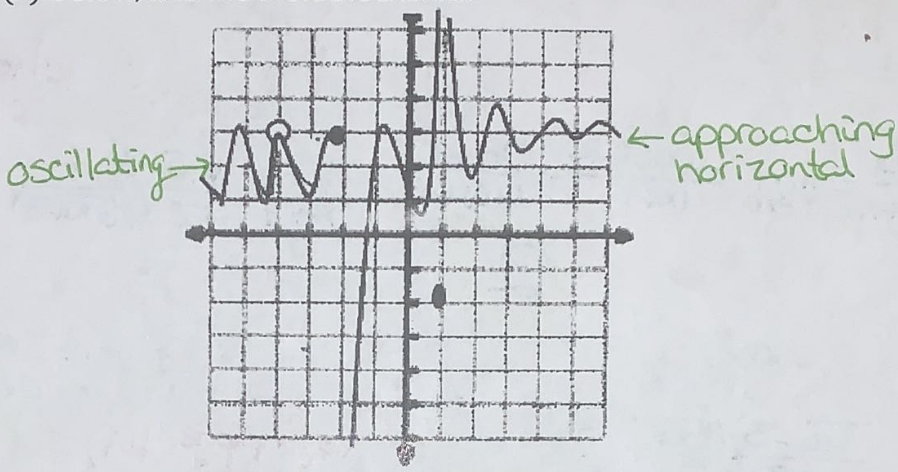
HONORS CALCULUS – FINAL EXAM REVIEW

1. Find the domain of the function:
$f(x) = \ln(3x + 6) - 3$ you can't take the log of 0 or a -#
$3x+6>0$
$x>-2$ $(-2,\infty)$

2. Find the domain of the function:
$f(x) = \sqrt{x-2} + 3$ you can't take an even root of a -#
$x-2\geq 0$
$x\geq 2$ $[2,\infty)$

3. Using the graph $g(x)$ below, find the indicated limits.



a. $\lim_{x\to-\infty} g(x)$ DNE

b. $\lim_{x\to-2^-} g(x)$ 3

c. $\lim_{x\to-2^+} g(x)$ $-\infty$

d. $\lim_{x\to-2} g(x)$ DNE

e. $\lim_{x\to-4} g(x)$ 3

f. $\lim_{x\to1} g(x)$ ∞

g. $\lim_{x\to\infty} g(x)$ 3

h. $g(1)$ -2

4. Sketch the graph with the following limits.

a. $\lim_{x\to-\infty} f(x) = \infty$ End behavior

b. $\lim_{x\to-2} f(x) = 4$ open pt.

c. $\lim_{x\to1^-} f(x) = 2$

d. $\lim_{x\to1^+} f(x) = 0$

e. $\lim_{x\to4^-} f(x) = 4$

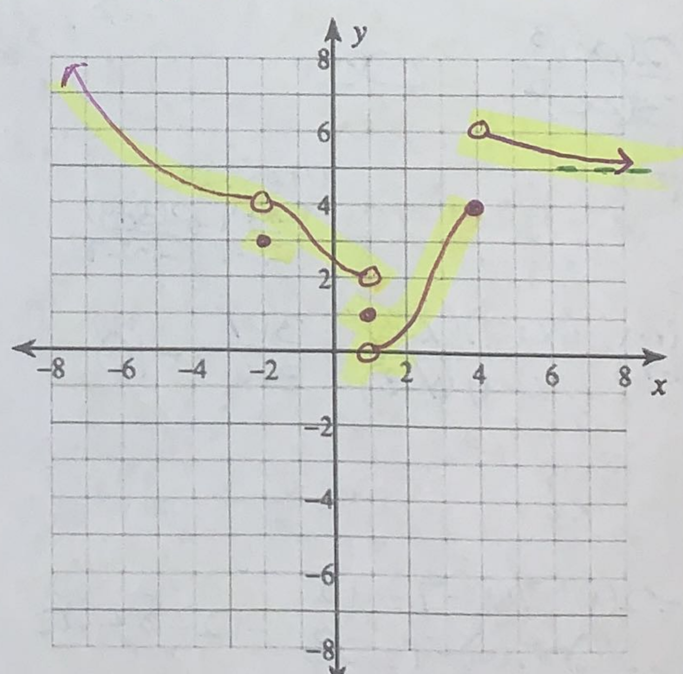
f. $\lim_{x\to4^+} f(x) = 6$

g. $\lim_{x\to\infty} f(x) = 5$ HA

h. $f(-2) = 3$ pt

i. $f(1) = 1$ pt

j. $f(4) = 4$ pt



Calculate each of the following limits:

