Mid-Term Review

1. Find the domain of $f(x) = \frac{1}{\sqrt{5-3x}}$

★ Can't divide by 0 or take the sq. rt. of a − #

$5-3x \geq 0$
$-3x \geq -5$
$x = 5/3$

$(-\infty, \frac{5}{3})$

not included

2. What is the average rate of change of $f(x) = \frac{1}{x+2}$ over [-3, 2]?

$f(-3) = -1 \quad f(2) = \frac{1}{4}$
$(-3, -1) \quad (2, \frac{1}{4})$
$x_1 \; y_1 \quad x_2 \; y_2$

$m = \frac{\frac{1}{4} - -1}{2 - -3} = \frac{\frac{5}{4}}{5} = \frac{5}{4} \div \frac{5}{1} = \frac{5}{4} \cdot \frac{1}{5}$

$m = \frac{1}{4}$

3. Evaluate $f(x) = \begin{cases} 5x-8, & x < 7 \\ \sqrt{x-7}, & 7 \leq x < 12 \\ \frac{4}{x}, & x \geq 12 \end{cases}$ for f(3), f(7), & f(12).

$f(3) = 5(3) - 8 = 7$
$f(7) = \sqrt{7-7} = 0$
$f(12) = \frac{4}{12} = \frac{1}{3}$

4. Find the exact value of $\cos\frac{3\pi}{4}$, $\tan\frac{7\pi}{6}$, & $\sin\frac{5\pi}{3}$.

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$
$\tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$
$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

5. Find the horizontal & vertical asymptote of $f(x) = \frac{3x^2+2x-16}{x^2-4}$

$= \frac{(3x+8)(x-2)}{(x+2)(x-2)}$ hole at $x=2$

$x+2=0$
$x=-2$

HA: $y=3$
VA: $x=-2$

6. If $f(x) = 2x^2 + 1$ and $g(x) = x + 2$, then find $(f \circ g)(x) =$

$2(x+2)^2+1$
$2(x+2)(x+2)+1$
$2(x^2+4x+4)+1$
$2x^2+8x+8+1 = 2x^2+8x+9$

7. Let f be defined as follows:

$f(x) = \begin{cases} \frac{x^2-25}{x+5}, & for\ x \neq -5 \\ 3, & for\ x = -5 \end{cases}$

$\frac{x^2-25}{x+5} = x-5$

$\frac{(x+5)(x-5)}{x+5}$

hole at $x=5$

Which of the following are true about f?

I. $f(x)$ is continuous at $x = -5$ no; hole at $x=5$

II. $\lim_{x \to -5} f(x)$ exists yes; approaches −10 from left + right

III. $f(-5)$ exists yes $f(-5) = 3$

II and III

8. $f(x) = \begin{cases} x^2, & for\ x > 2, \\ 5ax, & for\ x \leq 2 \end{cases}$

For what value of a is the function continuous?

$x^2 = 5ax$
$(2)^2 = 5a(2)$
$\frac{4}{10} = \frac{10a}{10}$

$a = 2/5$

9. Given a function is defined by $f(x) = \frac{2x+2}{x^2+5x+4}$, for what value(s) of x does the function have one or more vertical asymptotes?

$\frac{2(x+1)}{(x+1)(x+4)} = \frac{2}{x+4}$

VA: $x=-4$ only

10. Given a function defined by $f(x) = \frac{2x+1}{x^2+5x+4}$, for what values of x is the function discontinuous?

$f(x) = \frac{2x+1}{(x+1)(x+4)}$ $\quad x \neq -1 + -4$

discontinuous at
$x=-1 \quad x=-4$

11. $\lim_{x\to -5} \frac{-1}{(x+5)^2} = \frac{-1}{(-5+5)^2} = \text{undef}$

x	$f(x)$
-4.999	$-1{,}000{,}000$
-5	DNE
-5.001	$-1{,}000{,}000$

$-\infty$

12. $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} =$

$\frac{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}{\cancel{\sqrt{x}-3}}$ $\quad \lim_{x\to 9} \sqrt{9}+3 = 6$

13. $\lim_{x\to 0^-} \frac{1}{x} =$

x	y
$-.1$	-10
$-.01$	-100
$-.001$	-1000

$-\infty$

14. $\lim_{x\to 0} \frac{\sqrt{x+25}-5}{x} \cdot \frac{(\sqrt{x+25}+5)}{(\sqrt{x+25}+5)}$

