Honors Calculus Prerequisite Packet

Welcome to Honors Calculus! This is a semester-long course intended to prepare you for AP Calculus or Calculus I in college. There are certain math skills that you have learned in your previous courses that are necessary to be successful in this class. Some notes are provided, but you should also use other resources on the Internet to help familiarize yourself with these topics. Show all work & highlight your answers! You can do all of your work on separate paper, but be sure to attach it to this packet.

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. Find each of the following:

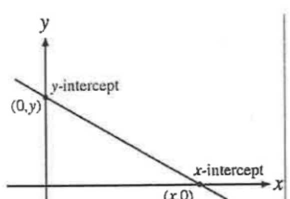
1. $f(2) =$ ________ **2.** $g(-3) =$ ________ **3.** $g(x+1) =$ ________ **4.** $g(f(x)) =$ ________

Let $f(x) = \sin(2x)$. Find the exact value of each of the following: ***(Hint –fill out your unit circle on p. 3 first.)***

5. $f\left(\frac{\pi}{4}\right) =$ ________ **6.** $f\left(\frac{2\pi}{3}\right) =$ ________

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

$x-$int. (*Let* $y = 0$)
$0 = x^2 - 2x - 3$
$0 = (x-3)(x+1)$
$x = -1$ *or* $x = 3$
$x-$intercepts $(-1,0)$ *and* $(3,0)$

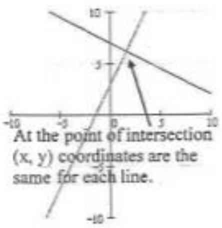
$y-$int. (*Let* $x = 0$)
$y = 0^2 - 2(0) - 3$
$y = -3$
$y-$intercept $(0,-3)$

Find the x- and y-intercepts for the following:

7. y = 2x – 5 **8. y = x² + x – 2** **9.** $y = \sqrt{x(16 - x^2)}$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP
Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2[nd] Trace) and hit INTERSECT.

Example: Find all points of intersection of $x^2 - y = 3$, $x - y = 1$

ELIMINATION METHOD
Subtract to eliminate y
$x^2 - x = 2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2$ or $x = -1$
Plug in $x = 2$ and $x = -1$ to find y
Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD
Solve one equation for one variable.
$y = x^2 - 3$
$y = x - 1$
Therefore by substitution $x^2 - 3 = x - 1$
$x^2 - x - 2 = 0$
From here it is the same as the other example

Find the points of intersection on the graph using the elimination or substitution method:

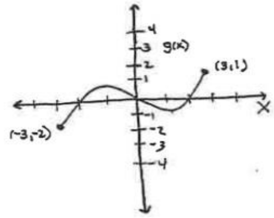
10. $x + y = 8$
$4x - y = 7$

11. $x = 3 - y^2$
$y = x - 1$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).
The domain is the set of inputs of the function. Input values run along the horizontal axis. The furthest-left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3. So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x) = \sqrt{4 - x^2}$
Write answers in interval notation.

DOMAIN
For $f(x)$ to be defined $4 - x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2, 2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0, \infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

12. $f(x) = x^2 - 5$

13. $f(x) = -\sqrt{x + 3}$

14. $f(x) = \frac{2}{x-1}$

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: $y = c$ (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

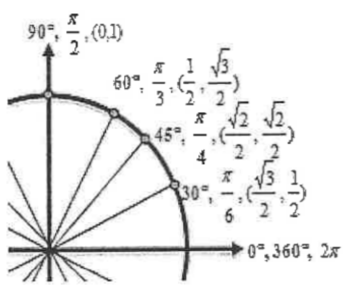
$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

15. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

16. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

17. Use point-slope form to write the equation of the line with a slope of $\frac{2}{3}$ that goes through the point (2, 5).

