Final Exam Review – Meaning of Derivatives and Derivative Rules

1. What is the difference between average rate of change on $[a, b]$ and instantaneous rate of change at $x = a$?

Use the limit definition of a derivative to evaluate.

2. $\lim_{h\to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$

3. $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$

4. Approximate the derivative using a graph: Figure 5 shows the graph of an object's position $s = s(t)$ on an $s - axis$ as a function of the time t. What is the object's approximate velocity in the positive s −direction at

a. $t = 3$

b. $t = 2$

c. $t = 6$

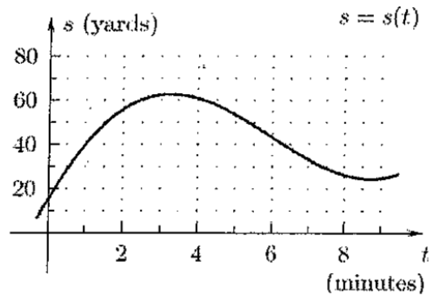


FIGURE 5

5. Suppose that the amount of water in gallons in a holding tank at t minutes is given by $V(t) = 2t^2 - 16t + 35$. Determine each of the following by finding the derivative of $V(t)$

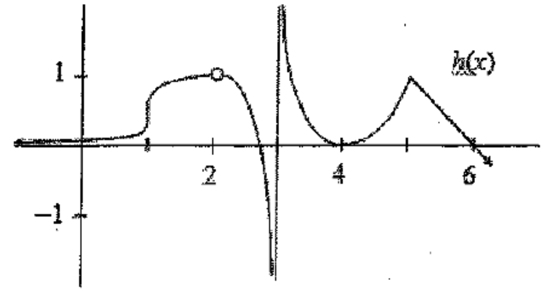
a. Find the average rate of change on $[1,5]$

b. Find the rate of change in the volume of water in the tank at $t = 1$ minute

c. Find the rate of change in the volume of water in the tank at $t = 5$ minutes

d. Is the volume of water in the tank ever not changing? If so, when?

6. Use the graph shown to describe where and why $h(x)$ is not differentiable.



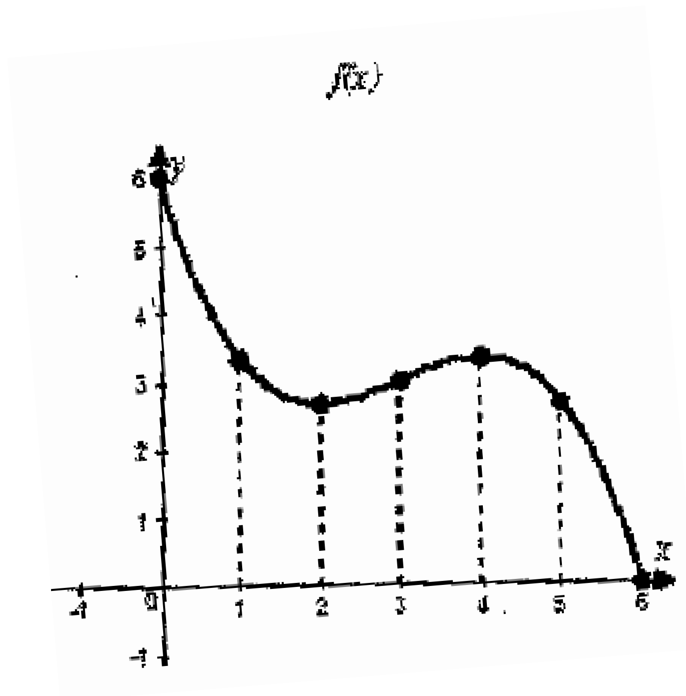
7. $f(x) = 4x^2 - 3x + 2$. Find the equation of the tangent line and normal line to the curve at the point where $x = -2$.

8. The line tangent to the graph of $y = f(x)$ at $x = 3$ passes through the points $(-1,5)$ and $(3,-1)$. Find $f(3)$ and $f'(3)$

9. Suppose $g(x) = 3x + 1$ is the equation of the tangent line to the graph of $y = f(x)$ at $a = 2$. What is $f(2)$ and $f'(2)$

10. Use the graph at the right to complete the table.

Condition	Domain Interval/Value
$f'(x) < 0$	
$f'(x) = 0$	
$f'(x) > 0$	
$f''(x) < 0$	
$f''(x) = 0$	
$f''(x) > 0$	



At what point is f speeding up? Slowing down?