5. $\lim_{x\to -1} -\dfrac{x-4}{x^2-6x+8} = -\dfrac{\cancel{x-4}}{\cancel{(x-4)}(x-2)}$

$\lim_{x\to -1} \dfrac{-1}{x-2} = \dfrac{-1}{-1-2} = \dfrac{1}{3}$

6. $\lim_{x\to 2^-} -\dfrac{5}{x-2}$ VA @ $x=2$

from left

x	y
1	5
1.9	50
1.99	500

∞

L'hopital's rule

7. $\lim_{x\to 0} \dfrac{\sin 2x}{2x^2+x} = \dfrac{0}{0}$

$\dfrac{d/dx \sin 2x}{d/dx\, 2x^2+x} = \lim_{x\to 0} \dfrac{2\cos(2x)}{4x+1} = \dfrac{2\cos 0}{4(0)+1}$

$=2$

8. $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9} = \dfrac{0}{0}$ L'hopital's or creative factoring

$\lim_{x\to 9} \dfrac{\cancel{\sqrt{x}-3}\ 1}{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}$

L'hopital's or Rationalize numerator

9. $\lim_{x\to 3} \dfrac{\sqrt{x+6}-3}{x-3} = \dfrac{0}{0}$

$\lim_{x\to 3} \dfrac{(\sqrt{x+6}-3)(\sqrt{x+6}+3)}{x-3\ (\sqrt{x+6}+3)}$

$\lim_{x\to 3} \dfrac{\cancel{x+6-9}\ 1}{\cancel{(x-3)}(\sqrt{x+6}+3} = \dfrac{1}{\sqrt{3+6}+3} = 1/6$

10. $\lim_{x\to\infty} \dfrac{x+1}{2x^2+2x+1}$ ← degree 1 ← degree 2 HA: $y=0$

Asymptote (HA) rules

0

look at graph

11. $\lim_{x\to\infty} \dfrac{2x^3}{3x^2-4}$ degree 3 degree 2 no HA SA (∞ or $-\infty$)

$\dfrac{2(\infty)^3}{3(\infty)^2} = \infty$

12. $\lim_{x\to -\infty} \dfrac{-4x^2}{7-2x^2}$ deg 2 deg 2 HA: $y = \dfrac{-4}{-2}$

2

Factor or L'hopital's

13. $\lim_{x\to 3} \dfrac{x^2+4x-21}{x^2-7x+12} = \dfrac{0}{0}$ L'hopital's $\dfrac{2x+4}{2x-7} = \dfrac{2(3)+4)}{2(3)-7}$

$\lim_{x\to 3} \dfrac{(x+7)\cancel{(x-3)}}{(x-4)\cancel{(x-3)}} = \dfrac{3+7}{3-4} = -10$

14. $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$

$\lim_{x\to 1} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 1+1 = 2$

15. Verify the conditions of the Intermediate Value Theorem and find the guaranteed c value such that $f(c) = 8$ for the function f (x) = $2x^2 - 7x - 7$ on the interval [1, 6].

$f(1) = 2(1)^2 - 7(1) - 7 = -12$

$f(6) = 2(6)^2 - 7(6) - 7 = 23$

$-12 < 8 < 23$

$8 = 2x^2 - 7x - 7$

$0 = 2x^2 - 7x - 15$

$0 = (2x+3)(x-5)$

$x \neq -3/2$ ← not in the interval $[1,6]$

$x = 5$

16. Find the value of a such that f(x) is continuous at all points. $f(x) = \begin{cases} 2ax+2, & x \le -3 \\ 3ax-4, & x > -3 \end{cases}$

$2ax+2 = 3ax-4$

$2a(-3)+2 = 3a(-3)-4$

$-6a+2 = -9a-4$

$3a = -6$

$a = -2$

Determine the asymptotes (VA, HA, SA) of each function:

17. $y = \frac{3x^2+4}{x^2-5x+6} = \frac{3x^2+4}{(x-3)(x-2)}$

VA: $x=3$ & $x=2$

HA: $y=3$

SA: none

18. $y = \frac{4x^3-4x^2+x-1}{x^2-1}$ $\frac{(4x^2-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$ Hole at $x=-1$