$\frac{\cancel{x+25-25}}{\cancel{x}(\sqrt{x+25}+5)} = \lim_{x\to 0} \frac{1}{\sqrt{0+25}+5} = \frac{1}{10}$

15. $\lim_{x\to -2} \frac{|x+2|}{x+2} =$ from right $\frac{x+2}{x+2}$ or fromleft $\frac{-(x+2)}{x+2}$

1 or -1

-1

16. $\lim_{x\to\infty} \frac{x^2-x}{-4x} = \frac{\cancel{x}(x-1)}{-4\cancel{x}}$

$\lim_{x\to\infty} \frac{x-1}{-4} = \frac{+\infty-1}{-4} = \frac{+}{-}$ so $-\infty$

or $\frac{100{,}000-1}{-4} = -$

17. If $f(x) = -\frac{4}{\sqrt[4]{x}}$, then $f'(16) =$

$f(x) = -4x^{-1/4}$

$f'(x) = x^{-5/4}$

$f'(16) = \frac{1}{4\sqrt{16}^5} = \frac{1}{32}$

18. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)(3)-(3x)(2x)}{(x^2+1)^2}$

$= \frac{3x^2+3-6x^2}{(x^2+1)^2} = \frac{-3x^2+3}{(x^2+1)^2}$

19. If $y = -\frac{4}{\sqrt[3]{x+5}}$, then $\frac{dy}{dx} =$

$y = -4(x+5)^{-1/3}$

$y' = -\frac{4}{3}(x+5)^{-4/3} \cdot 1$

$y' = \frac{4}{3(x+5)^{4/3}}$ or $\frac{4}{3\sqrt[3]{x+5}^4}$

20. Find the derivative of $y = \sqrt[3]{x^2+x}$

$y = (x^2+x)^{1/3}$

$y' = \frac{1}{3}(x^2+x)^{-2/3} \cdot (2x+1)$

$y' = \frac{2x+1}{3\sqrt[3]{x^2+x}^2}$

21. Find the derivative of $y = (x^2+2x+5)^6$

$y' = 6(x^2+2x+5)^5(2x+2)$

$y' = (12x+12)(x^2+2x+5)^5$

22. Find $f'(x)$ fiven $f(x) = \sin^3(4x)$

$f(x) = (\sin(4x))^3$

$f'(x) = 3(\sin(4x))^2 \cdot \cos(4x) \cdot 4$

$f'(x) = 12\sin^2(4x)\cos(4x)$

23. Given $y = \sin(2x^5)$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = \cos(2x^5) \cdot (10x^4)$

$\frac{dy}{dx} = 10x^4 \cos(2x^5)$

24. If $y = \cos(e^x)$, then $\frac{dy}{dx} =$

$-\sin(e^x)e^x$

or $-e^x \sin e^x$

25. Find $\frac{dy}{dx}$ if $y = x^2 \cdot e^x$

$x^2 e^x + e^x 2x$

or

$xe^x(x+2)$

26. Find $\frac{d}{dx}$ given $y = \ln\frac{5}{5-x}$

$\frac{1}{\frac{5}{5-x}}\left(\frac{(5-x)(0)-5(-1)}{(5-x)^2}\right)$

$\frac{\cancel{5-x}}{\cancel{5}} \cdot \frac{\cancel{5}}{(5-x)^{\cancel{2}}} = \frac{1}{5-x}$

27. If $y = e^{x^4-3x^2}$, then $y' =$

$e^{x^4-3x^2}(4x^3-6x)$

28. $\frac{d}{dx} e^{\ln(\cos x)} =$

$\frac{d}{dx}\cos x = -\sin x$

29. $\frac{d}{dx}\ln(e^{4x^2}+3)$

$\frac{1}{e^{4x^2+3}} \cdot e^{4x^2} \cdot \ln e \cdot 8x$

$\frac{8xe^{4x^2}}{e^{4x^2}+3}$

30. Find $\frac{dy}{dx}$ given $y = \frac{x^3}{3^x}$

$\frac{3^x \cdot 3x^2 - x^3 \cdot 3^x \ln 3 \cdot 1}{(3^x)^2}$

$\frac{\cancel{3^x}x(3x-x^2\ln 3)}{(3^x)^{\cancel{2}}} = \frac{x(3x-x^2\ln 3)}{3^x}$

