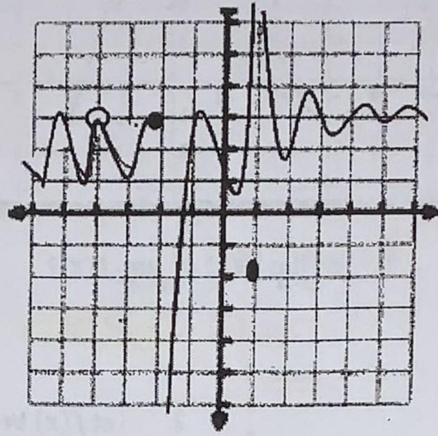
Final Exam Review – Limits

Limits from Graphs

Using the graph $g(x)$ below, find the indicated limits.



a. $\lim_{x\to-\infty} g(x)$ DNE

b. $\lim_{x\to-2^-} g(x)$ 3

c. $\lim_{x\to-2^+} g(x)$ $-\infty$

d. $\lim_{x\to-2} g(x)$ DNE

e. $\lim_{x\to-4} g(x)$ 3

f. $\lim_{x\to1} g(x)$ ∞

g. $\lim_{x\to\infty} g(x)$ 3

h. $g(1)$ -2

Limits from Tables

From this table of values, evaluate the following limits.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$f(x)$	1235	3025	44235	Undefined	6.004	6.006	6.012
$g(x)$	4.532	4.213	4.013	8	8.0015	8.1016	8.546
$h(x)$	6.275	6.191	6.013	12	5.997	5.987	5.971

$\lim_{x\to0^+} f(x)$ 6	$\lim_{x\to0^-} f(x)$ ∞	$\lim_{x\to0} f(x)$ DNE
$\lim_{x\to0^+} g(x)$ 8	$\lim_{x\to0^-} g(x)$ 4	$\lim_{x\to0} g(x)$ DNE
$\lim_{x\to0^+} h(x)$ 6	$\lim_{x\to0^-} h(x)$ 6	$\lim_{x\to0} h(x)$ 6

Discontinuities

Describe the type(s) of discontinuities.

a. $f(x) = \begin{cases} x-3, & x\le 2 \\ 2x+1, & x>2 \end{cases}$

$\lim_{x\to2^-} f(x) = -1$

$\lim_{x\to2^+} f(x) = 5$

Jump Discontinuity at $x=2$

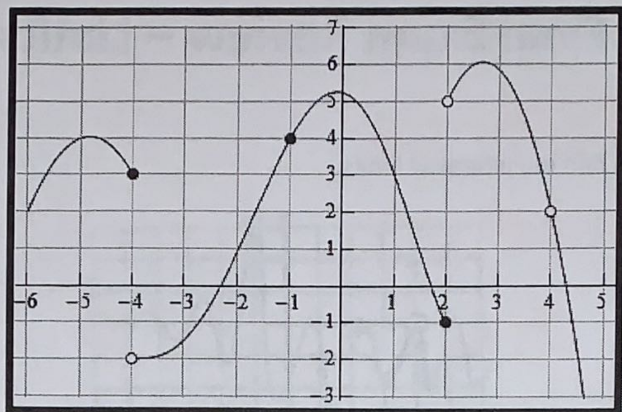
b. $f(x) = \frac{|x+4|}{x+4}$

Jump Disc. at $x=-4$

c. $f(x) = \frac{15x}{x^2+5x} = \frac{15x}{x(x+5)}$

Infinite Discontinuity at $x=-5$

Removable Discontinuity at $x=0$



a. Is $\lim_{x \to -4^+} f(x) = f(-4)$?

$3 \neq -2$; no

b. Is $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$?

$2 = 2$; yes

c. Is $\lim_{x \to 4} f(x) = f(4)$?

$2 \neq$ DNE ; no

More Limit Problems

1. $\lim_{x \to -5^-} \frac{4x+20}{|x+5|}$

$\lim_{x \to -5^-} \frac{4(x+5)}{-(x+5)} = -4$

2. Let $f(x)$ be a continuous function. If $f(x) = \frac{(2x-x^2)}{x^2+x-6}$ when $x^2 + x + 6 \neq 0$, what is $f(2)$?

$f(x) = \frac{-x(\cancel{x-2})}{(x+3)(\cancel{x-2})}$

$f(x) = \frac{-x}{x+3}$

hole at $x = 2$

$\therefore f(2)$ DNE

3. Let f be the function defined below, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$$f(x) = \begin{cases} 6 + cx, & x < 2 \\ -4 + 2\ln\left(\frac{x}{2}\right), & x \geq 2 \end{cases}$$

$6 + cx = -4 + 2\ln\left(\frac{x}{2}\right)$

$6 + 2c = -4 + 2\ln\left(\frac{2}{2}\right)$

$6 + 2c = -4$

$2c = -10$

$c = -5$

4. Let f be the function defined below, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$$f(x) = \begin{cases} x^2 \cos\left(\frac{\pi}{2}x\right), & x < 2 \\ x^2 + cx - 16, & x \geq 2 \end{cases}$$

$x^2 \cos\left(\frac{\pi}{2}x\right) = x^2 + cx - 16$

$(2)^2 \cos\left(\frac{\pi}{2} \cdot 2\right) = (2)^2 + 2c - 16$

$-4 = 2c - 12$

$2c = 8$

$c = -4$

5. Let f be the function given by $f(x) = \frac{(x+5)^2(x+1)}{(x+5)(x-3)}$. For which of the values of x is f not continuous?

