Final Exam – Integration Review

1. $\int 4\sin 5x \; dx$

$\frac{-4\cos 5x}{5} + C$

2. $\int e^{3x} dx$

$\frac{e^{3x}}{3} + C$

3. $\int 7\sec^2(2-3x) dx$

$-\frac{7}{3}\tan(2-3x) + C$

4. $\int \csc(\pi x)\cot(\pi x) \; dx$

$\frac{-\csc(\pi x)}{\pi} + C$

5. $\int (2x-5)^4 dx$

$\frac{(2x-5)^5}{10} + C$

6. $\int \frac{e^{-5x}}{2} dx$

$\frac{e^{-5x}}{-10} + C$ or $\frac{-1}{10e^{5x}} + C$

7. $\int 2\sqrt{9x-5}\, dx$

$\frac{2}{3} \cdot \frac{2(9x-5)^{3/2}}{9} + C$

$\frac{4}{27}(9x-5)^{3/2} + C$

8. $\int \tan 7x \; dx$

$-\frac{1}{7}\ln|\cos 7x| + C$

9. $\int \frac{3}{\sqrt{1-16x^2}} dx$

$3\int \frac{1}{\sqrt{1-(4x)^2}} dx$ $\quad u = 4x$, $du = 4dx$, $\frac{du}{4} = dx$

$\frac{3}{4}\int \frac{1}{\sqrt{1-u^2}} du$

$\frac{3}{4}\sin^{-1}(4x) + C$

10. $\int \frac{x\,dx}{1+25x^4}$

$\int \frac{x\,dx}{1+(5x^2)^2}$ $\quad u = 5x^2$, $du = 10x\,dx$, $\frac{1}{10}du = x\,dx$

$\frac{1}{10}\int \frac{1}{1+u^2} du$

$\frac{1}{10}\tan^{-1}(5x^2) + C$

11. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}} dx$

$2\int \sec^2 u\, du$ $\quad u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$, $2du = \frac{1}{\sqrt{x}}dx$

$2\tan\sqrt{x} + C$

12. $\int e^{2x}\cos(e^{2x}) \; dx$

$\frac{1}{2}\int \cos u\, du$ $\quad u = e^{2x}$, $du = 2e^{2x}dx$, $\frac{1}{2}du = e^{2x}dx$

$\frac{1}{2}\sin e^{2x} + C$

$\frac{u}{5} = x+3$

13. $\int (x+3)\sqrt{5x+15}\,dx$

$u = 5x+15$
$du = 5dx$
$\frac{1}{5}du = dx$

$\frac{1}{5}\int \frac{u}{5} u^{1/2} du$
$\frac{1}{25}\int u^{3/2}$
$\frac{2}{125}u^{5/2}+C \rightarrow \frac{2}{125}(5x+15)^{5/2}+C$

14. $\int 7x^2 e^{x^3}\,dx$

$u = x^3$
$du = 3x^2 dx$
$\frac{1}{3}du = x^2 dx$

$\frac{7}{3}\int e^u du$
$\frac{7}{3}e^{x^3}+C$

15. $\int x \cdot 3^{5x^2}\,dx$

$u = 5x^2$
$du = 10x\,dx$
$\frac{1}{10}du = x\,dx$

$\frac{1}{10}\int 3^u du$
$\frac{1}{10}\frac{3^u}{\ln 3}+C$
$\frac{3^{5x^2}}{10\ln 3}+C$

16. $\int \frac{10x}{8-x^2}\,dx$

$u = 8-x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\cdot 10\int \frac{1}{u} du$
$-5\ln|u|+C$
$-5\ln|8-x^2|+C$

17. $\int \frac{\sec^2(\frac{\pi}{x})}{x^2}\,dx$

$u = \frac{\pi}{x}$
$du = -\frac{\pi}{x^2}dx$
$-\frac{1}{\pi}du = \frac{1}{x^2}dx$

$-\frac{1}{\pi}\int \sec^2 u\,du$
$-\frac{1}{\pi}\tan(\frac{\pi}{x})+C$

18. $\int e^{3x}\sec(e^{3x})\tan(e^{3x})\,dx$

$u = e^{3x}$
$du = 3e^{3x}dx$
$\frac{1}{3}du = e^{3x}dx$

$\frac{1}{3}\int \sec u \tan u\,du$
$\frac{1}{3}\sec(e^{3x})+C$

19. $\int \frac{\sin(\ln x)}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$\int \sin u\,du$
$-\cos(\ln x)+C$

20. $\int \sin\theta \cos^3\theta\,d\theta$

$u = \cos\theta$
$du = -\sin\theta\,d\theta$

$-\int u^3 du$
$-\frac{\cos^4\theta}{4}+C$

21. $\int \frac{(\ln x)^3}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$\int u^3 du$
$\frac{(\ln x)^4}{4}+C$
or
$\frac{\ln^4 x}{4}+C$

22. $\int \tan^4(3x)\sec^2(3x)\,dx$

$u = \tan(3x)$
$du = 3\sec^2(3x)dx$
$\frac{du}{3} = \sec^2(3x)dx$

$\frac{1}{3}\int u^4 du$
$\frac{1}{3}\cdot\frac{u^5}{5}+C$
$\frac{\tan^5(3x)}{15}+C$