7.1 The Substitution Rule (u-Substitution) p.73

Integration method of u-Substitution – Undoes a chain rule!!!

Steps for Integrating by u-Substitution
1. Choose a new variable u (usually what is raised to an exp, under radical, in denom. etc.)
2. Determine the value dx
3. Make the substitution
4. Integrate resulting integral
5. Return to the initial variable x.

1. $\int \sqrt{2x+1}\,dx$ | $u = 2x+1$, $du = 2\,dx$, $dx = \frac{du}{2}$

$\int \sqrt{u}\,\frac{du}{2}$

$\frac{1}{2}\int u^{1/2}\,du$

$\frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$\frac{1}{3}u^{3/2} + C$

$\frac{1}{3}(2x+1)^{3/2} + C$

Now subst. into integral so entire prob is in terms of u. Then sub. back in for u

2. $\int (3x-4)e^{3/2x^2-4x}\,dx$ | $u = \frac{3}{2}x^2 - 4x$, $du = (3x-4)\,dx$

$\int e^u\,du$

$e^u + C$

$e^{3/2x^2-4x} + C$

3. $\int \frac{8x}{e^{x^2}}\,dx$ | $u = -x^2$, $du = -2x\,dx$, $\frac{du}{-2} = x\,dx$

$\int 8xe^{-x^2}\,dx$

$\frac{8}{-2}\int e^u\,du$

$-4e^u + C$

$-4e^{-x^2} + C$ or $\frac{-4}{e^{x^2}} + C$

4. $\int \sec\frac{x}{\pi}\tan\frac{x}{\pi}\,dx$ | $u = \frac{x}{\pi}$, $du = \frac{1}{\pi}dx$, $\pi\,du = dx$

$\int \sec u \tan u\,du$

$\pi\int \sec u\tan u\,du$

$\pi \sec u + C$

$\pi\sec\left(\frac{x}{\pi}\right) + C$

5. $\int (5x+3)^4\,dx$ | $u = 5x+3$, $du = 5\,dx$, $\frac{du}{5} = dx$

$\frac{1}{5}\int u^4\,du$

$\frac{1}{5}\cdot\frac{u^5}{5} + C$

$\frac{(5x+3)^5}{25} + C$

6. $\int \csc^2 11x\,dx$ | $u = 11x$, $du = 11\,dx$, $\frac{du}{11} = dx$

$\frac{1}{11}\int \csc^2 u\,du$

$\frac{1}{11}\cdot -\cot u + C$

$\frac{-\cot 11x}{11} + C$

7. $\int \sin(5+17x)\,dx$ | $u = 5+17x$, $du = 17\,dx$, $\frac{du}{17} = dx$

$\frac{1}{17}\int \sin u\,du$

$\frac{1}{17}\cdot -\cos u + C$

$\frac{-\cos(5+17x)}{17} + C$

8. $\int x\cos(x^2)\,dx$ | $u = x^2$, $du = 2x\,dx$, $\frac{du}{2x} = dx$

$1/2\int \cos u\,du$

$1/2 \sin u + C$

$1/2\sin x^2 + C$

9. $\int \frac{3x^2}{5-4x^3}\,dx$ | $u = 5-4x^3$, $du = -12x^2\,dx$, $\frac{du}{-4} = 3x^2\,dx$

$-\frac{1}{4}\int \frac{1}{u}\,du$

$-\frac{1}{4}\ln|5-4x^3| + C$

Change of Variables for Definite Integrals

If the function $y=g(x)$ has a continuous derivative on the closed interval $[a,b]$ and f is continuous on the range g, then

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Steps for Integrating with Change of Variable for Definite Integrals

1. Define u for change of variables.
2. Differentiate u to find du + solve for dx
3. Substitute in the integrand + simplify
4. Use substitution to change limits of integration. (Be careful not to reverse order!)
5. If x still occurs anywhere in the integrand, take your def. of u from step 1, solve for x in terms of u, sub in the integrand + simplify.
6. Integrate.
7. Evaluate with u at the upper + lower new limits + subtract

Set up the integral. (#1 + 2)

1. $\int_0^1 \sqrt{t^5+3t}\,(5t^4+3)\,dt$

$u = t^5+3t$

$du = (5t^4+3)\,dt$

$\int_0^1 u^{1/2}\,du$

$\int_0^4 u^{1/2}\,du$

New bounds:

