AB Calculus: Rules of Integration

Day	Date	Topic	Assignment
1	Monday, Nov. 9th	Keeper 7.1 - The Substitution Rule	U substitution (packet pgs. 1-3)
2	Tuesday, Nov. 10th	Keeper 7.2 – Integration Inverse Trigonometry	Arc Trig Integration (packet pgs. 4-5)
3	Wednesday, Nov. 11th	Optional Q & A Review U Sub and Arc Trig Integrals	Catch up on all keeper notes and homework. Complete review of U sub and arc trig for extra practice.
4	Thursday, Nov. 12th	Keeper 7.3 - Trigonometric Integrals	**Skills Check – K7.1** Trigonometric Integrals (packet pgs. 6-7)
5	Friday, Nov. 13th	Keeper 7.4 – Integration by Parts	**Skills Check – K7.1 – K7.3** Integration by Parts (packet pgs. 8-10)
6	Monday, Nov. 16th	Keeper 7.5 – Long Division and Partial Fractions	**Skills Check – Quick Integrals 3** Partial Fraction (packet pg. 11-12)
7	Tuesday, Nov. 17th	Review – Mix it Up!!	**Skills Check – K7.4 – 7.5** Catch up on all Homework and Keeper Notes
8	Wednesday, Nov. 18th	Optional Q & A Review Integration Rules	Complete Unit 7 Homework Packet Complete Unit 7 Additional Review
9	Thursday, Nov. 19th	Unit 7 Test – Integration Rules	

U Substitution

1. $\int x\cdot\cos(x^2)dx$

$u=x^2$

$du=2x\,dx$

$\frac{du}{2}=x\,dx$

$\frac{1}{2}\int\cos(u)\,du$

$\frac{1}{2}\sin(u)+C$

$\frac{1}{2}\sin x^2+C$

2. $\int x^2\cdot\sin(4x^3+8)\ dx$

$u=4x^3+8$

$du=12x^2dx$

$\frac{du}{12}=x^2dx$

$\frac{1}{12}\int\sin(u)\,du$

$-\frac{1}{12}\cos u+C$

$-\frac{1}{12}\cos(4x^3+8)+C$

3. $\int\frac{x^4}{x^5-17}\,dx$

$u=x^5-17$

$du=5x^4dx$

$\frac{du}{5}=x^4dx$

$\frac{1}{5}\int\frac{1}{u}\,du$

$\frac{1}{5}\ln|u|+C$

$\frac{1}{5}\ln|x^5-17|+C$

4. $\int 3x^2(x^3+1)^4\,dx$

$u=x^3+1$

$du=3x^2dx$

$\int u^4\,du$

$\frac{u^5}{5}+C$

$\frac{(x^3+1)^5}{5}+C$

5. $\int\frac{x^5}{\sqrt[3]{2x^3+7}}\,dx$ $\leftarrow x^2\cdot x^3$

$u=2x^3+7$

$du=6x^2dx$

$\frac{du}{6}=x^2dx$

$\frac{1}{6}\int\frac{\frac{u-7}{2}}{u^{1/3}}\,du$

$x^3=\frac{u-7}{2}$

$\frac{1}{12}\int\frac{u}{u^{1/3}}-\frac{7}{u^{1/3}}du$

$\frac{1}{12}\int u^{2/3}-7u^{-1/3}\,du$

$\frac{1}{12}\left(\frac{3}{5}u^{5/3}-\frac{21u^{2/3}}{2}\right)+C=\frac{1}{20}(2x^3+7)^{5/3}-\frac{7}{8}(2x^3+7)^{2/3}+C$