11. Given $f(x) = x^3 - 3x^2 - 9x + 6$, find the following:

a. Where is $f(x)$ increasing? Decreasing?

b. Where does $f(x)$ have a local max? Local min?

c. Where is $f(x)$ concave up? Concave down?

12. Find where the following functions have horizontal tangents:

a. $f(x) = 3x^2 - 15$

b. $f(x) = \sin(2x) - x$

c. $f(x) = 12x^2 - 3x^3$

d. $f(x) = \frac{3}{4}x^4 - \frac{3}{2}x^2$

13. Evaluate the derivative using derivative rules

a. $f(x) = (3x^2 - 4)(5x + 3)$

b. $y = \frac{6x^2}{\sin x}$

c. $f(x) = e^{2x}\sin(3x)$

d. $f(x) = e^{\cot x}$

e. $y = e^{x^2-3x}$

f. $y = \sqrt[3]{x^7} + \frac{1}{\sqrt[5]{x^2}} - \frac{2}{x}$

g. $g(x) = 5x^4 \ln(2x^3 + 6)$

h. $f(x) = \tan^{-1}(e^{3x})$

i. $y = \cos^6(3x - 2)$

j. $y = \ln(5x^2 - 3x) + 7\tan(x^3) - \frac{8}{x}$

14. Find the derivative using the chart.

a. Find $\frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$ when $x = -2$

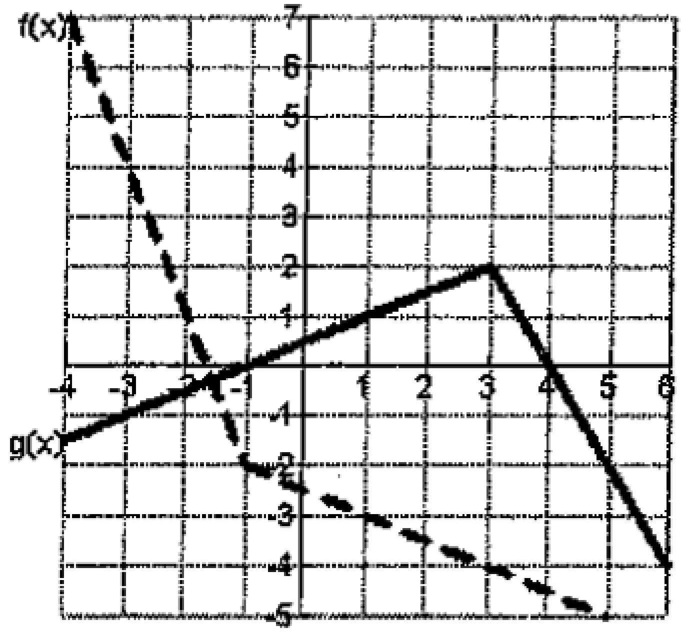
b. $\frac{d}{dx}\left[x^2 - f(\sqrt{x})\right]$ when $x = 4$

c. Find $\frac{d}{dx}[g^{-1}(x)]$ when $x = 3$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
–4	6	3	5	6
–2	3	-3	7	1
0	2	2	4	–2
1	-2	-5	-1	3
2	4	1	12	-3
4	5	–4	2	–4

15. f and g are functions whose graphs are given below. Find the following:

a. $p(x) = f(x) \cdot g(x)$, find $p'(5)$



b. $m(x) = \frac{x^2}{f(x)}$, find $m'(-2)$

c. $h(x) = 3g\big(f(x)\big)$, find $h'(2)$

16. If $f(x) = \frac{4}{(1-x)^2}$, find the linearization for $f(x)$ at 0. Use the linearization of $f(x)$ to approximate the value of $f(0.07)$.

17. The equation of motion of a particle is: $s(t) = t^3 - 15t^2 + 48t + 10$, where s is in meters and t is in seconds. Find the following when $0 < t < 9$. Be sure to include units where appropriate.

a. Find the velocity function in terms of t

b. When is the particle at rest?

c. When is the particle moving forward?

d. What is the acceleration function in terms of t?

e. When is the particle slowing down?

f. What is the displacement from $t = 0$ to $t = 9$?

g. What is the total displacement traveled from $t = 0$ to $t = 9$