VA: $x=1$

HA: none

SA: $y=4x-4$

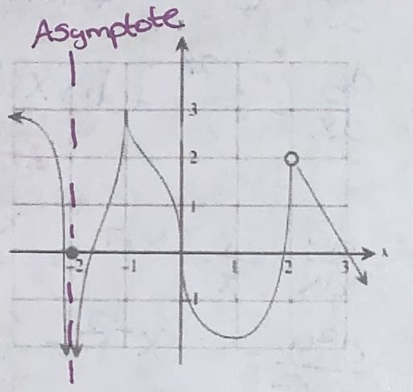
$x^2+0x-1 \overline{)\,4x^3-4x^2+x-1}$ quotient $4x-4$

$-4x^3-0x+4x$

$-4x+5x-1$

$+4x+0x+4$

19. The graph of f(x) is given below. Identify all the points where f(x) is not differentiable and give reasons why not.



$x=-2$ infinite discontinuity

$x=-1$ cusp

$x=0$ vertical tangent

$x=2$ removable

20. Find the Equation of the tangent line:
$f(x) = 4x^2 - 5x + 2$ at $x = 3$

$f(3) = 4(3)^2 - 5(3) + 2 = 23$ $(\overset{x}{3}, \overset{y}{23})$

$f'(x) = 8x - 5$

$f'(3) = 19$

$m = 19$

$y - 23 = 19(x-3)$

or $y = 19x - 34$

21. Find the Equation of the normal line: ← means ⊥
$f(x) = 4x^2 - 5x + 2$ at $x = 3$

$(3, 23)$ $\perp m = -\frac{1}{19}$

$y - 23 = -\frac{1}{19}(x-3)$

or $y = -\frac{1}{19}x + \frac{440}{19}$

22. Find the Derivative: $f(x) = \sqrt{2x+3}$

Chain Rule

$f(x) = (2x+3)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}(2x+3)^{-1/2} \cdot \cancel{2}$

$f'(x) = \frac{1}{\sqrt{2x+3}}$

23. Find the Derivative: $f(x) = \frac{x^2}{x^2+3}$

Quotient Rule

$f'(x) = \frac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2} = \frac{2x^3+6x-2x^3}{(x^2+3)^2}$

$f'(x) = \frac{6x}{(x^2+3)^2}$

24. $f(x) = (9-x^2)^{\frac{2}{3}}$, find $f'(x)$

Chain

$f'(x) = \frac{2}{3}(9-x^2)^{-1/3}(-2x)$

$f'(x) = \frac{-4x}{3\sqrt[3]{9-x^2}}$

25. Find $f'(x)$ of $f(x) = 2 - \frac{3}{x}$

Rewrite 1st

$f(x) = 2 - 3x^{-1}$

$f'(x) = 3x^{-2} = \frac{3}{x^2}$

26. Find the differential of $y = (2x-7)^3$

Chain

$y' = 3(2x-7)^2(2)$

$y' = 6(2x-7)^2$

27. If $y = \sqrt{\frac{1}{4x^2}}$, find $\frac{dy}{dx}$

Simplify 1st

$y = \frac{1}{2x}$ or $\frac{1}{2}x^{-1}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-2} = \frac{-1}{2x^2}$

