Keeper 1.1 – Equations of Lines, Piecewise and Transformations

Forms for the Equation of a Line		
Slope-Intercept	$y = mx + b$	m is the slope b is the y-intercept
Point-Slope	$y - y_1 = m(x - x_1)$	m is the slope (x_1, y_1) is a point on the line
Standard Form	$ax + by = c$	a is positive
Intercept Form	$\frac{x}{a} + \frac{y}{b} = 1$	a is the x-intercept b is the y-intercept
Vertical	$x = a$	Vertical line with a as the x-intercept
Horizontal	$y = b$	Horizontal line with b as the y-intercept

Two non-vertical lines with slopes m_1 and m_2 are:

Parallel
if the lines have the same slope,
$m_1 = m_2$.

Perpendicular
if the slopes are negative reciprocals,
$$m_2 = -\frac{1}{m_1}$$
or equivalently, if $m_1 \cdot m_2 = -1$.

Examples: Find the equation of the following lines:

1. Horizontal, passes through $(0, -2)$

$y = b$ x   y

$y = -2$

* Never circle/box your Answers!

2. Passes through $(-1,4)$ and $(2,7)$

• First find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{2+1} = \frac{3}{3} = 1$$

• Second use the point slope form to write equation

$y - 4 = 1(x+1)$

or

$y - 7 = 1(x-2)$

* No Need to simplify!

3. Parallel to $y = 3x - 4$ and contains $(1,2)$

Same Slope   Slope

$m = 3$

Point Slope form

$y - 2 = 3(x - 1)$

* No Need to Simplify to $y =$!

4. Perpendicular to $3x + 5y = 9$ and contains $(2,3)$

→ Negative Reciprocal Slope

$3x + 5y = 9$

$5y = -3x + 9$

$y = -\frac{3}{5}x + \frac{9}{5}$

↳ slope

$m = 5/3$

Point Slope form

$y - 3 = \frac{5}{3}(x - 2)$

Piecewise-Defined Function

A **piecewise-defined** function is one which is defined not by a single equation, but by two or more. Each equation is valid for some interval.

Examples: Graph and evaluate the function:

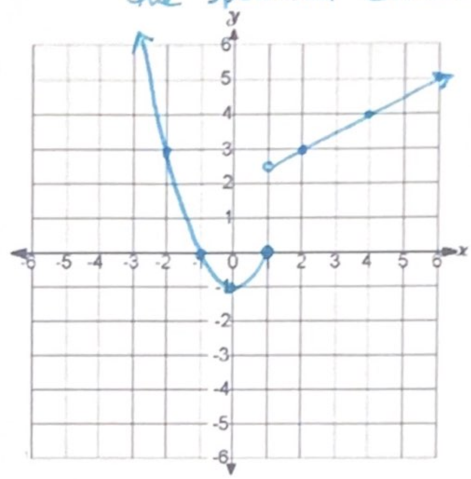
* graph each equation on the specified domain!

No calculator!

5. $f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ \frac{1}{2}x + 2, & x > 1 \end{cases}$

