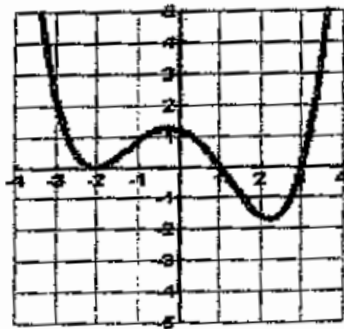


## Final Exam Review - Derivative Applications

1. Given the graph of  $f'(x)$ , find the following intervals or  $x$  values where: (estimate to the nearest  $\frac{1}{4}$  unit)



- a.  $f(x)$  is increasing. Justify.
- b.  $f(x)$  has horizontal tangents. Justify.
- c.  $f(x)$  is concave down. Justify.
- d.  $f(x)$  has a local minimum. Justify.
- e.  $f(x)$  has a point of inflection. Justify.
2. The radius of a circle is increasing at a rate of  $3 \text{ cm/sec}$ . How fast is the area of the circle changing when the radius is  $5 \text{ cm}$  long?
3. A road perpendicular to a highway leads to a farmhouse located  $1 \text{ mile}$  away. An automobile travels past the farmhouse at a speed of  $60 \text{ mph}$ . How fast is the distance between the automobile and the farmhouse increasing when the automobile is  $3 \text{ miles}$  past the intersection of the highway and the road?
4. Air is being pumped into a spherical balloon at  $5 \text{ cm}^3/\text{min}$ . Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is  $20 \text{ cm}$ .
5. A light is on the top of a  $12 \text{ ft}$  tall pole and a  $5 \text{ ft } 6 \text{ in}$  tall person is walking away from the pole at a rate of  $2 \text{ ft/sec}$ .
- a. At what rate is the tip of the shadow moving away from the pole when the person is  $25 \text{ ft}$  from the pole?
- b. At what rate is the tip of the shadow moving away from the person when the person is  $25 \text{ ft}$  from the pole?

6. Determine all the numbers  $c$  which satisfy the conclusion of the Mean Value Theorem for the following function  $f(x) = \frac{1}{4}x^3 + 1$  over the interval  $[-2,2]$ .
7. Find the absolute maximum and the absolute minimum of the function  $f(x) = x^3 - x^2 - x + 2$  on the interval  $[-10,2]$ . Justify your answer.
8. Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other is a maximum.
9. We want to build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?
10. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?
11. A manufacturer determines that  $x$  employees on a certain production line will produce  $y$  units per month where  $y = 75x^2 - 0.2x^4$ . To obtain maximum monthly production, how many employees should be assigned to the production line?
12. Find the point on the parabola  $y = x^2 + 1$  that is closest to the point  $(3,1)$ .
13. Find the dimensions of the rectangle with maximum area that has its base on the  $x$ -axis and its other two vertices along the  $x$ -axis and lying on the parabola  $y = 12 - x^2$ .