or $\frac{3x^2-x^3\ln 3}{3^x}$

31. If $y = \ln(2x^2-5)$, then $\frac{dy}{dx} =$

$\frac{1}{2x^2-5} \cdot (4x)$

$\frac{4x}{2x^2-5}$

32. Find $\frac{d^2y}{dx^2}$ for $y = \ln 5x^2$

$\frac{1}{5x^2} \cdot 10x = \frac{2}{x}$

$\frac{dy}{dx} = 2x^{-1}$

$\frac{d^2y}{dx^2} = \frac{-2}{x^2}$

33. Find the slope of the tangent line to the graph $f(x) = 2x(2x^2 - 1)$ at the point where $x = 1$

$f(x) = 4x^3 - 2x$

$f(1) = 2$ $\quad (1,2)$

$f'(x) = 12x^2 - 2$

$f'(1) = 12(1)^2 - 2 = 10$

$m = 10$

34. Find an equation of the tangent line to the curve $f(x) = -x^2 + 12$ passing through the point $(4,0)$

$f'(x) = -2x$

$f'(4) = -2(4) = -8 = m$

$y = -8(x-4)$

or

$y = -8x + 32$

35. Find the critical numbers of $f(x) = x^3 - 12x^2$

$f'(x) = 3x^2 - 24x$

$0 = 3x(x-8)$

$x = 0 \quad x = 8$

$f' \quad + \quad - \quad +$ (0, 8)

$x = 0$ + $x = 8$

36. Let $f(x) = x^2(x-3)$. Over what interval is the function decreasing?

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2) \quad x = 0 \quad x = 2$

$+ \quad - \quad +$ (0, 2)

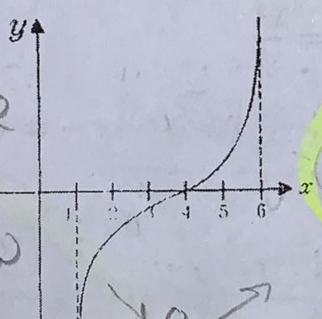
$(0,2)$

37. The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \leq x \leq 10$. For what value(s) does the function have a relative maximum?

above

on

below



$- \quad 0 \quad -$

1 4 6

none

38. Refer to the previous figure. For what value(s) does the function have a relative minimum?

$x = 4$

39. A particle's motion is described by $x(t) = 4t^3 - 5t^2$, $t \geq 0$, where t is in seconds and distance in meters. Find the velocity in the third second.

$v(t) = 12t^2 - 10t$

$v(3) = 12(3)^2 - 10(3)$

$v(3) = 78$ m/s

40. The position of a particle moving in a straight line at any time t is $x(t) = 2t^2 + 6t + 5$. What is the acceleration of the particle at $t = 3$?

$v(t) = 4t + 6$

$a(t) = 4$

$a(3) = 4$ m/s^2

41. Find all points of inflection for $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$ $\quad f(0) = 2$

$0 = 12x(x-2)$ $\quad f(2) = -14$

$+ \quad - \quad +$ at 0, 2

(0,2) and (2,-14)

42. Find the interval(s) on which the curve $y = x^3 - 3x^2 - 9x + 6$ is concave upward or concave downward.

$y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

$0 = 6(x-1)$

$- \quad 0 \quad +$ at 1

concave up at $(1, \infty)$

Concave down at $(-\infty, 1)$

43. Given that $f(x) = \frac{4}{x}$, determine where the function is concave up and concave down. $f(x) = 4x^{-1}$

$f'(x) = -\frac{4}{x^2}$ or $-4x^{-2}$

$f''(x) = \frac{8}{x^3}$ $\quad \frac{8}{x^3} = 0$ $\quad 0 \neq 8$

$- \quad +$ at 0

Concave up $(0, \infty)$

Concave down $(-\infty, 0)$

44. Given that $f(x) = -x^2 + 12x - 34$ has a relative maximum at $x = 6$, determine where $f'(x)$ is positive and negative.

