PARTICLE MOTION

Keeper 24

Honors Calculus

PARTICLE MOTION TERMINOLOGY

- s(t) is the <u>position</u> of the object moving along the x-axis
- s'(t) = v(t) the first derivative is <u>velocity</u>
- s''(t) = a(t) the second derivative is <u>acceleration</u>
- v(t) = 0 is when the particle is at rest
- + velocity means the particle is moving right (or up)
- - velocity means the particle is moving left (or down)
- If v(t) & a(t) have the same signs, then the particle is speeding up
- If v(t) & a(t) have different signs, then the particle is slowing down
- Displacement is the change in position from start to stop
- Total distance includes all of the distance traveled taking into consideration that the particle can change directions

1. $s(t) = t^3 - 6t^2 + 9t$

Beginner Level of Particle Motion

a) Find the velocity at time t.

$$V(t) = 3t^2 - 12t + 9$$

b) What is the velocity after 2 seconds?

$$V(2) = 3(2)^2 - 12(2) + 9$$
$$= -3 \, m/s$$

$$v(t) = 3t^2 - 12t + 9$$

c) Find the acceleration as a function of time t.

$$a(t) = 6t - 12$$

d) Find the acceleration at t=3 seconds.

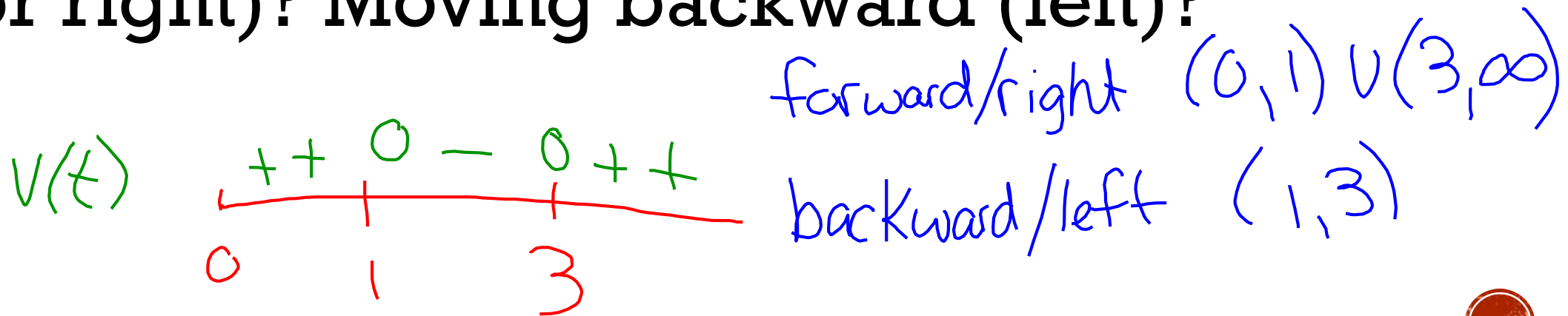$$a(3) = 6(3) - 12$$
$$= 6 \text{ m/s}^2$$

INTERMEDIATE LEVEL OF PARTICLE MOTION

e) When is the particle at rest? $0 = v(t)$

$$0 = 3t^2 - 12t + 9$$
$$0 = 3(t^2 - 4t + 3)$$
$$0 = 3(t-1)(t-3)$$

$t = 1 \text{ sec}$

$t = 3 \text{ sec}$

f) When is the particle moving forward (or right)? Moving backward (left)?

forward/right $(0,1) \cup (3,\infty)$

backward/left $(1,3)$

$v(t)$ $++$ 0 $-$ 0 $++$

0   1   3

MASTERY LEVEL OF PARTICLE MOTION

$$s(t) = t^3 - 6t^2 + 9t$$

displacement & total dist. are positive!

g) What is the displacement on [0,5] seconds?

$S(0) = 0$ $S(5) = 20$ $20 - 0 = \boxed{20m}$

h) Find the total distance traveled on [0,5].