closed circle → $x \le 1$ → quadratic equation down 1

open circle → $x > 1$ → linear y int 2 slope 1/2



Top equation ← a. Find $f(-2) = 3$

Top equation ← b. Find $f(0) = -1$

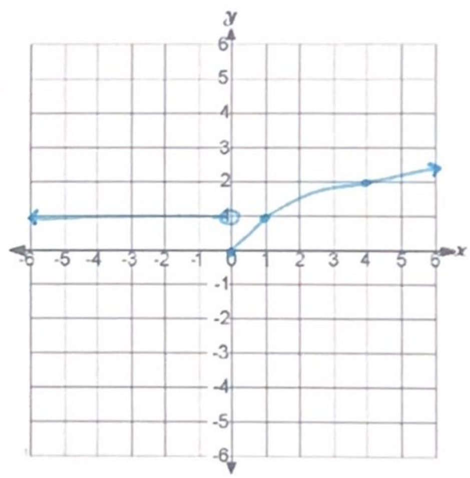
Top equation ← c. Find $f(1) = 0$

Bottom equation ← d. Find $f(s^2 + 2) = \frac{1}{2}(s^2+2)+2$

6. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$

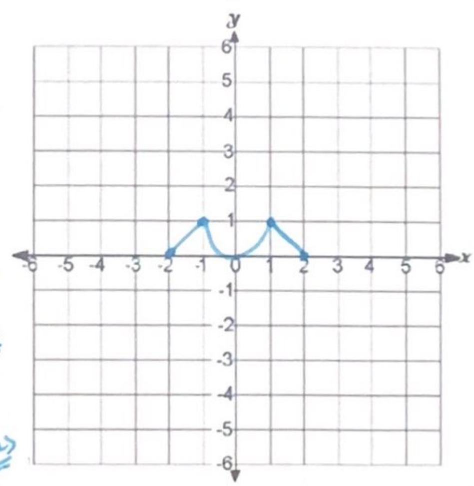
$x < 0$ → horizontal line at $y = 1$ open circle

$x \ge 0$ → Closed circle Parent square root graph



7. $f(x) = \begin{cases} x + 2, & -2 \le x \le -1 \\ 1 - \sqrt{1 - x^2}, & -1 < x < 1 \\ -x + 2, & 1 \le x \le 2 \end{cases}$

Bottom half of circle Centered at (0, 1) Radius of 1 open circles



Middle equation ← a. Find $f(0) = 0$

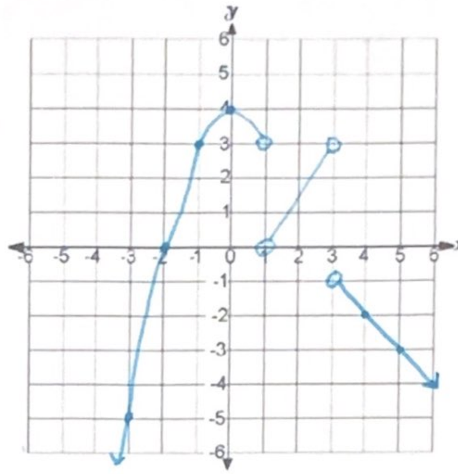
Top equation ← b. Find $f(-1.5) = .5$

Bottom equation ← c. Find $f(1.5) = .5$

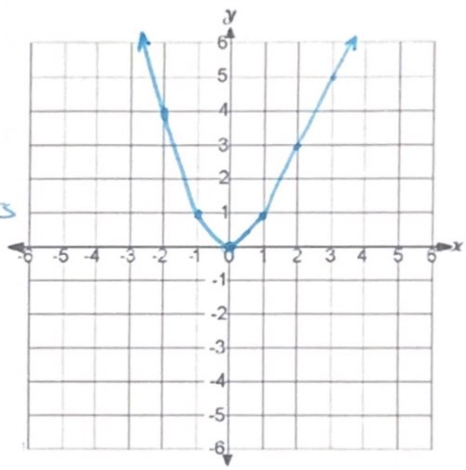
Not defined in the domain ← d. Find $f(3)$ Undefined

8. $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x - \frac{3}{2}, & 1 < x < 3 \\ -x + 2, & x > 3 \end{cases}$ → upside down Parabola up 4

open circle



9. $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$ → Parent quadratic → parent cubic



Domain: $\mathbb{R}$ ← symbol for All Real Numbers

Range: $[0, \infty)$

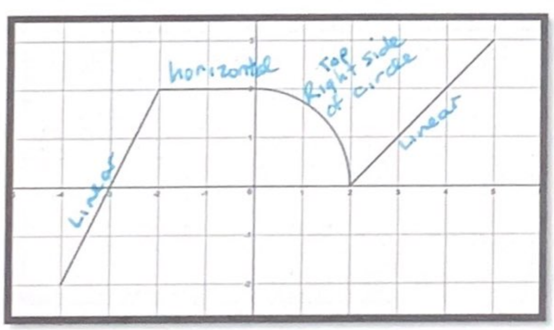
Middle Equation ← a. Find $f(1) = 1$

Top Equation ← b. Find $f(-2) = 4$

Top Equation ← c. Find $f(-3) = 9$

Write a formula for the following graphs:

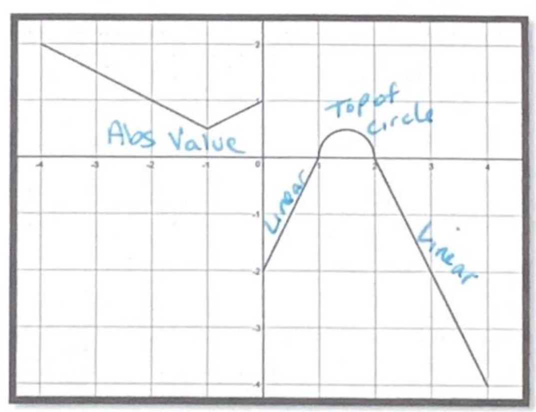
10.