$f'(x) = -2x + 12$

$0 = -2(x-6)$ $\quad x = 6$

$+ \quad -$ at 6

positive $(-\infty, 6)$

negative $(6, \infty)$

45. Find the point of inflection of $f(x) = x^3 - 3x^2 - x + 7$

$f'(x) = 3x^2 - 6x - 1$

$f''(x) = 6x - 6$

$0 = 6x - 6$

$x = 1$ $\quad f(1) = 1^3 - 3(1)^2 - 1 + 7$

$1 - 3 - 1 + 7 = 4$

$- \quad 0 \quad +$

(1,4)

46. Given a function defined by $f(x) = 3x^5 - 5x^3 - 8$, for what value(s) of x is there a point of relative minimum?

$f'(x) = 15x^4 - 15x^2$

$0 = 15x^2(x+1)(x-1)$

$x = 0 \quad x = -1 \quad x = 1$

$+ \quad - \quad - \quad +$ at -1, 0, 1

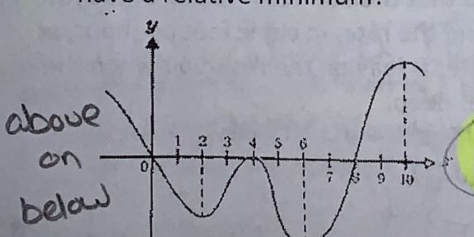
$f(1) = 3 - 5 - 8$

$-2 - 8$

-10

$X = -1$

47. The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \le x \le 10$. For what value(s) does the function have a relative minimum?



max $\quad$ min

$+ \quad 0 \quad - \quad 0 \quad - \quad +$

$0 \quad 4 \quad 8$

$X = 8$

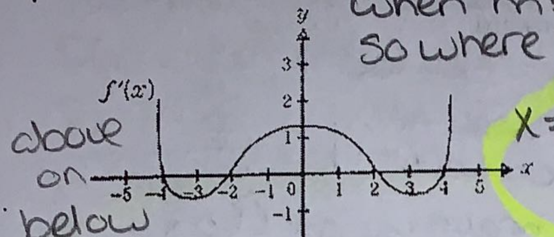
48. Refer to the previous figure. For what value(s) does the function have a relative maximum?

$X = 0$

49. The graph $f(x)$ has horizontal tangents when $x =$

when $m=0$

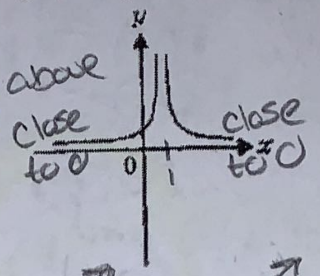
So where $f'(x)=0$

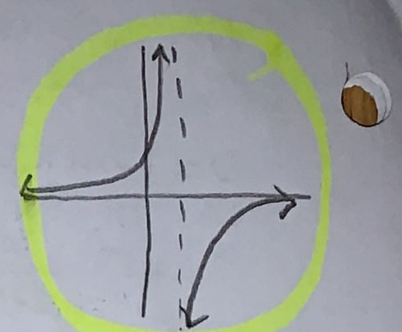


$x=-4,-2,$
$2,+4$

$+ \; 0 \; - \; 0 \; + \; 0 \; - \; 0 \; +$
$-4 \quad -2 \quad 2 \quad 4$

50. The graph of the derivative is shown. Draw the graph of f.

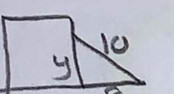




close to 0 $\;+\;+$ DNE $+\;+$ close to 0

f Incr. Incr

51. A ladder 10 feet long is leaning against a wall, with the foot of the ladder 8 feet away from the wall. If the foot of the ladder is being pulled away from the wall at 3 ft/sec how fast is the top of the ladder sliding down the wall?



K: $dx/dt = 3$ ft/sec

F: dy/dt

W: $x=8$ $y=6$ $L=10$

$8^2+y^2=10^2$

$y=6$

$x^2+y^2=L^2$

$2x\,dx/dt + 2y\,dy/dt = 2L\,dL/dt$

$2(8)(3)+2(6)dy/dt = 2(10)(0)$

$\frac{dy}{dt} = -4$ ft/sec

52. Find all value(s) of x (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval $[0,1]$.

$f(x)=(1+x)^{-1}$

$f'(c) = -1(1+c)^{-2}$

$f'(c) = \frac{-1}{(1+c)^2}$

$f(x)$ is continuous on $[0,1]$

$f(x)$ is differentiable on $(0,1)$

Therefore, there exists c in $(0,1)$ such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$f(0)=1$

$f(1)=\frac{1}{2}$

$\frac{-1}{(1+c)^2} = \frac{\frac{1}{2}-1}{1-0}$

$-2\left(-\frac{1}{2}(1+c)^2 = -1\right)$

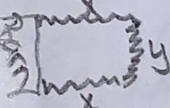
$\sqrt{(1+c)^2} = \sqrt{2}$

$1+c = \pm\sqrt{2}$

$c = -1 \pm \sqrt{2}$

$c = -1+\sqrt{2}$ only

53. A farmer has 20 feet of fence, and he wishes to make from it a rectangular pen for his pig Wilbur, using a barn as one of the sides. In square feet, What is the maximum area possible for his pet?