$u(1) = 1^5+3(1) = 4$

$u(0) = 0^5+3(0) = 0$

2. $\int_0^{\pi/3} \tan x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int_0^{\pi/3} u\,du$

New bounds

$u(\frac{\pi}{3}) = \sqrt{3}$

$u(0) = 1$

Extra variable

3. $\int x(3-x)^4\,dx$

$u = 3-x$

$du = -1\,dx$

$x = 3-u$

$-\int (3-u)u^4\,du$

$-\int 3u^4 - u^5\,du$

$-\left(\frac{3u^5}{5} - \frac{u^6}{6}\right) + C$

$-\frac{3(3-x)^5}{5} + \frac{(3-x)^6}{6} + C$

extra variable

4. $\int x^2(2x+5)^4\,dx$

$u = 2x+5$

$du = 2\,dx$

$\frac{du}{2} = dx$

$x = \frac{u-5}{2}$

$\frac{1}{2}\int \left(\frac{u-5}{2}\right)^2 u^4\,du$

$\frac{1}{2}\int \left(\frac{u^2-10u+25}{4}\right)u^4\,du$

$\frac{1}{8}\int (u^2-10u+25)u^4\,du$

$\frac{1}{8}\int (u^6-10u^5+25u^4)\,du$

$\frac{1}{8}\left(\frac{u^7}{7} - \frac{10u^6}{6} + \frac{25u^5}{5}\right) + C$

$\frac{(2x+5)^7}{56} - \frac{5(2x+5)^6}{24} + \frac{5(2x+5)^5}{8} + C$

7.2 Integration - Inverse Trigonometry

Inverse Trig Integration

$$\int \frac{1}{1+u^2}\,du = \arctan u + C \text{ or } \tan^{-1} u + C$$

$$\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin u + C \text{ or } \sin^{-1} u + C$$

$$\int \frac{1}{u\sqrt{u^2-1}}\,du = \operatorname{arcsec} u + C \text{ or } \sec^{-1} u + C$$

1. $\int \frac{dx}{\sqrt{1-4x^2}}$ looks like arcsinx Rewrite

$\int \frac{1}{\sqrt{1-(2x)^2}}\,dx$ | $u = 2x$, $du = 2\,dx$, $\frac{du}{2} = dx$

$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\frac{1}{2} \arcsin u + C$

$\frac{1}{2} \arcsin(2x) + C$

2. $\int \frac{dy}{y\sqrt{4y^2-1}}$ looks like arcsec x so rewrite

$\int \frac{1}{y\sqrt{(2y)^2-1}}\,dy$ | $u = 2y$, $du = 2\,dy$, $\frac{du}{2} = dy$, $y = \frac{u}{2}$

$\frac{1}{2}\int \frac{1}{\frac{u}{2}\sqrt{u^2-1}}\,du$

$\int \frac{1}{u\sqrt{u^2-1}}\,du$

$\sec^{-1} u + C = \sec^{-1}(2y) + C$

3. $\int \frac{12}{1+9x^2}\,dx$ looks like arctanx

$\int \frac{12}{1+(3x)^2}\,dx$ | $u = 3x$, $du = 3\,dx$, $\frac{du}{3} = dx$

$\frac{12}{3}\int \frac{1}{1+u^2}\,du$

$4\tan^{-1} u + C$

$4\tan^{-1} 3x + C$

4. $\int \frac{dx}{4+(x-3)^2}$ Factor out 4 from denominator!

$\frac{1}{4}\int \frac{1}{1+\frac{(x-3)^2}{4}}\,dx$ or $\frac{1}{4}\int \frac{1}{1+\left(\frac{x-3}{2}\right)^2}\,dx$

$\frac{1}{4}\cdot 2\int \frac{1}{1+u^2}\,du$ | $u = \frac{x-3}{2}$, $du = \frac{1}{2}\,dx$, $2\,du = dx$

$\frac{1}{2}\arctan u + C$

$\frac{1}{2}\arctan\left(\frac{x-3}{2}\right) + C$

5. $\int \frac{3\,dx}{2\sqrt{x}\,(1+x)}$

$u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$, $2\sqrt{x}\,du = dx$

$3\int \frac{1}{1+x}\,dx$

$3\int \frac{1}{1+u^2}\,du$

$3\tan^{-1} u + C$

$3\tan^{-1}\sqrt{x} + C$

7.3 Trigonometric Integrals

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Strategies:

