HOYA Block – Mini Review

Non - Calculator Section

1. Suppose $f(1) = -5, f'(1) = 4,$ and $f''(1) = 2$. Use linearization to approximate $f(0.75)$. Is the approximation an underestimate or overestimate? Justify your reasoning.

$y + 5 = 4(x-1)$

$L(x) = 4x - 9$

$L(3/4) = 4(3/4) - 9$

$= 3 - 9$

$= -6$

Since $f''(1) = 2$ $f(x)$ is conc up at $x = 1$ ∴ the tan line would be under the function creating an underestimate

2. Find the absolute extreme value(s) for $g(x) = e^{3x-x^3}$ on the closed interval $[0,2]$.

$g(x)$ is cont on $[0,2]$

$g(x)$ is diff on $(0,2)$

$g'(x) = (3 - 3x^2)e^{3x-x^3} = 0$

$3 = 3x^2$

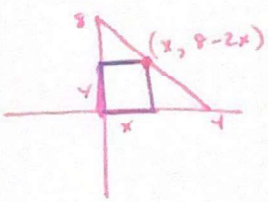
$x = \pm 1$

$g(0) = 1$

$g(1) = e^2$ ← Abs Max

$g(2) = e^{-2}$ ← Abs Min

3. Find the dimensions of the largest rectangle that can be inscribed within the region bounded by the line $y = 8 - 2x$ within he first quadrant.



$A = x \cdot y$

$A = x(8 - 2x)$

$A = 8x - 2x^2$

$A' = 8 - 4x = 0$

$4x = 8$

$x = 2$

+ + −

2 ← max

$y = 8 - 2x$

$= 8 - 2(2)$

$= 8 - 4$

$= 4$

2 units by 4 units

4. How close does the curve $y = \ln(x) - x$ come to the point $(2,0)$

$(2, 0)$

$(x, \ln x - x)$

$d = \sqrt{(x-2)^2 + (\ln x - x)^2}$

$d' = 2(x-2) + 2(\ln x - x)(\frac{1}{x} - 1) = 0$

$2x - 4 + (\frac{2}{x} - 2)(\ln x - x) = 0$

$x = 1.585$

− + +

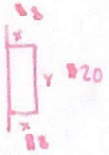
1.585 ← Min

$d = \sqrt{(1.585-2)^2 + (\ln(1.585) - 1.585)^2}$

$= 1.1986$

Calculator Section

5. An elementary school is building a fence around a rectangular playground. The playground is to border the school building and the side along the building does not need a fence. The side of the fence parallel to the building faces the road and will be built of wrought iron which cost 20 $/ft while the 2 sides perpendicular to the building will be built of a cheaper material that cost 8 $/ft. What is the largest area that can be fenced for a cost of $ 4000?



$8(2x) + 20(y) = 4000$

$16x + 20y = 4000$

$16x = 4000 - 20y$

$x = 250 - \frac{5}{4}y$

$A = x \cdot y$

$A = (250 - \frac{5}{4}y)y$

$A = 250y - \frac{5}{4}y^2$

$A' = 250 - \frac{5}{2}y = 0$

$\frac{5}{2}y = 250$

$y = 250 \cdot \frac{2}{5} = 100$

$x = 250 - \frac{5}{4}(100)$

$x = 250 - 125$

$x = 125$

$A = x \cdot y = 100 \cdot 125 = 12500\ ft^2$

6. An ambulance is racing north toward an intersection at 65 mph. A passenger car is traveling east away from the intersection at 40 mph. Find the rate of change of the distance between the ambulance and the passenger car when the ambulance is 0.4 miles from the intersection and the car is 0.3 miles from the intersection.

K: $\frac{dA}{dt} = -65$ mph

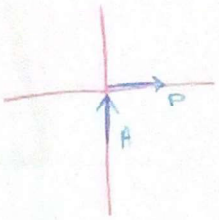
$\frac{dP}{dt} = 40$ mph

F: $\frac{dD}{dt}$

W: $A = .4$

$P = .3$

$D = .5$



$A^2 + P^2 = D^2$

$A\frac{dA}{dt} + P\frac{dP}{dt} = D\frac{dD}{dt}$

$.4(-65) + .3(40) = .5\frac{dD}{dt}$

$-14 = .5\frac{dD}{dt}$

$\frac{dD}{dt} = -28$ mph

7. Use linearization to approximate $\sqrt{16.443}$.

$x = 16 \quad f(x) = \sqrt{x}$

$f(16) = \sqrt{16} = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(16) = \frac{1}{2 \cdot 4} = \frac{1}{8}$

$y - 4 = \frac{1}{8}(x - 16)$

$L(x) = \frac{1}{8}x + 2$

$L(16.443) = \frac{1}{8}(16.443) + 2$

$= 4.055$

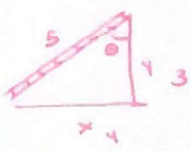
8. A 5-foot ladder is leaning against a house when its base starts to slide away. At what rate is the angle between the ladder and the wall changing when the base of the ladder is 4 feet from the wall if the ladder is sliding down the wall at a rate of 3 feet per second?

K: $\frac{dx}{dt} = -3$ ft/sec

F: $\frac{d\theta}{dt}$

W: $x = 4$

$y = 3$



$\cos\theta = \frac{x}{5}$

$-5\sin\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

$-5(\frac{4}{5})\frac{d\theta}{dt} = -3$

$-4\frac{d\theta}{dt} = -3$

$\frac{d\theta}{dt} = 3/4$ rad/sec