$20 = 2x+y$

$y = -2x+20$

$y=-2(5)+20$

$y=10$

$A=5(10)$

$A = 50\text{ ft}^2$

$A = xy$

$A = x(-2x+20)$

$A = -2x^2+20x$

$dA/dx = -4x+20$

$0 = -4x+20$

$x=5$

54. One person is walking south toward an intersection that is 60 ft away at a rate of 2 ft/s while a second person on a bicycle is riding east away from the same intersection at 10 ft/s. If the bicyclist is 80 ft from the intersection, how fast is the distance between he and the person walking increasing?

K: $dy/dt = -2$ ft/s $\quad dx/dt = 10$ ft/se

F: dd/dt

W: $x=80$ $y=60$ $d=100'$

$80^2+60^2 = d^2$

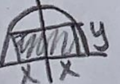
$x^2+y^2=d^2$

$2x\,dx/dt + 2y\,dy/dt = 2d\,dd/dt$

$2(80)(10)+2(60)(-2) = 2(100)\,dd/dt$

$dd/dt = 6.8$ ft/sec

55. A rectangle is inscribed between the parabola $y = 7 - x^2$ and the x-axis, with its base on the x-axis. Find the value of x that maximizes the area of the rectangle.



$y = 7-x^2$

$y = 7-\left(\sqrt{\tfrac{7}{3}}\right)^2$

$y = \frac{14}{3}$

$A = 2x \cdot y$

$A = 2x(7-x^2)$

$A = 14x - 2x^3$

$A' = 14-6x^2$

$0 = 14-6x^2$

$x = \sqrt{7/3}$

56. A circular conical reservoir, vertex down, has a depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of ½ ft/hr. Find the rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 feet deep.

$\frac{r}{h} = \frac{10}{20}$

$\frac{10h}{20} = \frac{20r}{20}$

$r = \frac{1}{2}h$

K: $dh/dt = -\frac{1}{2}$ ft/hr

F: dV/dt

W: $h = 8$ ft

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \frac{1}{4}h^3$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi(8)^2\left(-\frac{1}{2}\right)$

$\frac{dV}{dt} = -8\pi$ ft³/hr

Midterm Review Extra Word Problems

Solve each related rate problem.

1) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 5 m/min. How fast is the area of the spill increasing when the radius is 6 m?

K: $dr/dt = 5 m/min$
F: dA/dt
W: $r = 6m$

$A = \pi r^2$
$dA/dt = 2\pi r \, dr/dt$
$dA/dt = 2\pi(6)(5)$
$dA/dt = 60\pi \ m^2/min$

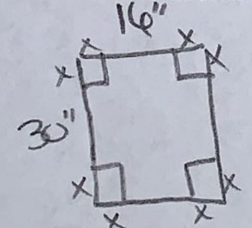
2) A spherical balloon is inflated so that its radius increases at a rate of 2 cm/sec. How fast is the volume of the balloon increasing when the radius is 9 cm?

K: $dr/dt = 2 cm/sec$
F: dV/dt
W: $r = 9 cm$

$V = \frac{4}{3}\pi r^3$
$dV/dt = 4\pi r^2 \, dr/dt$
$dV/dt = 4\pi(9)^2(2)$
$dV/dt = 648 \ cm^3/sec$

Solve each optimization problem.

3) A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?



Height = x
Length = 16 - 2x
Width = 30 - 2x

$V = LWH$
$V = x(16-2x)(30-2x)$
$V = 480x - 92x^2 + 4x^3$
$V' = 480 - 184x + 12x^2$
$0 = 4(3x^2 - 46x + 120)$
$0 = 4(3x-10)(x-12)$
$x = 10/3 \quad x = 12$

max
+ − +
10/3 12

$x = \frac{10}{3}$ in