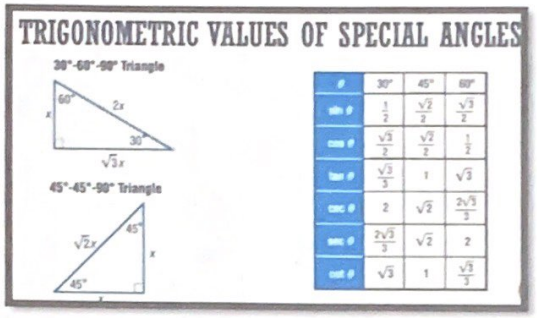
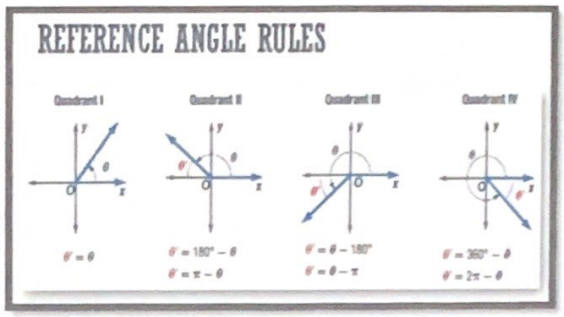
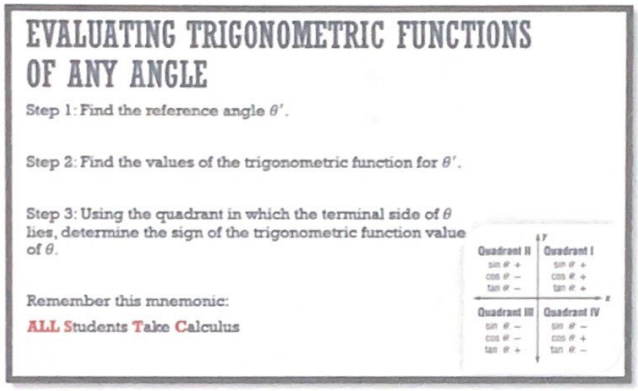
Keeper 1.3 – Trig, Inverse Trig, and Solving Trig Equations



θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
csc θ	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$
sec θ	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2
cot θ	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$





*****Unit Circle Values should be MEMORIZED!!! You should not have to draw a unit circle every time!**

Examples: Evaluate the Trig Function from MEMORY

1. $\sin\left(\frac{2\pi}{3}\right)$ $\frac{\sqrt{3}}{2}$
2. $\tan\left(-\frac{\pi}{4}\right)$ -1
3. $\sin\left(\frac{9\pi}{4}\right)$ $\frac{\sqrt{2}}{2}$
4. $\tan\left(\frac{4\pi}{3}\right)$ $\sqrt{3}$
5. $\sin\left(-\frac{2\pi}{3}\right)$ $-\frac{\sqrt{3}}{2}$
6. $sec\left(\frac{25\pi}{6}\right)$ $\frac{2}{\sqrt{3}}$ * No need to Rationalize
7. $sec\left(\frac{3\pi}{4}\right)$ $-\sqrt{2}$
8. $cos\left(-\frac{17\pi}{3}\right)$ $\frac{1}{2}$
9. $\csc\left(-\frac{31\pi}{6}\right)$ 2
10. $\csc\left(-\frac{5\pi}{6}\right)$ -2
11. $\tan\left(-\frac{13\pi}{6}\right)$ $-\frac{\sqrt{3}}{3}$
12. $\sin(-2\pi)$ 0

SUMMARY OF INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range
$y = \sin^{-1} x$ = $\arcsin x$, where $x = \sin y$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$y = \cos^{-1} x$ = $\arccos x$, where $x = \cos y$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$ = $\arctan x$, where $x = \tan y$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$

Examples: Evaluate the Expressions by MEMORY

13. $y = \sin^{-1} 0$ 0

14. $y = \tan^{-1} 1$ $\frac{\pi}{4}$

15. $y = \cos^{-1}(-1)$ π

Same problem different Notation!

16. $y = \arctan 0$ 0

17. $y = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$ $-\frac{\pi}{3}$

18. $y = arccos(-1)$ π

19. $y = \cot^{-1}(-1)$ $-\frac{\pi}{4}$

20. $y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{6}$

21. $y = \sec^{-1}(-\sqrt{2})$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

22. $y = \tan\left(\arccos\frac{3}{4}\right)$

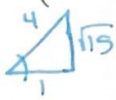
$3^2 + x^2 = 4^2$

$x^2 = 7$

$x = \sqrt{7}$

$\tan\left(\cos^{-1}\frac{3}{4}\right) = \frac{\sqrt{7}}{3}$

23. $y = \sin\left(\arccos\frac{1}{4}\right)$



$1^2 + x^2 = 4^2$

$x^2 = 15$

$x = \sqrt{15}$

$\sin\left(\cos^{-1}\left(\frac{1}{4}\right)\right) = \frac{\sqrt{15}}{4}$

24. $y = \cos(\tan^{-1}(-2))$

$1^2 + (-2)^2 = x^2$

$5 = x^2$

$x = \sqrt{5}$

$\cos\left(\tan^{-1}(-2)\right) = \frac{1}{\sqrt{5}}$

* No Need to Rationalize

HELPFUL HINTS FOR SOLVING TRIGONOMETRIC EQUATIONS

- Try to get equations in terms of one trig function by using identities.
- Be on the look-out for ways to substitute using identities.
- Try to get trig functions of the same angle.
- Get one side equals zero and factor.
- If the angle you are solving for is a multiple of θ, don't forget to add 2π to your answer for each multiple of θ.

Examples: Solve the trig equation for $0 \leq x < 2\pi$

25. $\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

26. $2 + \sec x = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

27. $\tan x\,(\csc x + 2) = 0$

$\tan x = 0$ $\quad \csc x + 2 = 0$

$x = 0, \pi$ $\quad \csc x = -2$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

28. $\tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

29. $\cos 3x = \frac{\sqrt{3}}{2}$

3 times Around unit circle

$3x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}, \frac{35\pi}{6}$

$x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

30. $\csc^2 x + 2\csc x = 0$

$\csc x(\csc x + 2) = 0$

$\csc x = 0$ → DNE $\csc x + 2 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

31. $\sin^2 x - 5\cos x = 5$

Pythagorean Identity

$1 - \cos^2 x - 5\cos x = 5$

$\cos^2 x + 5\cos x + 4 = 0$

$(\cos x + 4)(\cos x + 1) = 0$

$\cos x = -4$ DNE $\cos x = -1$

$x = \pi$

$x = \pi$

32. $2\cos^2 x + 3\sin x = 3$

Pythagorean identity

$2(1 - \sin^2 x) + 3\sin x = 3$

$2 - 2\sin^2 x + 3\sin x = 3$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ $\sin x - 1 = 0$

$\sin x = 1/2$ $\sin x = 1$

$x = \pi/6, \frac{5\pi}{6}$ $x = \pi/2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$