Product rule

28. Find the derivative of $y = (x^2 + 1)e^{3x}$

$y' = (x^2+1)(3e^{3x}) + e^{3x}(2x)$

$y' = 3e^{3x}(x^2+1) + 2xe^{3x}$

can be simplified or rewritten other ways

29. If $f(x) = x \cdot 5^{3x}$ find $f'(x)$ Product

$f'(x) = x(5^{3x} \ln 5 \cdot 3) + 5^{3x}(1)$

$f'(x) = 3x\,5^{3x} \ln 5 + 5^{3x}$

or

$f'(x) = 5^{3x}(3x \ln 5 + 1)$

30. Find the differential of $y = 3^{x^2+3x}$

$y' = 3^{x^2+3x} \ln 3\,(2x+3)$

31. $y = e^{2x} \ln x$, find $\frac{dy}{dx}$ Product

$\frac{dy}{dx} = e^{2x} \cdot \frac{1}{x} + \ln x\,(2e^{2x})$

$\frac{dy}{dx} = \frac{e^{2x}}{x} + \ln x\,(2e^{2x})$

32. Find $\frac{dy}{dx}$ of $y = e^{\ln(5x^2)}$ Prop. of Logs

$y = 5x^2 \ln e$

$y = 5x^2$

$\frac{dy}{dx} = 10x$

33. If $f(x) = \ln x^8$, then $\frac{d^2y}{dx^2} =$ ← means 2nd derivative

$\frac{dy}{dx} = \frac{1}{x^8} \cdot 8x^7 = \frac{8}{x}$ or $8x^{-1}$

$\frac{d^2y}{dx^2} = -8x^{-2} = \frac{-8}{x^2}$

34. Find the first TWO Derivatives:
$y = 2(x^2 - 1)^3$

$y' = 6(x^2-1)^2(2x)$

$y' = 12x(x^2-1)^2$

Product + chain $y'' = 12x[2(x^2-1)^1(2x)] + (x^2-1)^2(12)$

$y'' = 48x^2(x^2-1) + 12(x^2-1)^2$

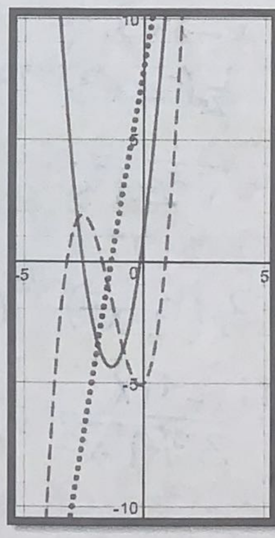
35. Find the $f', f'', and\ f'''$
$f(x) = x^3 + 2x^2 - 4x + 5$

$f'(x) = 3x^2 + 4x - 4$

$f''(x) = 6x + 4$

$f'''(x) = 6$

36. To the right, in no particular order, are the graphs of $f(x)$, $f'(x)$, and $f''(x)$. Decide which graph goes with each function.



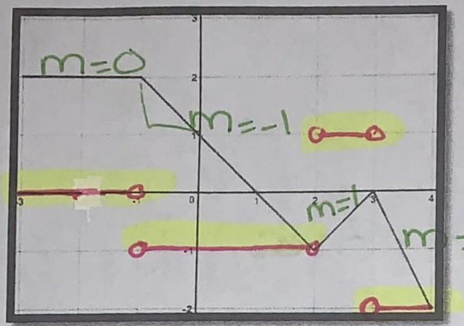
Dotted $f''(x)$

Dashed $f(x)$

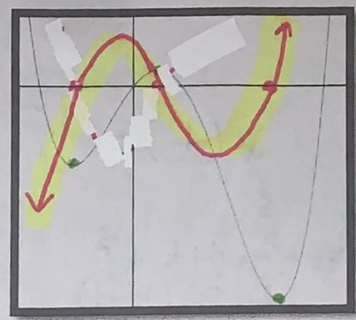
Solid $f'(x)$

For each of the following, sketch the graph of the derivative.

37.



38.



− 0 + 0 − 0 +

below on above on below on above

39. Find all relative extrema as well as the x-values at which they occur. State whether it is a minimum or a maximum. State where each function is increasing or decreasing. Sketch the function.

$f(x) = x^2 - 4x - 5$

$0=(x-5)(x+1)$

$x=5$ $x=-1$

$f'(x) = 2x-4$

$0 = 2(x-2)$

$x=2$

$f(2) = (2)^2 - 4(2) - 5$

Minimum x value: $x=2$

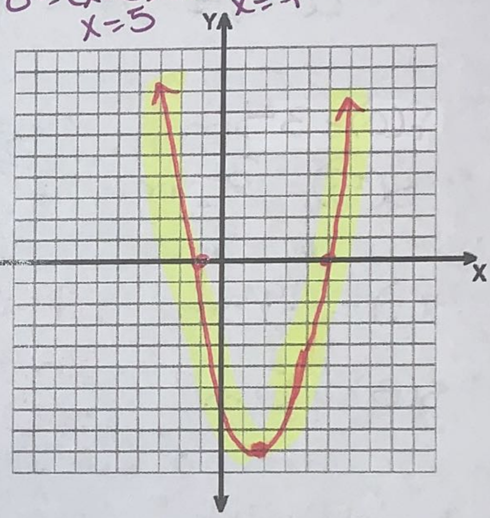
Minimum value: -9

Interval(s) of Increase: $(2,\infty)$

Interval(s) of decrease: $(-\infty,2)$

− − 0 + +

2



40. Find the absolute maximum and minimum value of each function over the indicated interval.

$f(x) = x^3 + x^2 - x + 1 \quad \left[-2, \frac{1}{2}\right]$

$f'(x) = 3x^2 + 2x - 1$

$0 = (3x-1)(x+1)$

$x = \frac{1}{3}$ $x=-1$

\+ − +

max −1 1/3 min

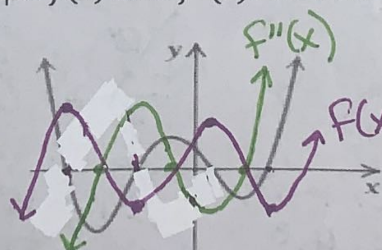
$f(-1) = 2$

max $(-1, 2)$

$f\left(\frac{1}{3}\right) = \frac{22}{27}$

min $\left(\frac{1}{3}, \frac{22}{27}\right)$

41. Given the graph of $f'(x)$, graph $f(x)$ and $f''(x)$. Be sure to label each graph.



above, bel, ab, bel, above

− 0 + 0 − 0 +

bel above bel above

42. The equation of a motion of a particle is: $s(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 5$, where s is in meters and t is in seconds. Find the following when $0 \le t \le 5$. Be sure to include units where appropriate.

a. Find the velocity function in terms of t

$v(t) = t^2 - 4t + 3$

b. When is the particle at rest?