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\cos\frac{\pi}{2} = 0$ $\tan\frac{\pi}{2} = und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

18. $\sin \pi$

19. $\cos\frac{3\pi}{2}$

20. $\sin(-\frac{\pi}{2})$

21. $\tan\frac{3\pi}{4}$

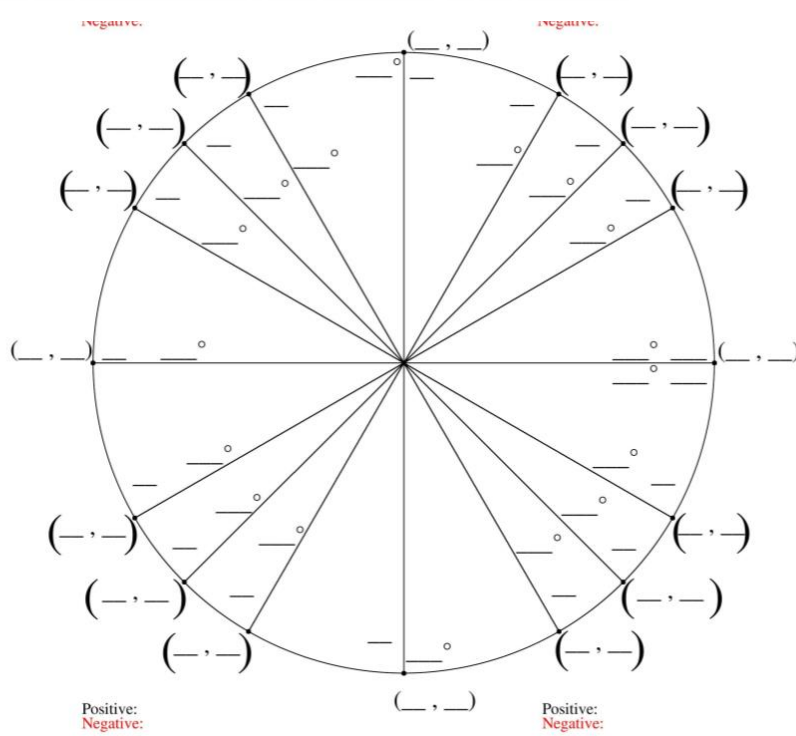
22. $\cos\frac{10\pi}{3}$

23. $\sec\frac{\pi}{6}$

24. $\csc\frac{7\pi}{6}$

25. $\tan\frac{\pi}{3}$

26. $\cot 0$



TRANSFORMATION OF FUNCTIONS

$h(x)=f(x)+c$	Vertical shift c units up	$h(x)=f(x-c)$	Horizontal shift c units right
$h(x)=f(x)-c$	Vertical shift c units down	$h(x)=f(x+c)$	Horizontal shift c units left
$h(x)=-f(x)$	Reflection over the x-axis		

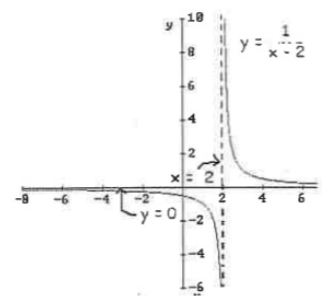
27. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How does $g(x)$ differ from $f(x)$?

28. Write an equation for the function that has the shape of $f(x) = x^3$ but shifted 6 units to the left and reflected over the x-axis.

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y=\frac{1}{x-2}$
Since when $x=2$ the function is in the form 1/0 then the vertical line $x=2$ is a vertical asymptote of the function.



HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.
Example: $y=\frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.
Exmaple: $y=\frac{2x^2+x-1}{3x^2+4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y=\frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)
Example: $y=\frac{2x^2+x-1}{3x-3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

State the vertical and horizontal asymptotes for each of the following:

29. $f(x)=\frac{1}{3x-2}$ **30.** $f(x)=\frac{4x^2}{x^2-4}$ **31.** $f(x)=\frac{2+x}{x^2(1-x)}$ **32.** $f(x)=\frac{x^3+x-1}{x^2-3x-4}$

