Enrichment for Integral Rules Unit

Related to U-Substitution

1. If $\int_9^{15} f(x)dx = 45$, the find $\int_3^5 f(3x)dx$.

2. If $\int_0^9 f(x)dx = 4$, then find $\int_0^3 xf(x^2)dx$.

3. $\int_0^{\frac{\pi}{4}} \frac{e^{tanx}}{cos^2 x}\ dx$

4. $\int_1^{\infty} \frac{x^2}{(x^3+2)^2} dx$ is an improper integral because its top bound is ∞. In AP Calculus BC we will learn that we may be able to evaluate the integral by noting: $\int_a^{\infty} f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$. If this limit exists and is some finite number, we say the improper integral converges. Determine if $\int_1^{\infty} \frac{x^2}{(x^3+2)^2} dx$ converges. If it does converge, determine what value it converges to.

Related to Trapezoidal Approximation (another numeric integration technique)

1. English mathematician Thomas Simpson (1710-1761) developed a rule for approximate integration resulting from using parabolas instead of the straight line segments creating trapezoids for our trapezoidal approximation technique. Simpson's Rule states the following:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+...+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$$

where n (the number of subintervals) is **even** and $\Delta x = \frac{b-a}{n}$.

Find the approximation for $\int_4^6 \ln(x^3+2)dx$ using Simpson's Rule with n = 10. (This problem is calculator active!)

2. A table of values of a function g is given. Use Simpson's Rule to estimate $\int_0^{1.6} g(x)dx$.

x	g(x)
0.0	12.1
0.2	11.6
0.4	11.3
0.6	11.1
0.8	11.7
1.0	12.2
1.2	12.6
1.4	13.0
1.6	13.2