$0 = t^2 - 4t + 3$
$0 = (t-3)(t-1)$

$t = 1$ sec & $t = 3$ sec

c. When is the particle moving right? Left?

+ 0 − 0 +
0 1 3 5

right: $[0, 1) \cup (3, 5]$
left: $(1, 3)$

d. What is the acceleration function in terms of t?

$a(t) = 2t - 4$

e. What is the particles displacement? end − beg.

$s(0) = 5$m
$s(5) = \frac{35}{3}$m

$\frac{35}{3} - 5 = \frac{20}{3}$m

f. What is the total distance traveled?

$s(1) = 19/3$m
$s(3) = 5$m

$s(1) - s(0) = 4/3$
$s(3) - s(1) = 4/3$
$s(5) - s(3) = 20/3$

add $\frac{28}{3}$m

g. When is the particle slowing down? Speeding Up?

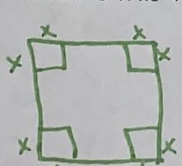
v: + | − − | + +
0 1 3 5

$a(t) =$
$2t - 4 = 0$
$t = 2$

a: − | − | + | +
0 1 2 3 5
dif same dif same

Slowing down: $(0, 1) \cup (2, 3)$
Speeding up: $(1, 2) \cup (3, 5)$

43. From a thin piece of cardboard 60 in. by 60 in., square corners are cut out so that the sides can be folded up to make an open box. What dimensions will yield a box of maximum volume? What is this volume?

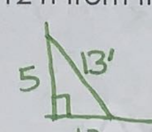


$L \& W = 60 - 2x$
$H = x$
$V = x(60 - 2x)^2$
$V = x(3600 - 240x + 4x^2)$
$V = 3600x - 240x^2 + 4x^3$
$V' = 3600 - 480x + 12x^2$
$0 = 12(300 - 40x + x^2)$
$0 = 12(30 - x)(10 - x)$
$x = 30 \quad x = 10$

+ − +
10 30
max

$H = 10$
$L + W = 60 - 2(10)$
$= 40$
40" x 40" x 10"
$V = 16{,}000 \text{ in}^3$

44. A pole 13 ft long leans against a vertical wall. If the lower end is moving away from the wall at the rate of 0.4 ft/sec, how fast is the upper end coming down when the lower end is 12 ft from the wall?



K: $dx/dt = 0.4$ ft/sec
F: dy/dt
W: $x = 12'$

$x^2 + y^2 = 13^2$
$12^2 + y^2 = 13^2$
$y = 5$

$x^2 + y^2 = 13^2$
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$
$2(12)(.4) + 2(5)\frac{dy}{dt} = 0$
$10\frac{dy}{dt} = 9.6$
$\frac{dy}{dt} = -\frac{24}{25}$ ft/sec or −.96 ft/sec

Differentiate implicitly to find $\frac{dy}{dx}$:

45. $2xy + 3 = 0$

product rule → $2x \cdot y$ (1st, 2nd)

$2x \cdot \frac{dy}{dx} + y \cdot 2 + 0 = 0$

$2x \frac{dx}{dy} = -2y$

$\frac{dy}{dx} = \frac{-2y}{2x} = \frac{-y}{x}$

46. $x^2 - y^2 = 1$

$2x - 2y\, dy/dx = 0$

$-2y\, dy/dx = -2x$

$dy/dx = \frac{-2x}{-2y}$

$dy/dx = \frac{x}{y}$

47. $2y^2 = \frac{5x-3}{5x+3}$ ← Quotient rule

$4y \frac{dy}{dx} = \frac{(5x+3)(5) - (5x-3)(5)}{(5x+3)^2}$

$4y \frac{dy}{dx} = \frac{\cancel{25x} + 15 - \cancel{25x} + 15}{(5x+3)^2}$

$\frac{dy}{dx} = \frac{\cancel{30}\ 15}{\cancel{4}2y(5x+3)^2} = \frac{15}{2y(5x+3)^2}$