- If $\sin x$ or $\cos x$ are raised to an odd power, pull aside one $\sin x$ or $\cos x$ & reverse $\sin x$ or $\cos x$ for your du. That should leave an even power that can be changed to the u value with $1-\sin^2 x$ or $1-\cos^2 x$
- If $\sin x$ or $\cos x$ are raised to an even power & they are alone without an odd power partner or anything to reserve for du, use the identity $\sin^2 u = \frac{1}{2}(1-\cos 2u)$ or $\cos^2 u = \frac{1}{2}(1+\cos 2u)$
- If $\tan x$ & $\sec x$ are in the integrand try to reserve $\sec^2 x\,dx$ or $\sec x \tan x\,dx$ as your du & change what is left to the appropriate u using $\tan^2 x = \sec^2 - 1$ or $1+\tan^2 x = \sec^2 x$

1. $\int \cos^3 dx$

$\int \cos x \cdot \cos^2 x\,dx$

$\int \cos x\,(1-\sin^2 x)\,dx$ $\quad u=\sin x$, $du = \cos x\,dx$

$\int 1-u^2\,du$

$u - \frac{u^3}{3} + C$

$\sin x - \frac{\sin^3 x}{3} + C$

2. $\int \sin^3 x \cos^2 x\,dx$

$\int \sin x \cdot \sin^2 x \cdot \cos^2 x\,dx$

$\int \sin x\,(1-\cos^2 x)\cos^2 x\,dx$ $\quad u=\cos x$, $du=-\sin x\,dx$

$-\int (1-u^2)u^2\,du$

$-\int (u^2-u^4)\,du$

$-\frac{u^3}{3} + \frac{u^5}{5} + C$

$-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

3. set aside

$\int \tan^2 x \sec^2 x\,dx$ $\quad u=\tan x$, $du=\sec^2 x\,dx$

$\int u^2\,du$

$\frac{u^3}{3} + C$

$\frac{\tan^3 x}{3} + C$

4. set aside

$\int \cos^3(2x)\sin(2x)\,dx$ $\quad u=\cos(2x)$, $du=-2\sin 2x\,dx$, $-\frac{du}{2} = \sin 2x\,dx$

$-\frac{1}{2}\int u^3\,du$

$-\frac{1}{2}\cdot\frac{u^4}{4} + C$

$-\frac{\cos^4(2x)}{8} + C$

5. $\int \sin^2 x \cos^2 x\,dx$ use power-reducing strategy

$\int \frac{1}{2}(1-\cos 2x)\cdot\frac{1}{2}(1+\cos 2x)\,dx$

$\frac{1}{4}\int 1-\cos^2(2x)\,dx$ ← use power-reducing again

$\frac{1}{4}\int 1-\left(\frac{1}{2}+\frac{1}{2}\cos 4x\right)dx$

$\frac{1}{4}\int 1-\frac{1}{2}-\frac{1}{2}\cos 4x\,dx$

$\frac{1}{4}\int \frac{1}{2}-\frac{1}{2}\cos 4x\,dx$

$\frac{1}{8}\int 1-\cos 4x\,dx$

$\frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C$

6. $\int \sin^4(2x)\,dx$ use power-reduc.

$\int \sin^2(2x)\cdot\sin^2(2x)\,dx$

$\int \frac{1}{2}(1-\cos 4x)\cdot\frac{1}{2}(1-\cos 4x)\,dx$

$\frac{1}{4}\int (1-\cos 4x)(1-\cos 4x)\,dx$

$\frac{1}{4}\int 1-2\cos 4x+\cos^2 4x\,dx$ use again

$\frac{1}{4}\int 1-2\cos 4x+\frac{1}{2}(1+\cos 8x)\,dx$

$1/4\int 1-2\cos 4x+\frac{1}{2}+\frac{1}{2}\cos 8x\,dx$

$1/4\int 3/2-2\cos 4x+\frac{1}{2}\cos 8x\,dx$

$\frac{1}{4}\left(\frac{3}{2}x - \frac{2\sin 4x}{4} + \frac{1}{2}\frac{\sin 8x}{8}\right) + C$

$\frac{3}{8}x - \frac{\sin 4x}{16} + \frac{\sin 8x}{64} + C$

7. $\int \sec^4(x)\,dx = \int \sec^2 x \cdot \sec^2 x\,dx$

$\int (1+\tan^2 x)\sec^2 x\,dx$ $\quad \boxed{u=\tan x,\ du=\sec^2 x\,dx}$

$\int (1+u^2)\,du$

$u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$

7.4 Integration By Parts

Undoes the product rule

$$\boxed{\int uv' = uv - \int v\,du}$$

How to choose u: L.I.A.T.E.