*****Horizontal asymptotes are very important in finding limits, so memorize these rules*****

PROPERTIES OF EXPONENTS

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^{-1} = \frac{1}{x^1}$	$5^{-1} = \frac{1}{5}$
4	$(x^m)(x^n) = x^{m+n}$	$(x^2)(x^3) = x^{2+3} = x^5$
5	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^3}{x^2} = x^{3-2} = x^1$
6	$(x^m)^n = x^{(m)(n)}$	$(x^3)^2 = x^{(3)(2)} = x^6$
7	$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
8	$(\frac{x}{y})^n = \frac{x^n}{y^n}$	$(\frac{x}{y})^3 = \frac{x^3}{y^3}$
9	$x^{-n} = \frac{1}{x^n}$	$x^{-2} = \frac{1}{x^2}$

Simplify:

33. $-2x^4 \bullet 4x^3$

34. $\left(2^2 x^4 y^5 z\right)^3$

35. $\left(3x^2 x^{-2}\right)\left(\left((-2)^3 z^2\right)^2 z^5\right)^0$

36. $\frac{-6x^4 y^{-3}}{9x^3 x y^7}$

37. $\frac{\left(3x^2 y\right)^5}{9x^{10} y^6}$

EXPONENTIAL FUNCTIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$ Get a common base

$2^{2x+2} = 2^{-3x+2}$ Simplify

$2x + 2 = -3x + 2$ Set exponents equal

$x = 0$ Solve for x

Solve:

38. $3^{3x+5} = 9^{2x+1}$

39. $\left(\frac{1}{6}\right)^x = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828… or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Evaluate the following:

40. $\log_7 7$

41. $\log_8 1$

42. ln e

43. $\log_{16} 4$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\log_b x^y = y \log_b x$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following:

44. $log_2 2^5$

45. lne^3

46. e^{ln3}

47. $2^{log_2 3}$

RADICALS & RATIONAL EXPONENTS

Product Property of Square Roots

$\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$

Examples

$\sqrt{4 \bullet 9} = \sqrt{36} = 6$ or $\sqrt{4 \bullet 9} = \sqrt{4} \bullet \sqrt{9} = 2 \bullet 3 = 6$

Simplify Square Roots

Simplify: $\sqrt{80} = \sqrt{2 \bullet 2 \bullet 2 \bullet 2 \bullet 5}$
$= \sqrt{2^2} \bullet \sqrt{2^2} \bullet \sqrt{5}$
$= 2 \bullet 2 \bullet \sqrt{5}$
$= 4\sqrt{5}$

$\sqrt{80} = \sqrt{16 \bullet 5}$
$= \sqrt{16} \bullet \sqrt{5}$
$= 4\sqrt{5}$

When using rational exponents, remember that the numerator is the power and the denominator is the root.

$\sqrt[n]{x}^{m} = x^{\frac{m}{n}}$ For example, $27^{\frac{2}{3}} = \sqrt[3]{27}^{2} = (3)^2 = 9$

Simplify. Write answer as a simplified radical.

48. $\sqrt{80}$

49. $\sqrt[3]{54x^2y^8}$

50. $25^{\frac{3}{2}}$

51. $\frac{\sqrt{5}}{\sqrt{15}}$

52. $8^{-\frac{1}{2}}$

DIVIDING FRACTIONS WITH A MONOMIAL DENOMINATOR

Divide each term in the numerator by the monomial in the denominator.

For example, $\frac{x^3+3x^2-2x+5}{x} = \frac{x^3}{x} + \frac{3x^2}{x} - \frac{2x}{x} + \frac{5}{x} = x^2 + 3x - 2 + \frac{5}{x}$

53. $\frac{5x^6-3x^4-4x^2+3}{x^2}$

54. $\frac{x^3-2x^2+4x+5}{\sqrt{x}}$ **1st change the radical to a rational exponent**

FACTORING

54. $x^2 - 4$

56. $x^2 + 7x + 12$

57. $3x^2 + 10x + 3$

58. $x^4 - 7x^2 + 12$

59. $100a^2 - 36b^2$

60. $a^3 - 8$

61. $3x^3 + 12x^2 - 2x - 8$

62. $-5x^2y + 10xy^3$