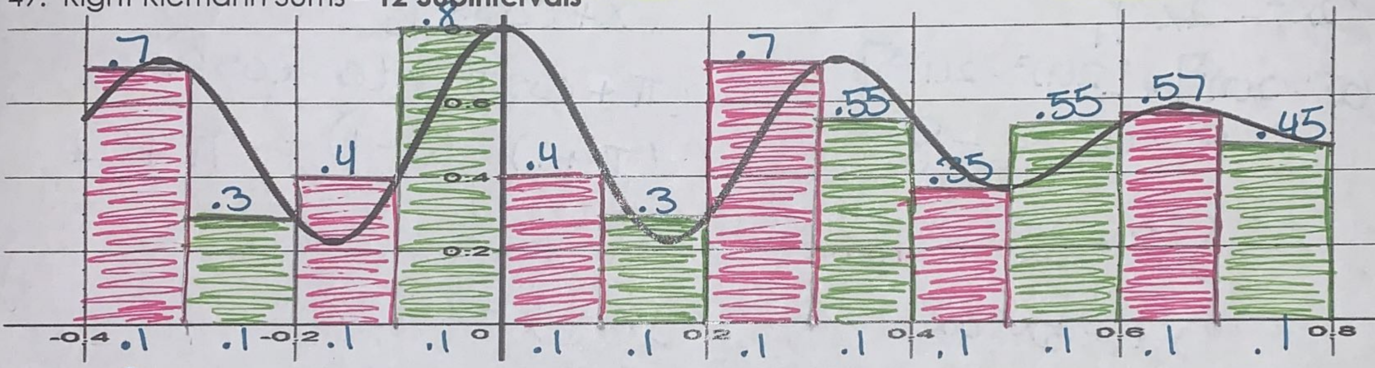
48. $3x^3 - y^3 = 7$

$9x^2 - 3y^2 \frac{dy}{dx} = 0$

$\frac{-3y^2 \frac{dy}{dx}}{-3y^2} = \frac{-9x^2}{-3y^2}$

$\frac{dy}{dx} = \frac{3x^2}{y^2}$

49. Right Riemann Sums – **12 Subintervals**



$0.8 - -0.4 = \frac{1.2}{12} = .1$

$A = .1(.7 + .3 + .4 + .8 + .4 + .3 + .7 + .55 + .35 + .55 + .57 + .45)$

$A = .607$

Integrate the following:

50. $\int (x^3 + 2)\, dx$

$\frac{x^4}{4} + 2x + C$

51. $\int \sin 8x\, dx$

$-\frac{\cos(8x)}{8} + C$

52. $\int \sqrt[3]{x^2}\, dx$

$\int x^{2/3}\, dx$

$\frac{3x^{5/3}}{5} + C$

53. $\int \frac{1}{x^3}\, dx$

$\int x^{-3}\, dx$

$\frac{x^{-2}}{-2} + C = \frac{-1}{2x^2} + C$

Rewrite 1st!

54. $\int \frac{x^2+1}{x^2}\,dx = \int \frac{x^2}{x^2} + \frac{1}{x^2}\,dx$

$\int 1 + x^{-2}\,dx$

$x - \frac{1}{x} + C$

multiply 1st!

55. $\int (x+1)(3x-2)\,dx$

$\int 3x^2 + x - 2\,dx$

$\frac{\cancel{3}x^3}{\cancel{3}} + \frac{x^2}{2} - 2x + C$

$x^3 + \frac{x^2}{2} - 2x + C$

56. $\int_{-1}^{1} 3x^2\,dx$

$x^3 \Big|_{-1}^{1} = (1)^3 - (-1)^3$

$1 + 1$

2

57. $\int_{-\pi}^{\frac{\pi}{2}} 2\cos x\,dx$

$2\sin x \Big|_{-\pi}^{\pi/2}$

$2\sin(\pi/2) - 2\sin(-\pi)$

$2(1) - 2(0) = 2$

58. $\int_{1}^{2} 4x - 6x^2\,dx$ $\quad \frac{4x^2}{2} - \frac{6x^3}{3}$

$2x^2 - 2x^3 \Big|_1^2$

$[2(2)^2 - 2(2)^3] - [2(1)^2 - 2(1)^3]$

$-8 - 0 = -8$

59. $\int_{0}^{\pi} 1 + \sin x$

$x - \cos x \Big|_0^{\pi}$

$(\pi - \cos\pi) - (0 - \cos 0)$

$(\pi + 1) - (0 - 1) = \pi + 2$

60. $\int \frac{2x+1}{2x}\,dx$ $\quad \int \frac{2x}{2x} + \frac{1}{2x}$ memorize