- **L**ogs (Ln)
- **I**nverse trig
- **A**lgebraic
- **T**rig
- **E**xponential

u is the one that comes 1st in LIATE

1. $\int \overset{A}{x} \cdot \overset{E}{e^x}\,dx$

$u = x \qquad dv = e^x\,dx$

$du = 1\,dx \qquad v = e^x$

$uv - \int v\,du$

$x e^x - \int e^x\,dx$

$x e^x - e^x + C$

2. $\int \ln x\,dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$x \cdot \ln x - \int x \cdot \frac{1}{x}\,dx$

$x \ln x - \int 1\,dx$

$x \ln x - x + C$ ← memorize this one

3. $\int \overset{A}{x^6} \cdot \overset{L}{\ln x}\,dx$

$u = \ln x \qquad dv = x^6$

$du = \frac{1}{x}dx \qquad v = \frac{x^7}{7}$

$\frac{\ln x \cdot x^7}{7} - \int \frac{x^7}{7} \cdot \frac{1}{x}\,dx$

$\frac{x^7 \ln x}{7} - \frac{1}{7}\int x^6\,dx$

$\frac{x^7 \ln x}{7} - \frac{1}{7} \cdot \frac{x^7}{7} + C$

$\frac{x^7 \ln x}{7} - \frac{x^7}{49} + C$

4. $\int \overset{A}{x^2} \cdot \overset{T}{\sin 4x}\,dx$

$u = x^2 \qquad dv = \sin 4x$

$du = 2x\,dx \qquad v = -\frac{\cos 4x}{4}$

$\frac{-x^2 \cos 4x}{4} - \int -\frac{\cos 4x}{4} \cdot 2x\,dx$

$\frac{-x^2 \cos 4x}{4} + \frac{1}{2}\int \overset{u}{x} \cdot \overset{dv}{\cos 4x}\,dx$

Int by Parts again

$u = x$

$du = dx$

$dv = \cos 4x$

$v = \frac{\sin 4x}{4}$

$\frac{-x^2 \cos 4x}{4} + \frac{1}{2}\left(\frac{x \sin 4x}{4} - \int \frac{\sin 4x}{4}\,dx\right)$

$-\frac{x^2 \cos 4x}{4} + \frac{1}{2}\left(\frac{x \sin 4x}{4} + \frac{\cos 4x}{16}\right) + C$

$\frac{-x^2 \cos 4x}{4} + \frac{x \sin 4x}{8} + \frac{\cos 4x}{32} + C$

5. $\int \overset{T}{\cos(3x)}\overset{E}{e^x}\,dx$

$u = \cos(3x) \qquad dv = e^x$

$du = -3\sin(3x) \qquad v = e^x$

$\cos(3x)e^x - \int -3e^x \sin(3x)\,dx$

$e^x \cos(3x) + 3\int \overset{E}{e^x}\,\overset{T}{\sin(3x)}\,dx$ ← by parts again

$u = \sin(3x) \qquad dv = e^x$

$du = 3\cos(3x)\,dx \qquad v = e^x$

$e^x \cos(3x) + 3\left(e^x \sin(3x) - 3\int e^x \cos(3x)\,dx\right)$

$e^x \cos(3x) + 3e^x \sin 3x - 9\int e^x \cos 3x\,dx$

*original integral!

$\int e^x \cos 3x\,dx = e^x \cos 3x + 3e^x \sin 3x - 9\int e^x \cos 3x\,dx$

$10\int e^x \cos 3x\,dx = e^x \cos 3x + 3e^x \sin 3x$

$\int e^x \cos 3x\,dx = \frac{e^x \cos 3x + 3e^x \sin 3x}{10} + C$

Called a cycling problem

*cycling problems: generally do the formula twice, set expression equal to problem, add to other side + divide coefficient continue 7.4 — p. 80

Int. by Parts Table method can be used when u is algebraic

Evaluate with table method:

6. $\int \overset{A}{x} \cdot \overset{T}{\cos(3x)}\,dx$ *same as #5

deriv to 0 ↓ | | integrate ↓

	u	dv
+	x	$\cos 3x$
−	1	$\frac{\sin 3x}{3}$
+	0	$-\frac{\cos 3x}{9}$

$$\frac{x\sin 3x}{3} + \frac{\cos 3x}{9} + C$$

7. $\int \overset{A}{x^2}\, \overset{E}{e^x}\,dx$

	u	e^x
+	x^2	e^x
−	$2x$	e^x
+	2	e^x
	0	e^x

$$x^2e^x - 2xe^x + 2e^x + C$$

8. $\int \overset{A}{(x^2+2x+1)}\,\overset{T}{\sin x}\,dx$

	u	dV
+	x^2+2x+1	$\sin x$
−	$(2x+2)$	$-\cos x$
+	2	$-\sin x$
	0	$\cos x$

$$-(x^2+2x+1)\cos x + (2x+2)\sin x + 2\cos x + C$$

Continue example 4 from p.77

7th Rewrite integral (using step 2) & Integrate

$$\int \frac{-1}{2x-1} + \frac{3}{x+1}\,dx$$

$$-\int \frac{1}{2x-1}\,dx + 3\int \frac{1}{x+1}\,dx$$

$$\boxed{\begin{array}{ll} u_1 = 2x-1 & u_2 = x+1 \\ du_1 = 2\,dx & du_2 = dx \\ \frac{du_1}{2} = dx & \end{array}}$$

$$-\frac{1}{2}\int \frac{1}{u}\,du + 3\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u| + 3\ln|u| + C = -\frac{1}{2}\ln|2x-1| + 3\ln|x+1| + C$$

7.5 Long Division + Partial Fractions

Polynomial Long Division: When integrating a fraction that has a polynomial in the numerator + denominator, if the degree of the numerator is ≥ the degree of denominator, use long or synthetic division to simplify the fraction before integrating.

ALGEBRA II STEPS FOR POLYNOMIAL LONG DIVISION

Step 1: Make sure the polynomial is written in descending order. If any terms are missing, use a zero to fill in the missing term (this will help with the spacing).

Step 2: Divide the term with the highest power inside the division symbol by the term with the highest power outside the division symbol.

Step 3: Multiply (or distribute) the answer obtained in the previous step by the polynomial in front of the division symbol.

Step 4: Subtract and bring down the next term.

Step 5: Repeat Steps 2, 3, and 4 until there are no more terms to bring down.

Step 6: Write the final answer. The term remaining after the last subtract step is the remainder and must be written as a fraction in the final answer.

1. $\int \frac{x^2}{x+1}\,dx$

Long Division:

$$\begin{array}{r} x - 1 + \frac{1}{x+1} \\ x+1 \overline{\smash{)}\, x^2 + 0x + 0} \\ \underline{-x^2 - 1x} \\ -x + 0 \\ \underline{+x + 1} \\ 1 \end{array}$$

$$\int x - 1 + \frac{1}{x+1}\,dx$$

$$\frac{x^2}{2} - x + \int \frac{1}{x+1}\,dx$$

$$\boxed{\begin{array}{l} u = x+1 \\ du = dx \end{array}}$$

$$\frac{x^2}{2} - x + \int \frac{1}{u}\,du$$

$$\frac{x^2}{2} - x + \ln|u| + C$$

$$\frac{x^2}{2} - x + \ln|x+1| + C$$

2. $\int \frac{x}{x+2}\,dx$

Long ÷

$$\begin{array}{r} 1 - \frac{2}{x+2} \\ x+2 \overline{\smash{)}\, x + 0} \\ \underline{-x - 2} \\ -2 \end{array}$$

$$\int 1 - \frac{2}{x+2}\,dx$$

$$x - 2\int \frac{1}{x+2}\,dx$$

$$\boxed{\begin{array}{l} u = x+2 \\ du = dx \end{array}}$$

$$x - 2\int \frac{1}{u}\,du$$

$$x - 2\ln|u| + C$$

$$x - 2\ln|x+2| + C$$

3. $\int \frac{x^3 - 2x + 5}{x-2}\,dx$

Synth. ÷

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -2 & 5 \\ & \downarrow & 2 & 4 & 4 \\ \hline & 1 & 2 & 2 & 9 \end{array}$$

$$x^2 + 2x + 2 + \frac{9}{x-2}$$

$$\int x^2 + 2x + 2 + \frac{9}{x-2}\,dx$$

$$\frac{x^3}{3} + x^2 + 2x + 9\int \frac{1}{x-2}\,dx$$

$$\frac{x^3}{3} + x^2 + 2x + 9\ln|x-2| + C$$

ALGEBRA II STEPS FOR PARTIAL FRACTION DECOMPOSITION

1. Set up the partial fraction decomposition with the unknown constants A, B, C, etc., in the numerator of the composition.
2. Multiply both sides of the resulting equation by the least common denominator.
3. Simplify the right-hand side of the equations.
4. Write both sides in descending powers, equate coefficients of like powers of x, and equate constant terms.
5. Solve the resulting linear system for A, B, C, etc.
6. Substitute the values for A, B, C, etc., into the equation in the first step.

