Keeper 1.5 – Solving Inequalities and Absolute Value

SIGN GRAPH METHOD

- If there are any denominators, set the factor in the denominators equal to 0 to get critical numbers.
- Get more critical numbers by pretending the inequality is an equation and solving.
- Place all critical numbers on a number line. These critical numbers will form regions for which you will test a random number in the region to determine if the inequality is satisfied for this number. If the region is satisfied for this tested number, you will include the region in the solution. If the inequality is not satisfied for the number tested, you will not include the region in the solution.

Solve the following inequalities

1. $|4 - 3x| > 1$

$4-3x > 1$ $\quad$ $4-3x < -1$

$-3x > -3$ $\quad$ $-3x < -5$

$x < 1$ $\quad$ $x > 5/3$

$(-\infty, 1) \cup (5/3, \infty)$

2. $2 - \frac{12+11x}{6x} \geq -\frac{x}{6}$

$$2 + \frac{x}{6} - \frac{12+11x}{6x} \geq 0$$

$$\frac{12x}{6x} + \frac{x^2}{6x} - \frac{12+11x}{6x} \geq 0$$

$$\frac{x^2 + x - 12}{6x} \geq 0$$

$$\frac{(x+4)(x-3)}{6x} \geq 0$$

$-$ | -4 | $+$ | 0 | $-$ | 3 | $+$

$[-4, 0) \cup [3, \infty)$

3. $x^2(3x-5)(x^2-4) \leq 0$

$0 \quad 5/3 \quad \pm 2$

$-$ | -2 | $+$ | 0 | $+$ | $5/3$ | $-$ | 2 | $+$

$(-\infty, -2] \cup [5/3, 2]$

4. $-4(3y+1)(y^2-81) \geq 0$

$-1/3 \quad \pm 9$

$+$ | -9 | $-$ | $-1/3$ | $+$ | 9 | $-$

$(-\infty, -9] \cup [-1/3, 9]$

5. $\frac{7}{x} - 2 \ge \frac{5}{x} - 4$

$\frac{7}{x} - \frac{5}{x} + 2 \ge 0$

$\frac{2}{x} + \frac{2x}{x} \ge 0$

$\frac{2x+2}{x} \ge 0$

$+ \quad - \quad +$

$-1 \quad 0$

$(-\infty, -1] \cup (0, \infty)$

6. $(x+3)(x-3) \le -4x(x+3)$

$x^2 - 9 + 4x(x+3) \le 0$

$x^2 - 9 + 4x^2 + 12x \le 0$

$5x^2 + 12x - 9 \le 0$

$(5x-3)(x+3) \le 0$

$+ \quad - \quad +$

$-3 \quad 3/5$

$[-3, 3/5]$

7. $\frac{3}{x+4} \le \frac{5}{2(2x-2)}$

$\frac{3}{x+4} - \frac{5}{2(2x-2)} \le 0$

$\frac{6(2x-2) - 5(x+4)}{2(2x-2)(x+4)} \le 0$

$\frac{7x - 32}{2(2x-2)(x+4)} \le 0$

$- \quad + \quad - \quad +$

$-4 \quad 1 \quad 32/7$

$(-\infty, -4) \cup (1, 32/7]$

8. $x^3 < -3x^2$

$x^3 + 3x^2 < 0$

$x^2(x+3) < 0$

$- \quad + \quad +$

$-3 \quad 0$

$(-\infty, -3)$

9. $\left|\frac{x+3}{3x-2}\right| \le 2$

$\frac{x+3}{3x-2} \le 2$

$\frac{x+3-2(3x-2)}{3x-2} \le 0$

$\frac{x+3-6x+4}{3x-2} \le 0$

$\frac{-5x+7}{3x-2} \le 0$

$- \quad + \quad -$

$2/3 \quad 7/5$

$\frac{x+3}{3x-2} \ge -2$

$\frac{x+3+2(3x-2)}{3x-2} \ge 0$

$\frac{7x-1}{3x-2} \ge 0$

$+ \quad - \quad +$

$1/7 \quad 2/3$

merge intervals $(-\infty, 1/7) \cup (7/5, \infty)$

10. $|x^2 + 3x - 4| > 0$

↑ Abs Valu → Aways Pos!

$(x+4)(x-1) \ne 0$

$x \ne -4, 1$

$(-\infty, -4) \cup (-4, 1) \cup (1, 4)$

DEFINITION OF ABSOLUTE VALUE

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \quad \text{or in terms of a function} \quad |f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

REWRITING ABSOLUTE VALUE AS PIECEWISE

1. Determine when the inside of the absolute value is equal to 0.
2. Determine over which intervals the inside of the absolute value is positive and over which it is negative. Using a number line makes it easy.
3. Set up the piecewise function as indicated in the definition.

Examples: Write as a piecewise function.

17. $f(x) = |x^2 - 5x - 6|$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = -1, 6$

$+ \quad -1 \quad - \quad 6 \quad +$

$$f(x) = \begin{cases} x^2 - 5x - 6, & x < -1 \\ -(x^2 - 5x - 6), & -1 \leq x \leq 6 \\ x^2 - 5x - 6, & x > 6 \end{cases}$$

18. $f(x) = |x - 3|$

$- \quad 3 \quad +$

$$f(x) = \begin{cases} -(x-3), & x \leq 3 \\ x - 3, & x > 3 \end{cases}$$

19. $f(x) = |x^2 - 11x + 24|$

$x^2 - 11x + 24$

$(x-8)(x-3)$

$+ \quad 3 \quad - \quad 8 \quad +$

$$f(x) = \begin{cases} x^2 - 11x + 24, & x < 3 \\ -(x^2 - 11x + 24), & 3 \leq x \leq 8 \\ x^2 - 11x + 24, & x > 8 \end{cases}$$

20. $f(x) = \left|\frac{x^2-4x+4}{x^2-2x}\right|$

hole at $x = 2$

$f(x) = \left| \frac{(x-2)(x-2)}{x(x-2)} \right|$

$+ \quad 0 \quad - \quad 2 \quad +$

$$f(x) = \begin{cases} \frac{x^2-4x+4}{x^2-2x}, & x < 0 \\ -\left(\frac{x^2-4x+4}{x^2-2x}\right), & 0 \leq x < 2 \\ \frac{x^2-4x+4}{x^2-2x}, & x > 2 \end{cases}$$

SQUARE ROOTS OF PERFECT SQUARE FUNCTIONS

Definition: $\sqrt{(f(x))^2} = |f(x)|$ **Example:** $\sqrt{(x-5)^2} = |x-5|$

Examples: Rewrite the square root as an absolute value and then as a piecewise function.

21. $\sqrt{x^2}$ $= |x|$

$$= \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

22. $\sqrt{x^4}$ $= |x^2|$

$$= x^2$$

23. $\sqrt{(2x-3)^2}$ $= |2x-3|$

$- \quad +$

$3/2$

$$= \begin{cases} -(2x-3), & x \leq 3/2 \\ 2x-3, & x > 3/2 \end{cases}$$

24. $\sqrt{(x+4)^2}$ $= |x+4|$

$- \quad +$

-4

$$= \begin{cases} -(x+4), & x < -4 \\ x+4, & x \geq -4 \end{cases}$$