4 pieces!

$$f(x) = \begin{cases} 2x + 6, & x \le -2 \\ 2, & -2 < x < 0 \\ \sqrt{4 - x^2}, & 0 \le x \le 2 \\ x - 2, & x > 2 \end{cases}$$

11.

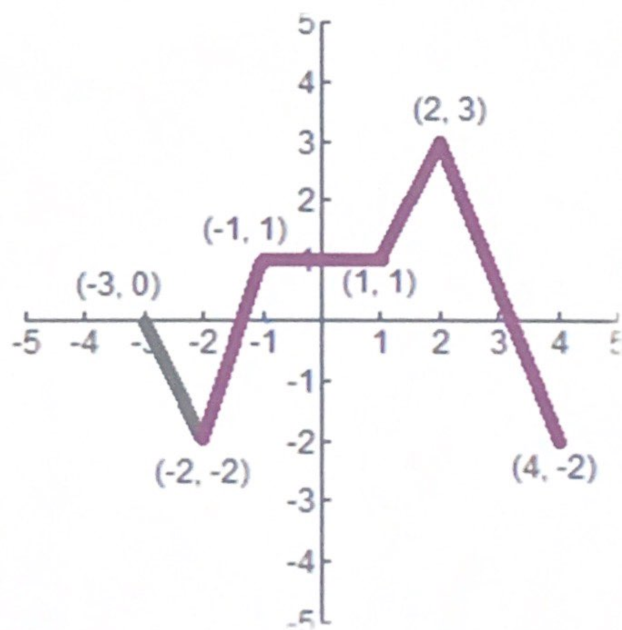


4 pieces!

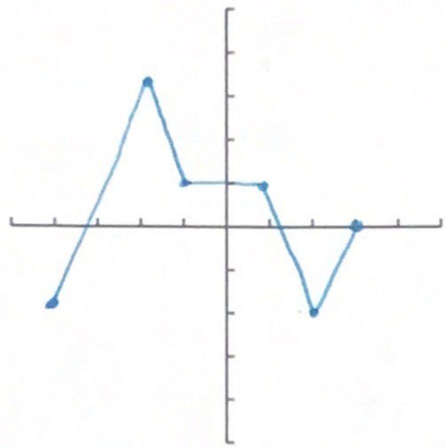
$$f(x) = \begin{cases} \frac{1}{2}|x+1| + \frac{1}{2}, & x < 0 \\ 2x - 2, & 0 \le x \le 1 \\ \sqrt{1/4 - (x - 3/2)^2}, & 1 < x < 2 \\ -2x + 4, & x \ge 2 \end{cases}$$

Transformation Rules for Functions		
Function Notation	Type of Transformation	Change to Coordinate Point
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) – d	Vertical translation down d units	(x, y) → (x, y – d)
f(x + c)	Horizontal translation left c units	(x, y) → (x – c, y)
f(x – c)	Horizontal translation right c units	(x, y) → (x + c, y)
–f(x)	Reflection over x-axis	(x, y) → (x, –y)
f(–x)	Reflection over y-axis	(x, y) → (–x, y)
af(x)	Vertical stretch for |a|> 1	(x, y) → (x, ay)
	Vertical compression for 0 < |a| < 1	
f(bx)	Horizontal compression for |b| > 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for 0 < |b| < 1	