$\int 1 + \frac{1}{2}x^{-1}\,dx$

$x + \frac{1}{2}\ln|x| + C$

61. $\int \frac{x^3-x-1}{x^2}\,dx$

$\int x - \frac{1}{x} - x^{-2}\,dx$

$\frac{x^2}{2} - \ln|x| + \frac{1}{x} + C$

u-sub

62. $\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\,d\theta$

$u = \cos\theta$

$du = -\sin\theta\,d\theta$

$d\theta = \frac{du}{-\sin\theta}$

$\int_0^{\pi/2} \sin\theta\, u\,d\theta$

$\int_0^{\pi/2} \sin\theta\, u\,\frac{du}{-\sin\theta}$

$u_{\pi/2} = \cos\frac{\pi}{2} = 0$

$u_0 = \cos 0 = 1$

$-\int_0^{\pi/2} u\,du$

$-\frac{u^2}{2}\Big|_1^0 = \frac{0^2}{2} - -\frac{(1)^2}{2}$

$0 + \frac{1}{2} = \frac{1}{2}$

A T

63. $\int_0^{\pi} x\cos x\,dx$ Integration by Parts w/ table

u	dV
$+x$	$\cos x$
-1	$\sin x$
$+0$	$-\cos x$

$x\sin x + \cos x \Big|_0^{\pi}$

$(\pi\sin\pi + \cos\pi) - (0\sin 0 + \cos 0)$

$(0 - 1) - (0 + 1)$

$-1 - 1$

-2

u-sub

64. $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

$u = 1+\sin x$

$du = \cos x\, dx$

$dx = \frac{du}{\cos x}$

$\int \frac{\cos x}{u^{1/2}} dx$

$\int \frac{\cancel{\cos x}}{u^{1/2}} \frac{du}{\cancel{\cos x}}$

$\int u^{-1/2} du$

$2u^{1/2} + C = 2\sqrt{1+\sin x} + C$

u-sub

65. $\int \frac{e^x}{e^x-1} dx$

$u = e^x - 1$

$du = e^x dx$

$dx = \frac{du}{e^x}$

$\int \frac{e^x}{u} dx$

$\int \frac{\cancel{e^x}}{u} \frac{du}{\cancel{e^x}}$

$\int u^{-1} du = \ln|u| + C$

$\ln|e^x+1| + C$

66. $\int \overset{A}{x^2} \overset{E}{e^x} dx$ By Parts (Table)

u	dv
$+ x^2$	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x

$x^2e^x - 2xe^x + 2e^x + C$

or

$e^x(x^2-2x+2) + C$

67. $\int \overset{A}{x^3} \overset{L}{\ln x}\, dx$ Integration by Parts (can't use table)

$uv - \int v\,du$

$u = \ln x \quad dv = x^3$

$du = \frac{1}{x}dx \quad v = \frac{x^4}{4}$

$\ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$

$\frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3 dx$

$\frac{x^4 \ln x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C$

FTC

68. $\frac{d}{dx}\int_x^{x+2} 4t+1\, dt$ Plug in upper times deriv. of upper - plug in lower times deriv. of lower

$(4(x+2)+1)(1) - (4x+1)(1)$

$4x+8+1-4x-1 = 8$

69. $\frac{d}{dx}\int_0^{x^3} \sin t^2\, dt$

$\sin(x^3)^2(3x^2)$

$3x^2 \sin(x^6)$

Find the average value over the given interval.

70. $f(x) = x^2 - 8x + 12; [2,5]$

$\frac{1}{5-2}\int_2^5 x^2 - 8x + 12\, dx$

$\frac{1}{3}\left(\frac{x^3}{3} - 4x^2 + 12x \Big|_2^5\right)$

$\frac{1}{3}(-9) = -3$

71. $f(x) = x^3 - x^2; [-1,2]$

$\frac{1}{2--1}\int_{-1}^2 x^3 - x^2\, dx$

$\frac{1}{3}\left(\frac{x^4}{4} - \frac{x^3}{3} \Big|_{-1}^2\right)$

$\frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4}$

Find all values c that satisfy the Mean Value Theorem for Integrals.

