

Integration Rules Mixed Practice

Evaluate the indefinite and definite integrals using a u-substitution or integration by parts.

1. $\int \frac{x^2}{\sqrt{x^3+3}} dx$ $u = x^3+3$
 $du = 3x^2 dx$
 $dx = \frac{du}{3x^2}$

$\int \frac{x^2}{u^{1/2}} \frac{du}{3x^2} = \frac{1}{3} \int u^{-1/2} du$
 $\frac{1}{3} \cdot u^{1/2} \cdot \frac{2}{1} = \frac{2}{3} u^{1/2} + C = \frac{2\sqrt{x^3+3}}{3} + C$

2. $\int_0^{\pi} \sin^2 x \cos x dx$ $u = \sin x$
 $du = \cos x dx$
 $dx = \frac{du}{\cos x}$

$\int_0^{\pi} u^2 \cos x \frac{du}{\cos x} = \int_0^{\pi} u^2 du = \frac{u^3}{3} \Big|_0^{\pi} = \frac{0^3}{3} - \frac{1^3}{3} = -\frac{1}{3}$
 $u_{\pi} = 0$
 $u_{\frac{\pi}{2}} = 1$

3. $\int x(1-3x^2)^4 dx$ $u = 1-3x^2$
 $du = -6x dx$
 $dx = \frac{du}{-6x}$

$\int x \cdot u^4 \frac{du}{-6x} = -\frac{1}{6} \int u^4 du = -\frac{1}{6} \cdot \frac{u^5}{5} + C = -\frac{(1-3x^2)^5}{30} + C$

4. $\int_0^1 x e^{-x^2} dx$ $u = -x^2$
 $du = -2x dx$
 $dx = \frac{du}{-2x}$

$\int_0^1 x e^u \frac{du}{-2x} = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u \Big|_0^1 = -\frac{1}{2} e^1 - (-\frac{1}{2} e^0) = -\frac{1}{2}e + \frac{1}{2}$
 $u_1 = -1$
 $u_0 = 0$

5. $\int \frac{x+3}{x^2+6x-5} dx$ $u = x^2+6x-5$
 $du = 2x+6 dx$
 $dx = \frac{du}{2x+6}$ or $\frac{du}{2(x+3)}$

$\int \frac{x+3}{u} \frac{du}{2(x+3)} = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C = \frac{\ln|x^2+6x-5|}{2} + C$

6. $\int (x^2-1)e^x dx$

u	dv
x^2-1	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x

$(x^2-1)e^x - 2xe^x + 2e^x + C$

7. $\int x^2 \sqrt{x^3+3} dx$ $u = x^3+3$
 $du = 3x^2 dx$
 $dx = \frac{du}{3x^2}$

$\int x^2 \sqrt{u} \frac{du}{3x^2} = \frac{1}{3} \int u^{1/2} du = \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2\sqrt{x^3+3}^3}{9} + C$

8. $\int x^2 \sin 2x dx$

u	dv
x^2	$\sin 2x$
$-2x$	$-\frac{\cos 2x}{2}$
$+2$	$-\frac{\sin 2x}{4}$
-0	$\frac{\cos 2x}{8}$

$-\frac{x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C$

9. $\int x^4 e^{-x} dx$

u	dv
x^4	e^{-x}
$-4x^3$	$-e^{-x}$
$+12x^2$	e^{-x}
$-24x$	$-e^{-x}$
$+24$	e^{-x}
-0	$-e^{-x}$

$$\frac{-x^4}{e^x} - \frac{4x^3}{e^x} - \frac{12x^2}{e^x}$$

$$-\frac{24x}{e^x} - \frac{24}{e^x} + C$$

10. $\int_0^1 2x e^{2x} dx$

u	dv
$2x$	e^{2x}
-2	$\frac{e^{2x}}{2}$
$+0$	$\frac{e^{2x}}{4^2}$

$$x e^{2x} - \frac{e^{2x}}{2} \Big|_0^1$$

$$(1e^2 - \frac{e^2}{2}) - (0 - \frac{e^0}{2})$$

$$e^2 - \frac{e^2}{2} + \frac{1}{2}$$

11. $\int x \sin(3x^2) dx$

$$\int x \sin u \frac{du}{6x}$$

$$\frac{1}{6} \int \sin u$$

$$\frac{1}{6} \cdot -\cos u + C = -\frac{\cos(3x^2)}{6} + C$$

$$u = 3x^2$$

$$du = 6x dx$$

$$dx = \frac{du}{6x}$$

12. $\int x^3 e^x dx$

u	dv
x^3	e^x
$-3x^2$	e^x
$+6x$	e^x
-6	e^x
$+0$	e^x

$$x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$$

13. $\int \frac{\sin x}{\sqrt{1-\cos x}} dx$

$$\int \frac{\sin x du}{\sqrt{u} \sin x}$$

$$\int u^{-1/2} du$$

$$2u^{1/2} + C = 2\sqrt{1-\cos x} + C$$

$$u = 1 - \cos x$$

$$du = \sin x dx$$

$$dx = \frac{du}{\sin x}$$

14. $\int x^2 e^{x^3} dx$

$$\int x^2 e^u \frac{du}{3x^2}$$

$$\frac{1}{3} e^u + C$$

$$\frac{e^{x^3}}{3} + C$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$dx = \frac{du}{3x^2}$$

