

Keeper 1.3 – Trig, Inverse Trig, and Solving Trig Equations

TRIGONOMETRIC VALUES OF SPECIAL ANGLES

30°-60°-90° Triangle

θ	30°	45°	60°
$\sin \theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan \theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\csc \theta$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$
$\sec \theta$	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2
$\cot \theta$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

45°-45°-90° Triangle

REFERENCE ANGLE RULES

Quadrant I

$\theta = \theta$

Quadrant II

$\theta = 180^\circ - \theta$
 $\theta = \pi - \theta$

Quadrant III

$\theta = \theta - 180^\circ$
 $\theta = \theta - \pi$

Quadrant IV

$\theta = 360^\circ - \theta$
 $\theta = 2\pi - \theta$

EVALUATING TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

Step 1: Find the reference angle θ' .

Step 2: Find the values of the trigonometric function for θ' .

Step 3: Using the quadrant in which the terminal side of θ lies, determine the sign of the trigonometric function value of θ .

Remember this mnemonic:
ALL Students Take Calculus

Quadrant II	Quadrant I
$\sin \theta +$	$\sin \theta +$
$\cos \theta -$	$\cos \theta +$
$\tan \theta -$	$\tan \theta +$
Quadrant III	Quadrant IV
$\sin \theta -$	$\sin \theta -$
$\cos \theta -$	$\cos \theta +$
$\tan \theta +$	$\tan \theta -$

*****Unit Circle Values should be MEMORIZED!!! You should not have to draw a unit circle every time!**

Examples: Evaluate the Trig Function from MEMORY

1. $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$
2. $\tan\left(-\frac{\pi}{4}\right) = -1$
3. $\sin\left(\frac{9\pi}{4}\right) = \frac{\sqrt{2}}{2}$
4. $\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$
5. $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$
6. $\sec\left(\frac{25\pi}{6}\right) = \frac{2}{\sqrt{3}}$
7. $\sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$
8. $\cos\left(-\frac{17\pi}{3}\right) = \frac{1}{2}$
9. $\csc\left(-\frac{31\pi}{6}\right) = 2$
10. $\csc\left(-\frac{5\pi}{6}\right) = -2$
11. $\tan\left(-\frac{13\pi}{6}\right) = -\frac{\sqrt{3}}{3}$
12. $\sin(-2\pi) = 0$

** No need to Rationalize*

SUMMARY OF INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range
$y = \sin^{-1} x$ = arcsin x , where $x = \sin y$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$y = \cos^{-1} x$ = arccos x , where $x = \cos y$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$ = arctan x , where $x = \tan y$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$

Examples: Evaluate the Expressions by MEMORY

13. $y = \sin^{-1} 0 = 0$

14. $y = \tan^{-1} 1 = \frac{\pi}{4}$

15. $y = \cos^{-1}(-1) = \pi$

↑
Same problem
different Notation!
↓

16. $y = \arctan 0 = 0$

17. $y = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

18. $y = \arccos(-1) = \pi$

19. $y = \cot^{-1}(-1) = -\frac{\pi}{4}$

20. $y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

21. $y = \sec^{-1}(-\sqrt{2})$
 $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

22. $y = \tan\left(\arccos\frac{3}{4}\right)$



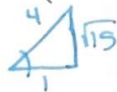
$$3^2 + x^2 = 4^2$$

$$x^2 = 7$$

$$x = \sqrt{7}$$

$$\tan\left(\cos^{-1}\frac{3}{4}\right) = \frac{\sqrt{7}}{3}$$

23. $y = \sin\left(\arccos\frac{1}{4}\right)$



$$1^2 + x^2 = 4^2$$

$$x^2 = 15$$

$$x = \sqrt{15}$$

$$\sin\left(\cos^{-1}\left(\frac{1}{4}\right)\right) = \frac{\sqrt{15}}{4}$$

24. $y = \cos(\tan^{-1}(-2))$



$$1^2 + (-2)^2 = x^2$$

$$5 = x^2$$

$$x = \sqrt{5}$$

$$\cos(\tan^{-1}(-2)) = \frac{1}{\sqrt{5}}$$

* No Need to Rationalize

HELPFUL HINTS FOR SOLVING TRIGONOMETRIC EQUATIONS

- Try to get equations in terms of one trig function by using identities.
- Be on the look-out for ways to substitute using identities.
- Try to get trig functions of the same angle.
- Get one side equals zero and factor.
- If the angle you are solving for is a multiple of θ , don't forget to add 2π to your answer for each multiple of θ .

Examples: Solve the trig equation for $0 \leq x < 2\pi$

25. $\cos x + 1 = 0$

$$\cos x = -1$$

$$x = \pi$$

26. $2 + \sec x = 0$

$$\sec x = -2$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

27. $\tan x (\csc x + 2) = 0$

$$\tan x = 0 \quad \csc x + 2 = 0$$

$$\csc x = -2$$

$$x = 0, \pi$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$$

28. $\tan^2 x - 3 = 0$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

29. $\cos 3x = \frac{\sqrt{3}}{2}$ ↖ 3 times Around unit circle

$$3x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}, \frac{35\pi}{6}$$

$$x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$$

30. $\csc^2 x + 2 \csc x = 0$

$$\csc x (\csc x + 2) = 0$$

$$\csc x = 0 \quad \csc x + 2 = 0$$

$$\downarrow \quad \sin x = -\frac{1}{2}$$

DNE

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

31. $\sin^2 x - 5 \cos x = 5$

$$1 - \cos^2 x - 5 \cos x = 5$$

Pythagorean Identity

$$\cos^2 x + 5 \cos x + 4 = 0$$

$$(\cos x + 4)(\cos x + 1) = 0$$

$$\cos x = -4 \quad \cos x = -1$$

DNE

$$x = \pi$$

$$x = \pi$$

↖ Pythagorean Identity

32. $2 \cos^2 x + 3 \sin x = 3$

$$2(1 - \sin^2 x) + 3 \sin x = 3$$

$$2 - 2 \sin^2 x + 3 \sin x = 3$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$2 \sin x - 1 = 0 \quad \sin x - 1 = 0$$

$$\sin x = \frac{1}{2} \quad \sin x = 1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$