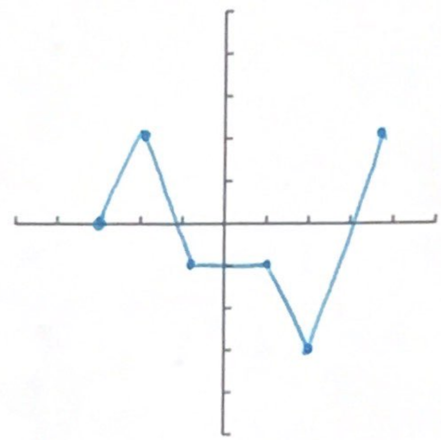
Examples: Let $f(x)$ be the function shown. Draw a graph and write a description of the transformation



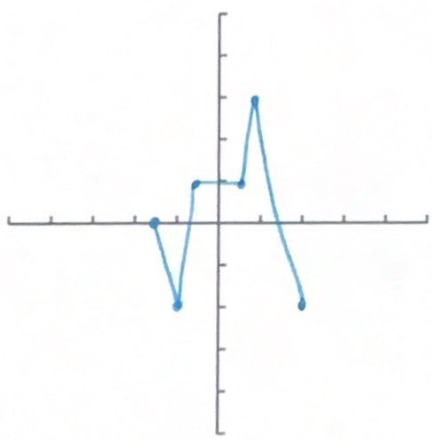
12. $y = f(-x)$ Y axis Reflection



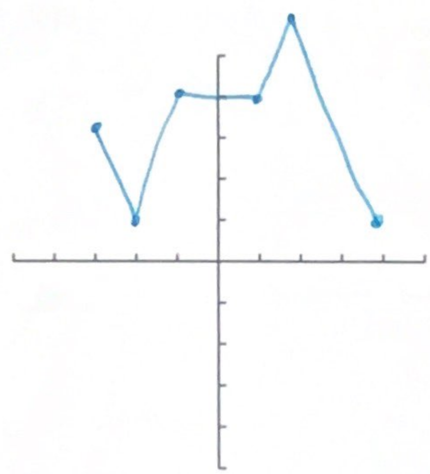
13. $y = -f(x)$ X axis Reflection



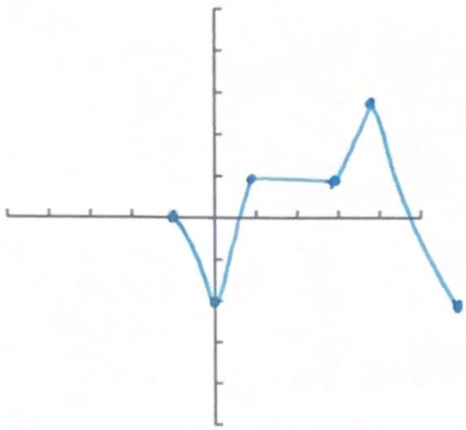
14. $y = f(2x)$ Horizontal Compression



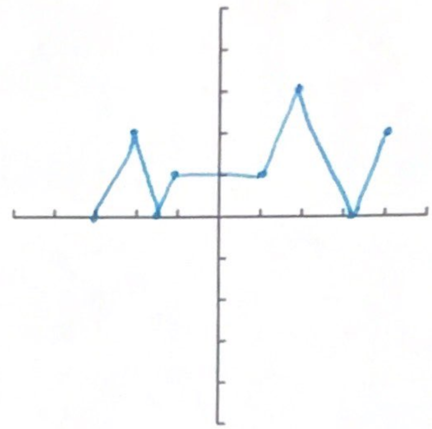
15. $y = f(x) + 3$ up 3



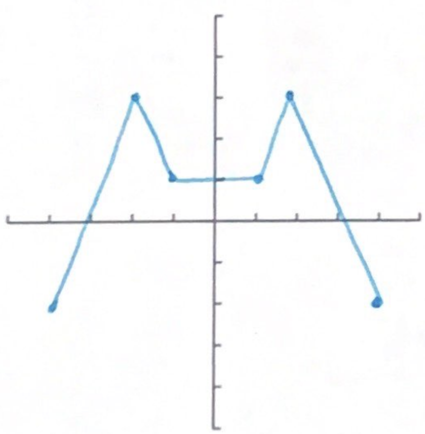
16. $y = f(x-2)$ Right 2



17. $y = |f(x)|$ → Y values turn positive if Negative



18. $y = f(|x|)$ Negative x values Mimic positive x values



19. $y = |f(x)| - 1$ Same as # 17 down 1