15. $\int \frac{\cos x}{\sqrt{\sin(x)}} dx$

$$\int \frac{\cos x du}{\sqrt{u} \cos x}$$

$$\int u^{-1/2}$$

$$2u^{1/2} + C = 2\sqrt{\sin x} + C$$

$$u = \sin x$$

$$du = \cos x dx$$

$$dx = \frac{du}{\cos x}$$

16. $\int x \ln(3x) dx$

$$uv - \int v du$$

$$\ln(3x) \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$\frac{x^2 \ln(3x)}{2} - \frac{1}{2} \int x dx$$

$$\frac{x^2 \ln(3x)}{2} - \frac{x^2}{4} + C$$

$$u = \ln(3x) \quad dv = x$$

$$du = \frac{1}{3x} \cdot 3 \quad v = \frac{x^2}{2}$$

$$du = \frac{1}{x} dx$$

17. $\int_0^1 x^2 e^x dx$

u	dv
x^2	e^x
$-2x$	e^x
$+2$	e^x
0	e^x

$x^2 e^x - 2x e^x + 2e^x \Big|_0^1$
 $(1e^1 - 2(1)e^1 + 2e^1) - (0 - 0 + 2e^0)$
 $1e - 2e + 2e - 2$

$e - 2$

18. $\int (x^2 - 1)e^x dx$

u	dv
$x^2 - 1$	e^x
$-2x$	e^x
$+2$	e^x
0	e^x

$(x^2 - 1)e^x - 2x e^x + 2e^x + C$

19. $\int_1^e 2x \ln x dx$

u.v - $\int v du$
 $\ln x \cdot x^2 - \int x^2 \cdot \frac{1}{x} dx$
 $x^2 \ln x - \int x dx$

$u = \ln x$ $dv = 2x$
 $du = \frac{1}{x} dx$ $v = \frac{2x^2}{2}$
 $v = x^2$

$x^2 \ln x - \frac{x^2}{2} \Big|_1^e = (e^2 \ln e - \frac{e^2}{2}) - (\frac{1^2 \ln 1}{2} - \frac{1}{2})$

$e^2 - \frac{e^2}{2} + \frac{1}{2}$

20. $\int x \cos x dx$

u	dv
x	$\cos x$
-1	$\sin x$
$+0$	$-\cos x$

$x \sin x + \cos x + C$

21. $\int \cot^4 x \csc^2 x dx$

$\int u^4 \csc^2 x \frac{du}{-\csc^2 x}$
 $-\int u^4 du = -\frac{u^5}{5} + C$

$u = \cot x$
 $du = -\csc^2 x dx$
 $dx = \frac{du}{-\csc^2 x}$

$-\frac{\cot^5 x}{5} + C$

22. $\int x e^{x^2} dx$

$\int x e^u \frac{du}{2x}$

$u = x^2$
 $du = 2x dx$
 $dx = \frac{du}{2x}$

$\frac{1}{2} e^u + C$
 $\frac{e^{x^2}}{2} + C$

23. $\int_3^6 \frac{x}{3\sqrt{x^2 - 8}} dx$

$\int_3^6 \frac{x}{3\sqrt{u}} \frac{du}{2x}$

$u = x^2 - 8$
 $du = 2x dx$
 $dx = \frac{du}{2x}$

$\frac{1}{6} \int_3^6 u^{-1/2} du$

$u_6 = 28$
 $u_3 = 1$

$\frac{1}{3} \cdot \frac{2}{3} u^{1/2} = \frac{\sqrt{u}}{3} \Big|_1^{28}$

$\frac{\sqrt{28}}{3} - \frac{\sqrt{1}}{3} = \frac{2\sqrt{7} - 1}{3}$

24. $\int_0^3 \frac{1}{\sqrt{1+x}} dx$

$\int_0^3 (1+x)^{-1/2} dx$
 $2(1+x)^{1/2} \Big|_0^3$

$2\sqrt{1+3} - 2\sqrt{1+0}$

$2\sqrt{4} - 2\sqrt{1}$

$4 - 2 = 2$