Basic Related Rates Review

1. Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 cm^3/s$. How fast is the radius of the balloon increasing when the diameter is 50cm?

K: $\frac{dV}{dt} = 100 cm^3/s$

F: $\frac{dr}{dt}$

W: $d = 50cm$, $r = 25cm$

$V = \frac{4}{3}\pi r^3$

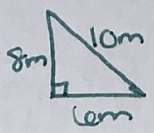
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$100 = 4\pi (25)^2 \frac{dr}{dt}$

$\frac{25}{625\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{25\pi}$ cm/sec

2. A ladder 10m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6m from the wall?



K: $\frac{dx}{dt} = 1 m/s$

F: $\frac{dy}{dt}$

W: $x = 6m$, $y = 8m$, $L = 10m$

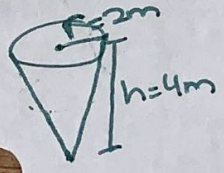
$x^2 + y^2 = L^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$6(1) + 8\left(\frac{dy}{dt}\right) = 0$

$\frac{dy}{dt} = -\frac{3}{4} m/s$

3. A water tank has the shape of an inverted cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate if $2m^3/min$, find the rate at which the water level is rising when the tank is 3m deep.



K: $\frac{dV}{dt} = 2 m^3/min$

F: $\frac{dh}{dt}$

W: $h = 3m$

$\frac{r}{h} = \frac{2}{4}$

$2h = 4r$

$\frac{1}{2}h = r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

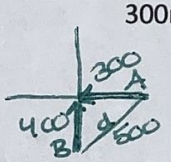
$dV = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$2 = \frac{1}{4}\pi (3^2)\frac{dh}{dt}$

$\frac{dh}{dt} = 2 \cdot \frac{4}{9\pi}$

$\frac{dh}{dt} = \frac{8}{9\pi} m/min$

4. Car A is traveling west at 50km/h and car B is traveling north at 60km/h. Both cars are headed for the intersection of the two roads. At what rate are the two cars approaching each other when car A is 300m and car B is 400m from the intersection?



K: $\frac{dA}{dt} = -50 Km/hr$ $\frac{dB}{dt} = -60 Km/hr$

F: $\frac{dd}{dt}$

W: $A = 300m$ or $.3Km$, $B = 400m$ or $.4Km$, $d = 500m$ or $.5Km$

$A^2 + B^2 = d^2$

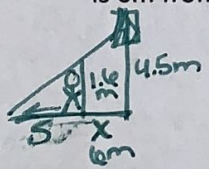
$A\frac{dA}{dt} + B\frac{dB}{dt} = d\frac{dd}{dt}$

$.3(-50) + .4(-60) = .5\frac{dd}{dt}$

$-15 - 24 = .5\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{-39}{5} = -78 Km/hr$

5. Sven, who is 1.6 m tall, walks away from the base of a 4.5 m high lamppost at a rate of 1.2 m/s. At what rate is the length of his shadow increasing when he is 6m from the lamppost?



K: $\frac{dx}{dt} = 1.2 m/s$

F: $\frac{ds}{dt}$

W: $x = 6cm$

$\frac{1.6}{s} = \frac{4.5}{x+s}$

$4.5s = 1.6x + 1.6s$

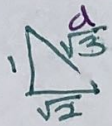
$2.9s = 1.6x$

$2.9\frac{ds}{dt} = 1.6\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{1.6(1.2)}{2.9}$

$\frac{ds}{dt} = 0.662 m/s$

6. The sides of a cube are increasing at a rate of 1cm/s. How fast is the diagonal of the cube changing when the side length is 1cm?



K: $\frac{dx}{dt} = 1 cm/s$

F: $\frac{dd}{dt}$

W: $s = 1cm$ $b = \sqrt{2}$ $d = \sqrt{3}$

$\sqrt{1^2 + 1^2} = \sqrt{2}$

$\sqrt{1^2 + \sqrt{2}^2} = \sqrt{1+2} = \sqrt{3}$

$x^2 + (\sqrt{2}x)^2 = d^2$

$x^2 + 2x^2 = d^2$

$3x^2 = d^2$

$d = \sqrt{3}x$

$\frac{dd}{dt} = \sqrt{3}\frac{dx}{dt}$

$\frac{dd}{dt} = \sqrt{3}(1)$

$\frac{dd}{dt} = \sqrt{3}$ cm/sec