Derivative Applications Practice Test

1. Hermione is looking to climb a 17 foot ladder that is leaning against a rack of books. She needs to grab "A History of Hogwarts" book from the top shelf, but the ladder starts sliding away at a rate of 4 feet/sec. How fast is the top of the ladder sliding down the bookcase when the foot of the ladder is 8 feet from the bookcase.

K: $dx/dt = 4$ ft/sec

F: $dy/dt =$

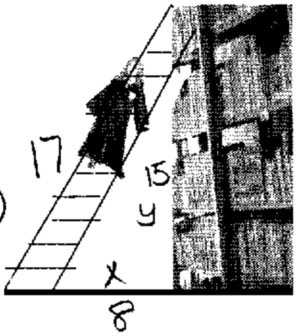
W: $x = 8$

$x^2 + y^2 = L^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2L\frac{dL}{dt}$

$2(8)(4) + 2(15)\frac{dy}{dt} = 2(17)(0)$

$\frac{dy}{dt} = -\frac{32}{15}$ ft/sec



$8^2 + y^2 = 17^2$

$64 + y^2 = 289$

$y^2 = 225$

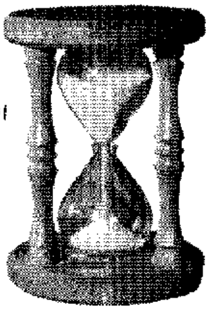
$y = 15$

2. The top cone of the Wicked Witches' Hourglass, which is counting down to Dorothy's death, has a diameter of 10 inches and a height of 12 inches. If the sand is 8 inches deep and sinking at a rate of 5 inches per hour, at what rate is the volume changing?

$d = 10''$

$r = 5''$

$h = 12''$



K: $dh/dt = -5''/hr$

F: dV/dt

W: $h = 8''$

$\frac{r}{h} = \frac{5}{12}$

$5h = 12r$

$r = \frac{5}{12}h$

$V = \frac{1}{3}\pi r^2 h$

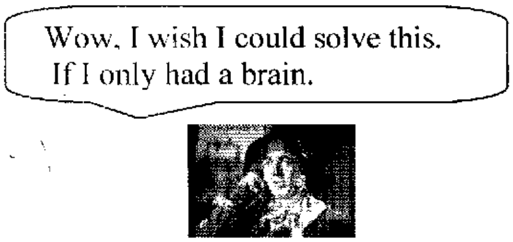
$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$

$V = \frac{1}{3}\pi\frac{25}{144}h^3$

$\frac{dV}{dt} = \frac{25}{144}\pi h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{25}{144}\pi(8)^2(-5)$

$dV/dt = \frac{-500\pi}{9}$ in³/hr



3. Harry Potter is walking through Hogsmeade Village one fine evening, heading back to Hogwarts castle. Harry walks past a lamppost that is 20 feet tall at a constant rate of 4 feet per second. Recently enduring a growth spurt, Harry is now 6 feet tall.

a) Determine the rate at which Harry's shadow is lengthening when three seconds have passed since he walked by the light. $ds/dt = 12/7$ ft/sec

b) Determine the rate at which the tip of Harry's shadow is lengthening.

$\frac{d\,tip}{dt} = \frac{ds}{dt} + \frac{dx}{dt}$

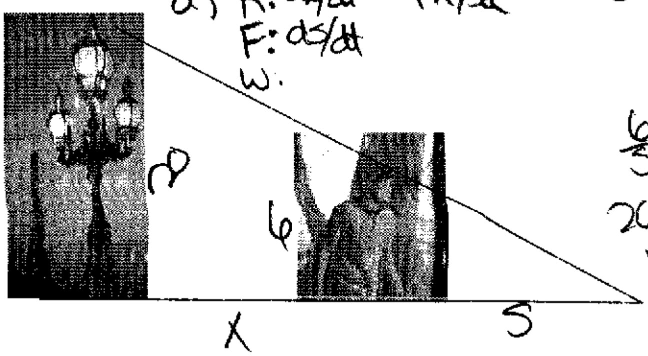
$= \frac{12}{7} + \frac{4}{1}$

$\frac{d\,tip}{dt} = \frac{40}{7}$ ft/sec

a) K: $dx/dt = 4$ ft/sec

F: ds/dt

W:



$\frac{6}{s} = \frac{20}{x+s}$

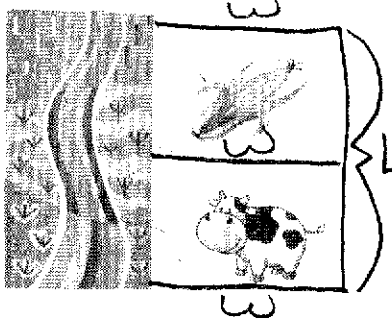
$20s = 6x + 6s$

$14s = 6x$

$14\frac{ds}{dt} = 6\frac{dx}{dt}$ (÷14)

$ds/dt = 3/7\,(4)$ $\frac{ds}{dt} = \frac{12}{7}$

4. A farmer has 1000 ft of fencing and wants to fence off a rectangular field that borders a relatively straight river. He needs no fence along the river. He also need to have the field divided by the fence into 2 sections so his animals don't eat his vegetables. What are the dimensions of the field that has the largest area?



① $P = 3W + L$

$1000 = 3W + L$

$L = 1000 - 3W$

② $A = L \cdot W$

$A = (1000 - 3W)W$

$A = 1000W - 3W^2$

$\frac{dA}{dt} = 1000 - 6W$

$0 = 1000 - 6W$

$W = 166\tfrac{2}{3}'$

③ $L = 1000 - 3(166\tfrac{2}{3})$

$L = 500'$

$500' \times 166\tfrac{2}{3}'$

5. UPS has contracted you to design an open–top rectangular box with a square base that has a volume of 10,000 cubic inches.

$V = x^2 y$

$10{,}000 = x^2 y$

$y = \frac{10{,}000}{x^2}$

a) What dimensions yield the minimum surface area?

$SA = x^2 + 4xy$

$SA = x^2 + 4x\left(\frac{10{,}000}{x^2}\right)$

$SA = x^2 + 40{,}000x^{-1}$

$\frac{dSA}{dt} = 2x - 40{,}000x^{-2}$

$\frac{dSA}{dt} = 2x - \frac{40{,}000}{x^2}$

$0 = 2x - \frac{40{,}000}{x^2}$

$\frac{40{,}000}{x^2} = \frac{2x}{1}$

$2x^3 = 40{,}000$

$x^3 = 20{,}000$

$x = \sqrt[3]{20{,}000} \approx 27.14''$

$y = \frac{10{,}000}{(27.14)^2}$

$y = 13.58''$

$27.14'' \times 13.58''$

b) What is the minimum surface area?

$SA = (27.14)^2 + 4(27.14)(13.58)$

$SA = 2210.82 \text{ in}^2$

6. Morpheus sells 1000 packages of sleeping pills every month at a price of $12 per package. Suppose that for each $1 increase in price, 10 less packages would be sold. At what price should Morpheus sell each package in order to maximize his revenue? Also, what would his maximum revenue be?

$R(x) = (1000 - 10x)(12 + 1x)$

$R(x) = 12000 + 1000x - 120x - 10x^2$

$R(x) = 12000 + 880x - 10x^2$

$R'(x) = 880 - 20x$

$0 = 880 - 20x$

$20x = 880$

$x = 44$

price = \$12 + \$44

price = \$56

Max Revenue

$R(44) = (1000 - 10 \cdot 44)(12 + 44)$

\$31,360