$(0,1) = S(1) - S(0) = |4 - 0| = 4m$ $S(1) = 4$

$(1,3) = S(3) - S(1) = |0 - 4| = 4m$ $S(3) = 0$

$(3,5) = S(5) - S(3) = |20 - 0| = 20m$ $4 + 4 + 20 = \boxed{28m}$

MASTERY LEVEL OF PARTICLE MOTION

i) Find the velocity when acceleration is $24\ \dfrac{m}{second^2}$

$a(t)$

$a(t) = 6t - 12$

$24 = 6t - 12$

$t = 6$

$V(t) = 3t^2 - 12t + 9$

$V(6) = 3(6)^2 - 12(6) + 9$

$= 45\ m/s$

j) Find when the particle is speeding up and slowing down.

$a(t) = 6t - 12$

$0 = 6t - 12$
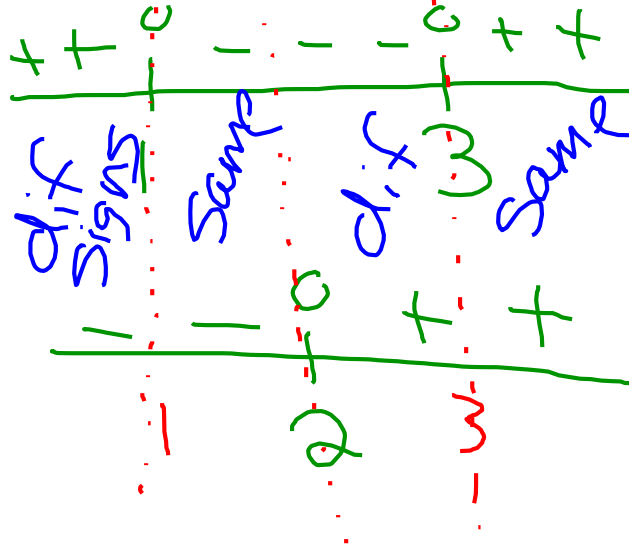
$12 = 6t$

$2 = t$

$V(t)$

$a(t)$

$+\ +\ \circ\quad -\ -\ -\ -\circ\quad +\ +$

dif signs   same   dif 3   same

$-\ -\ -\ -\circ\quad +\quad +$

1   2   3

(Same signs)   (dif signs)

Speed ↑ $(1,2) \cup (3,\infty)$

Slow ↓ $(0,1) \cup (2,3)$

2. $s(t) = t^3 - 12t^2 + 45t \quad [0,7]$

a) What is the velocity function? What is the velocity at $t = 2$ seconds?

$v(t) = 3t^2 - 24t + 45$

$v(2) = 3(2)^2 - 24(2) + 45$

$v(2) = 9 \text{ m/s}$

b) When is the particle at rest?

$0 = 3t^2 - 24t + 45$

$0 = 3(t^2 - 8t + 15)$

$0 = 3(t-5)(t-3)$

$t = 5 \text{ sec} \text{ & } 3 \text{ sec}$

c) When is the particle moving right? Moving left?

$v(t)$

$\overset{++}{\underset{0}{\,}} \quad \overset{--}{\underset{3}{\,}} \quad \overset{++}{\underset{5}{\,}} \quad \underset{7}{\,}$

right $(0,3) \cup (5,7)$

left $(3,5)$

CONTINUE $s(t) = t^3 - 12t^2 + 45t$ $[0,7]$

$v(t) = 3t^2 - 24t + 45$

d) What is the acceleration function? What is the acceleration at $t = 1$ second?

$a(t) = 6t - 24$

$a(1) = 6(1) - 24$
$= -18 \ m/s^2$

e) What is the displacement and total distance traveled for the indicated interval?

$s(0) = 0$
$s(7) = 70$

displacement $= 70 - 0$
$70m$

$(0,3) = 54 - 0 = 54m$
$(3,5) = |50 - 54| = 4m$
$(5,7) = 70 - 50 = 20m$

$s(3) = 54$
$s(5) = 50$

total dist $= 78m$

$CONTINUE$ $s(t) = t^3 - 12t^2 + 45t$ $[0,7]$
$$v(t) =$$
$$a(t) =$$

f) When is the particle speeding up? Slowing down?

g) Find the velocity when the acceleration is 0.