Final Exam Review – Meaning of Integration

Find the antiderivative of the following.

1. $\int \frac{\sqrt[3]{x^5}+2}{x}\,dx$
$\int x^{2/3} + 2x^{-1}\,dx$
$\frac{3}{5}x^{5/3} + 2\ln|x| + C$

2. $\int e^2 \csc x \cot x\,dx$
$-e^2\csc x + C$

3. $\int (2x-7)^2\,dx$
$\frac{(2x-7)^3}{6} + C$

4. $\int \frac{dx}{\cos^2 x}$
$\int \sec^2 x\,dx$
$\tan x + C$

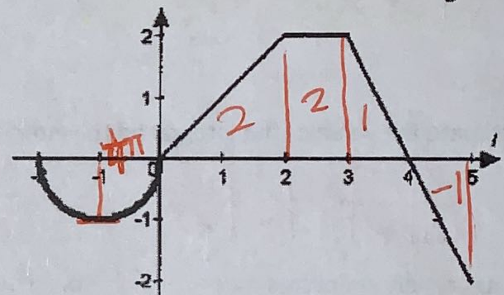
2. A particle moves along a straight line with acceleration $a(t) = 7 + 8t - 6t^2$. The velocity at $t = 1$ second is $3\ m/sec$. Its position at time $t = 0$ is 2 meters. Find both the velocity function and the position function.

$v(t) = \int 7 + 8t - 6t^2\,dt$
$v(t) = 7t + 4t^2 - 2t^3 + C$
$3 = 7(1) + 4(1)^2 - 2(1)^3 + C$
$C = 6$

$v(t) = -2t^3 + 4t^2 + 7t + 6$
$s(t) = -\frac{1}{2}t^4 + \frac{4}{3}t^3 + \frac{7}{2}t^2 + 6t + C$
$s(t) = -\frac{1}{2}t^4 + \frac{4}{3}t^3 + \frac{7}{2}t^2 + 6t + 2$

3. Suppose $A(x) = \int_3^x f(t)dt$. Use the graph of f shown to answer the following.

a. $A(-1) = -\frac{\pi}{4} + 4$
b. $A(4) = 1$
c. $A(5) = 0$
pt. d. $A'(-1) = -1$
pt. e. $A'(4) = 0$
slope f. $A''(1) = 1$
slope g. $A''(2.5) = 0$
slope h. $A''(4) = -2$



i. Identify the value(s) of x for which A has local maxima and/or minima. Identify which type of extrema occurs at these x value(s).

$A' = f(x) = 0$
Local min: $x = 0$ Local max: $x = 4$

4. Evaluate the definite integrals.

a. $\int_4^{e^5} \frac{3}{t}\,dt$
$3\ln|t|\Big|_4^{e^5}$
$3\ln e^5 - 3\ln 4$
$15 - 3\ln 4$

b. $\int_{\frac{\sqrt{3}}{3}}^{1} \frac{1}{1+x^2}\,dx$
$\arctan x \Big|_{\sqrt{3}/3}^{1}$
$\arctan(1) - \arctan\left(\frac{\sqrt{3}}{3}\right)$
$\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

c. $\int_0^7 |6 - x|\,dx$
$\frac{1}{2}(6)(6) + \frac{1}{2}(1)(1)$
$18 + \frac{1}{2}$
$\frac{37}{2}$

d. $\int_1^4 \left(\frac{2}{\sqrt{t}} - \frac{1}{2}\right) dt$
$\int_1^4 \left(2t^{-1/2} - \frac{1}{2}\right) dt$
$4t^{1/2} - \frac{t}{2}\Big|_1^4$
$4\sqrt{4} - \frac{1}{2}(4) - 4\sqrt{1} + 1\left(\frac{1}{2}\right)$
$8 - 2 - 4 + \frac{1}{2}$
$\frac{5}{2}$

5. Use the Fundamental Theorem of Calculus to simplify:

a. If $F(x) = \int_1^{x^2} \frac{1}{1+t^2}\, dt$,

Find:

$F(1)$ $\int_1^1 \frac{1}{1+t^2} dt = 0$

$F'(x)$ $\frac{1}{1+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$

$F'(1)$ $\frac{2(1)}{1+1^4} = 1$

b. $\frac{d}{dx}\int_{x^3}^{4} \sin(t^2)\, dt$

$-\frac{d}{dx}\int_4^{x^3} \sin(t^2) dt$

$-\sin(x^3)^2 \cdot 3x^2$

$-3x^2 \sin(x^6)$

c. If $F(x) = \int_{x^3}^{\tan x} \sqrt{1+t}\, dt$, find $F'(x)$

$\sec^2 x \sqrt{1+\tan x} - 3x^2\sqrt{1+x^3}$

d. $\frac{d}{dx}\int_3^{x^3} \sqrt{t^2 - 4t}\, dt$

$3x^2\sqrt{x^6 - 4x^3}$

e. Find $f'(x)$ when $f(x) = \int_{x^2}^{e^x} \frac{1}{\sqrt{1-t^2}} dt$

$\frac{e^x}{\sqrt{1-e^{2x}}} - \frac{2x}{\sqrt{1-x^4}}$

6. Estimate the value of the integral on the interval $[-1,2]$ using the following with 3 equal subintervals.

x	-1	0	1	2
f(x)	0	1	8	27

$$\int_{-1}^{2} (x+1)^3 dx$$

(-1,0) (0,1) (1,2)

L

a. Using left endpoints

$1(0+1+8) = 9$

b. Using the midpoint

$1(f(-.5)+f(.5)+f(1.5))$

$\frac{1}{8} + \frac{27}{8} + \frac{125}{8}$

$153/8$

c. Using trapezoids

$\frac{1}{2}(f(-1)+2f(0)+2f(1)+f(2))$

$\frac{1}{2}(0+2+16+27)$

$45/2$

7. Express the limit as a definite integral on the given interval.

a. $\lim_{n\to\infty} \sum_{k=1}^{3n} \frac{1}{n}\left[\sec^2\left(\frac{k}{n}\right)\right]$

$a + k\Delta x = \sec^2(\frac{k}{n})$

so $a = 0$, $\Delta x = \frac{1}{n}$

$+ f(x) = \sec^2 x$

$\Delta x = \frac{b-a}{n}$

$\frac{1}{n} = \frac{b-0}{n}$

$b = 1$

$\int_0^1 \sec^2 x\, dx$

b. $\lim_{n\to\infty} \sum_{k=1}^{3n} \frac{1}{n}\left[\sec\left(\frac{k}{n}\right)\tan\left(\frac{k}{n}\right)\right]$

$\int_0^1 \sec x \tan x\, dx$

c. $\lim_{n\to\infty} \sum_{i=1}^{n} [6(x_i)^2 - 4x_i]\Delta x_i$ $[2,3]$

$\int_2^3 6x^2 - 4x\, dx$

d. $\lim_{n\to\infty} \sum_{k=1}^{3n} \frac{1}{n}\left[\csc^2\left(\frac{k}{n}\right)\right]$

$\int_0^1 \csc^2 x\, dx$