72. $f(x) = x - 1; [-6,-1]$

$(-1+6)(c-1) = \int_{-6}^{-1} x - 1\, dx$

$5c - 5 = -\frac{45}{2}$

$5c = -\frac{35}{2}$

$c = -\frac{7}{2}$

73. $f(x) = -x^2 - 1; [-1,2]$

$(2--1)(-c^2-1) = \int_{-1}^2 -x^2 - 1\, dx$

$-3c^2 - 3 = -6$

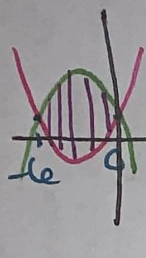
$-3c^2 = -3$

$c^2 = 1$

$c = \pm 1$

Find the area of the region enclosed by the curves.

74. $y = -\frac{x^2}{2} - 3x + \frac{3}{2}, y = \frac{x^2}{2} + 3x + \frac{3}{2}$



$A = \int_{-6}^{0} \left(-\frac{x^2}{2} - 3x + \frac{3}{2}\right) - \left(\frac{x^2}{2} + 3x + \frac{3}{2}\right)$

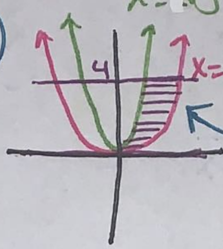
$A = \int_{-6}^{0} (-x^2 - 6x)\, dx$

$A = -\frac{x^3}{3} - 3x^2 \Big|_{-6}^{0}$

$A = 36$

75. $x = \sqrt{y}, x = 2\sqrt{y}, y = 0, y = 4$

$y = x^2 \quad y = \frac{x^2}{4}$

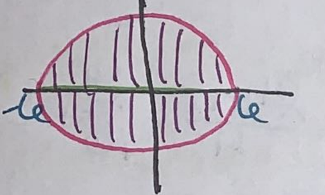


only this area bc ↺

$A = \int_0^4 2\sqrt{y} - \sqrt{y}\, dy$

$A = \frac{16}{3}$ or 5.33

76. The base of a solid is the region enclosed by the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. Cross-sections perpendicular to the x-axis are squares.



$144\left(\frac{x^2}{36} + \frac{y^2}{16} = 1\right)$

$4x^2 + 9y^2 = 144$

$\frac{9y^2}{9} = \frac{-4x^2 + 144}{9}$

$y^2 = -\frac{4}{9}x^2 + 16$

$y = \pm\sqrt{-\frac{4}{9}x^2 + 16}$

top - bottom

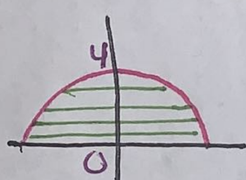
$b = \sqrt{-4/9x^2 + 16} - -\sqrt{-4/9x^2 + 16} = 2\sqrt{-4/9x^2 + 16}$

$V = \int_{-6}^{6} \left(2\sqrt{-4/9x^2 + 16}\right)^2 dx$

$V = 512$

77. The base of a solid is the region enclosed by $y = \sqrt{16 - x^2}$ and $y = 0$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

$A = \frac{1}{2}b^2$



$y = \sqrt{16 - x^2}$

$y^2 = 16 - \cancel{x^2}$

$x^2 = 16 - y^2$

$x = \pm\sqrt{16 - y^2}$

$V = \frac{1}{2}\int_0^4 \left(2\sqrt{16 - y^2}\right)^2 dy$

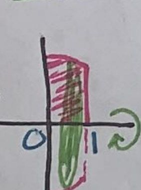
$V = \frac{256}{3}$

$b = \sqrt{16 - y^2} - -\sqrt{16 - y^2}$

$b = 2\sqrt{16 - y^2}$

78. Find the volume of the solid that results when the region enclosed by the curves is revolved about the x-axis. $y = -x^2 + 4, y = 0, x = 0, x = 1$

Disk

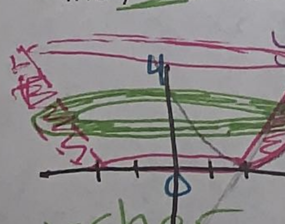


$V = \pi \int_0^1 (-x^2 + 4)^2 dx$

$V = \frac{203\pi}{15}$

79. Find the volume of the solid that results when the region enclosed by the curves is revolved about the y-axis. $x = \sqrt{y} + 2, x = \frac{y}{2} + 2$

$y = (x - 2)^2 \quad y = 2x - 4$



washer

$\sqrt{y} = \frac{y}{2} + 2$

$y = \left(\frac{y}{2} + 2\right)^2$

$y = \frac{y^2}{4} + 2y + 4$

$4y = y^2 + 8y + 16$

$0 = y^2 + 4y + 16$

$V = \pi \int_0^4 (\sqrt{y} + 2)^2 - \left(\frac{y}{2} + 2\right)^2 dy$

$V = 8\pi$