$x = -5$ Removable Disc.

$x = 3$ Infinite Disc.

6. $\lim_{x \to 4} \frac{x - \sqrt{x+12}}{x-4}$

$\lim_{x \to 4} x - \sqrt{x+12} = 0$

$\lim_{x \to 4} x - 4 = 0$

LH $\lim_{x \to 4} 1 - \frac{1}{2\sqrt{x+12}} = 1 - \frac{1}{2\sqrt{16}}$

$1 - \frac{1}{8}$

$\frac{7}{8}$

7. $\lim\limits_{x\to 0} \frac{\left(\frac{2}{x+3}-\frac{2}{3}\right)}{x}$

$\lim\limits_{x\to 0} \frac{2}{x+3}-\frac{2}{3}=0$

$\lim\limits_{x\to 0} x=0$

LH $\lim\limits_{x\to 0} \frac{-2}{(x+3)^2} = \frac{-2}{9}$

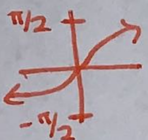
8. $\lim\limits_{x\to 2} \frac{3x^2+5x-2}{x^2-4}$

$\lim\limits_{x\to 2} \frac{(3x-1)(\cancel{x+2})}{(x-2)(\cancel{x+2})}$

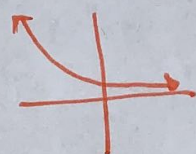
$\lim\limits_{x\to 2^-} \frac{3x-1}{x-2}=-\infty$ $\lim\limits_{x\to 2^+} \frac{3x-1}{x-2}=\infty$ not answer

DNE

9. $\lim\limits_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$

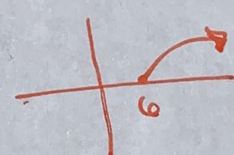


10. $\lim\limits_{x\to-\infty} e^{-x} = \infty$ reflect over y-axis

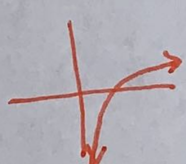


11. $\lim\limits_{x\to\infty} e^{-x} = 0$

12. $\lim\limits_{x\to 6^-} \sqrt{x-6}$ DNE



13. $\lim\limits_{x\to 0^+} \ln x = -\infty$



14. $\lim\limits_{x\to 0^-} \ln x$ DNE

15. $\lim\limits_{x\to\infty} \ln x = \infty$

16. $\lim\limits_{x\to-\infty} \sin x$ DNE

(oscillates)

17. $\lim_{x\to\infty} \frac{-4x+2x^3}{8x^3+4x^2-3} = \frac{2}{8} = \frac{1}{4}$

HA

18. $\lim_{x\to\infty} \frac{5x^2-2x^4-4}{2x-3x^2+x^3} = -\infty$

SA $y=-2x$

19. $\lim_{x\to-\infty} \frac{\sqrt{4x^2-3x+5}}{7x+4} = $ like $\frac{\pm 2x}{7x}$

$-\frac{2}{7}$

20. $\lim_{x\to 2^-} -\frac{5}{x-2}$ $\quad \frac{-5}{1.999-2}$

∞

21. $\lim_{x\to 0^-} -\frac{3}{x^3}$ $\quad \infty$

x	y
-1	3
$-\frac{1}{10}$	3000
$-\frac{1}{100}$	3,000,000

22. A function $f(x)$ has a vertical asymptote at $x = 3$. Which of the following statements could be used to justify this vertical asymptote?

a. $\lim_{x\to 0} f(x) = 3$
b. $\lim_{x\to 3} f(x) = 0$
c. $\lim_{x\to 3^+} f(x) = \infty$
d. $\lim_{x\to 3^-} f(x) = 0$
e. $\lim_{x\to \infty} f(x) = -5$

23. If $y = 5$ is a horizontal asymptote of a rational function f, then which of the following must be true?

a. $\lim_{x\to 5^+} f(x) = \infty$
b. $\lim_{x\to \infty} f(x) = 5$
c. $\lim_{x\to 0} f(x) = 5$
d. $\lim_{x\to 5} f(x) = 0$
e. $\lim_{x\to -\infty} f(x) = -5$

24. The graph of which function has $y = 2$ as an asymptote?

a. $y = e^{-x} + 2$
b. $y = \ln(x-2)$
c. $y = -\frac{2x^2}{4+x^2}$
d. $y = -\frac{2}{1-x}$
e. $y = \frac{4x}{2+x}$