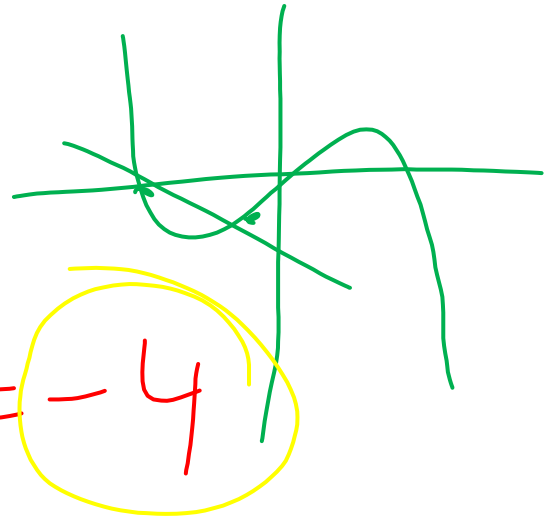
$x_1 \; f(x_1) \quad x_2 \; f(x_2)$

$y_1 \qquad y_2$

$f(-2) = -(-2)^3 + 3(-2) = 2$

$f(-1) = -(-1)^3 + 3(-1) = -2$

$m_{[-2,-1]} = \dfrac{-2-(2)}{-1-(-2)} = \dfrac{-4}{1} = -4$



b. $[0, 1]$

$x \quad y$

$f(0) = 0$

$f(1) = 2$

$x \quad y$

$m = \dfrac{2-0}{1-0} = 2$

$\dfrac{0-2}{0-1} = 2$

2. Find the average rate if change on the interval $[-6,2]$.

$$f(x) = \frac{x+5}{x-4}$$

$f(-6) = \frac{-6+5}{-6-4} = \frac{-1}{-10}$

$f(-6) = \frac{1}{10}$ $\left(-6, \frac{1}{10}\right)$ $x_1\ y_1$

$f(2) = \frac{7}{-2}$ $\left(2, -\frac{7}{2}\right)$ $x_2\ y_2$

$-\frac{35}{10} - \frac{1}{10} = -\frac{36}{10}$

$m = \frac{\left(-\frac{7}{2.5} - \frac{1}{10}\right)}{(2-6)} = \frac{-\frac{18}{5}}{8}$

$-\frac{\cancel{18}^{9}}{5} \cdot \frac{1}{\cancel{8}_{4}} = \frac{-9}{20}$

3. Find the average rate if change on the interval $[-4,4]$.

$$f(x) = \sqrt{x+8}$$

$f(-4) = 2$

$f(4) = \sqrt{12}$

$= 2\sqrt{3}$

$m_{[-4,4]} = \dfrac{2\sqrt{3}-2}{8}$

$\dfrac{\sqrt{3}-1}{4}$

$\dfrac{2(\sqrt{3}-1)}{8}$

1.3 WRITING AN EQUATION OF A LINE

Slope-Intercept Form: Given the slope m and the y-intercept b,

$$y = mx + b$$

***Point-Slope Form:** Given the slope m and a point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

Standard Form: $\mathbf{Ax + By = C}$

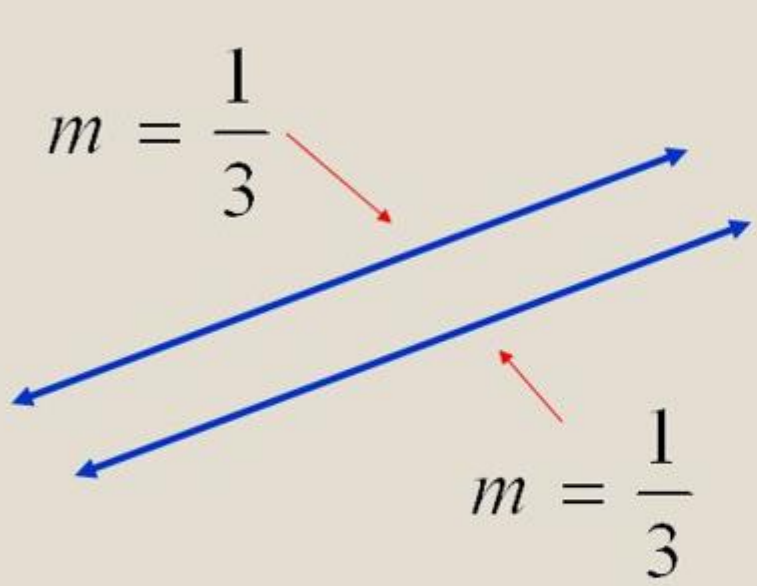
1. Write the point-slope form of the equation of the line that passes through $(2,3)$ and has a slope of $-\frac{1}{2}$.

x_1, y_1

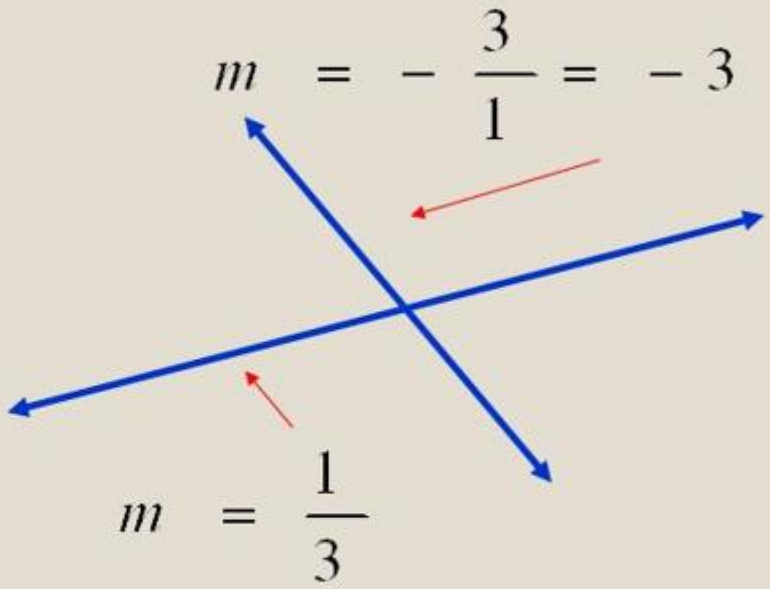
$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{2}(x - 2)$$

PARALLEL AND PERPENDICULAR LINES



Parallel lines have the SAME slope.



Perpendicular lines have an OPPOSITE, RECIPROCAL slope.

2. Write the point-slope equation of the line that passes through $(3,2)$ and is perpendicular to the line $y = -3x + 2$.

$m = -3$

$\perp m = \frac{1}{3}$

$(3, 2)$

x_1, y_1

$y - 2 = \frac{1}{3}(x - 3)$

3. Write the equation of the line that passes through $(3,2)$ and is parallel to the line $y = -3x + 2$. Then convert to slope-intercept form.

$y = mx + b$

Pt-Slope Form: $y - 2 = -3(x - 3)$

$y - 2 = -3(0 - 3)$

$y - 2 = 9$

$y = 11$

y-int

b

Slope-Int: $y - 2 = -3x + 9$

$y = -3x + 11$