Use Partial Fraction Decomposition when you can't divide but your denominator is factorable.

Case	Fraction $\frac{N(x)}{D(x)}$	Form of denominator, D(x)	Partial Fraction Form (where A, B and C are unknown constants)
1	$\frac{N(x)}{(ax+b)(cx+d)}$	Linear Factors (distinct)	$\frac{A}{ax+b} + \frac{B}{cx+d}$
2	$\frac{N(x)}{(ax+b)^2}$	Repeated Linear Factors like $(x+2)^2$	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
	$\frac{N(x)}{(ax+b)(cx+d)^2}$	Linear and Repeated Linear Factors	$\frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$
3	$\frac{N(x)}{(ax+b)(x^2+c^2)}$	Linear and Quadratic (which cannot be factorised) Factors	$\frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$

4. $\int \frac{5x-4}{2x^2+x-1}\,dx$

1st Factor denominator:

$(2x-1)(x+1)$

2nd Set up Partial Fraction:

$$\frac{A}{2x-1} + \frac{B}{x+1}$$

3rd Common Denominator:

$$\frac{x+1}{x+1}\cdot\frac{A}{2x-1} + \frac{B}{x+1}\cdot\frac{2x-1}{2x-1}$$

$$\frac{Ax + A + 2Bx - B}{2x^2 + x - 1}$$

4th Set the numerators equal:

$Ax + A + 2Bx - B = 5x - 4$

5th Set like terms equal to each other:

$A + 2B = 5$ and $A - B = -4$

6th Solve the system of equations:

$$\begin{array}{rl} A + 2B &= 5 \\ -(A - B &= -4) \\ \hline 3B &= 9 \\ B &= 3 \end{array} \qquad \begin{array}{rl} A - B &= -4 \\ A - 3 &= -4 \\ A &= -1 \end{array}$$

*Continued on left page!

5. $\int \frac{2}{x^2+3x-4}\,dx$

1. x^2+3x-4
$(x+4)(x-1)$

2. $\frac{(x-1)}{(x-1)} \cdot \frac{A}{x+4} + \frac{B}{x-1} \cdot \frac{(x+4)}{(x+4)}$

3. $\frac{Ax-A+Bx+4B}{(x+4)(x-1)}$

4. $Ax-A+Bx+4B=2$

5. $A+B=0$ + $-A+4B=2$

6. $A+B=0$
$-A+4B=2$
$5B=2$
$B=\frac{2}{5}$

$A+B=0$
$A+\frac{2}{5}=0$
$A=-\frac{2}{5}$

7. $\int \frac{-2/5}{x+4} + \frac{2/5}{x-1}\,dx$

$\int \frac{2/5}{x-1}\,dx - \int \frac{2/5}{x+4}\,dx$

switched order so positive 1st

$\frac{2}{5}\ln|x-1| - \frac{2}{5}\ln|x+4| + C$

* Use quick integral since denominator's have L.C. = 1

6. $\int \frac{x^2-x+1}{x^2+x}\,dx$

long ÷

$$1 + \frac{-2x+1}{x^2+x}$$

$$x^2+x+0 \overline{\big)\, x^2-x+1}$$
$$\underline{-x^2-x-0}$$
$$-2x+1$$

$\int 1 + \frac{-2x+1}{x^2+x}\,dx$

$x + \int \frac{-2x+1}{x^2+x}\,dx$ ← Partial fractions

1. $x^2+x = x(x+1)$

2. $\frac{A}{x} + \frac{B}{x+1}$

3. $\frac{Ax+A+Bx}{x(x+1)}$ ← not necessary to write

4. $Ax+A+Bx = -2x+1$

5. $A+B=-2$ and $A=1$

6. $A=1$ so $A+B=-2$
$1+B=-2$
$B=-3$

7. $x + \int \frac{1}{x} + \frac{-3}{x+1}\,dx$

$x + \ln|x| - 3\ln|x+1| + C$