6. $\int(1+\sin(x))^{\frac{5}{2}}\cos(x)\,dx$

$u=1+\sin x$

$du=\cos x\,dx$

$\int u^{5/2}\,du$

$\frac{2u^{7/2}}{7}+C$

$\frac{2}{7}(1+\sin x)^{7/2}+C$

7. $\int xe^{-x^2}dx$

$u=-x^2$

$du=-2x\,dx$

$\frac{du}{-2}=x\,dx$

$-\frac{1}{2}\int e^u\,du$

$-\frac{1}{2}e^u+C$

$-\frac{1}{2}e^{-x^2}+C$ or $\frac{-1}{2e^{x^2}}+C$

8. $\int 4\cos(6x)\ dx$

$4\int\cos(6x)\,dx$

$\frac{4\sin 6x}{6}+C$

$\frac{2\sin 6x}{3}+C$

9. $\int\frac{\ln(x)}{x}\,dx$

$u=\ln x$

$du=\frac{1}{x}\,dx$

$\int u\,du$

$\frac{u^2}{2}+C$

$\frac{(\ln x)^2}{2}+C$

10. $\int x^2e^{4x^3}dx$

$u=4x^3$

$du=12x^2dx$

$\frac{du}{12}=x^2dx$

$\frac{1}{12}\int e^u\,du$

$\frac{1}{12}e^u+C$

$\frac{1}{12}e^{4x^3}+C$

11. $\int \frac{x^2 dx}{(x^3-1)^2}$

$u=x^3-1$
$du=3x^2dx$
$\frac{du}{3}=x^2dx$

$\frac{1}{3}\int u^{-2}du$

$\frac{1}{3}\cdot\frac{u^{-1}}{-1}+C$

$\frac{-1}{3(x^3-1)}+C$

12. $\int x(3-x^2)^5dx$

$u=3-x^2$
$du=-2x\,dx$
$\frac{du}{-2}=x\,dx$

$-\frac{1}{2}\int u^5du$

$-\frac{1}{2}\cdot\frac{u^6}{6}+C$

$-\frac{(3-x^2)^6}{12}+C$

13. $\int \sin 3x\ dx$

$-\frac{\cos 3x}{3}+C$

14. $\int t^3\sqrt{t^4+2}\ dt$

$u=t^4+2$
$du=4t^3du$
$\frac{du}{4}=t^3du$

$\frac{1}{4}\int u^{1/2}du$

$\frac{1}{4}\cdot\frac{2}{3}u^{3/2}+C$

$\frac{1}{3}(t^4+2)^{3/2}+C$

15. $\int \sin(7x+5)dx$

$-\frac{\cos(7x+5)}{7}+C$

16. $\int \sec^2\left(\frac{x}{4}\right)\ dx$

$4\tan\left(\frac{x}{4}\right)+C$

17. $\int \frac{y+2y^2}{\sqrt{y}}\ dy$

$\int \frac{y}{y^{1/2}}+\frac{2y^2}{y^{1/2}}\ dy$

$\int y^{1/2}+2y^{3/2}\ dy$

$\frac{2}{3}y^{3/2}+\frac{4}{5}y^{5/2}+C$

18. $\int x\cdot\sin(2x^2)dx$

$u=2x^2$
$du=4x\,dx$
$\frac{du}{4}=x\,dx$

$\frac{1}{4}\int\sin(u)\,du$

$-\frac{1}{4}\cos(u)+C$

$-\frac{1}{4}\cos(2x^2)+C$

19. $\int \cos\sqrt{x}\,\frac{dx}{\sqrt{x}}$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

$2du=\frac{dx}{\sqrt{x}}$

$2\int \cos u\,du$

$2\sin u+C$

$2\sin\sqrt{x}+C$

20. $\int 5(4x-7)^3dx$

$u=4x-7$

$du=4\,dx$

$\frac{du}{4}=dx$

$\frac{5}{4}\int u^3\,du$

$\frac{5u^4}{16}+C$

$\frac{5}{16}(4x-7)^4+C$

21. $\int \cos^2 2y\cdot\sin 2y\ dy$

$u=\cos 2y$

$du=-2\cos 2y\,dy$

$\frac{du}{-2}=\cos 2y\,dy$

$-\frac{1}{2}\int u^2\,du$

$-\frac{1}{2}\cdot\frac{u^3}{3}+C$

$-\frac{1}{6}\cos^3(2y)+C$

22. $\int \sec(2x+1)\tan(2x+1)\ dx$

$\frac{\sec(2x+1)}{2}+C$

23. $\int\sqrt{2+\sin 3t}\,\cos 3t\ dt$

$u=2+\sin 3t$

$du=3\cos 3t\,dt$

$\frac{du}{3}=\cos 3t\,dt$

$\frac{1}{3}\int u^{1/2}\,dt$

$\frac{1}{3}\cdot\frac{2u^{3/2}}{3}+C$

$\frac{2}{9}(2+\sin 3t)^{3/2}+C$

24. $\int t^2\left(\frac{1}{2}t^3-4\right)dt$

$u=\frac{1}{2}t^3-4$

$du=\frac{3}{2}t^2\,dt$

$\frac{2}{3}du=t^2\,dt$

$\frac{2}{3}\int u\,du$

$\frac{2}{3}\cdot\frac{u^2}{2}+C$

$\frac{1}{3}\left(\frac{1}{2}t^3-4\right)^2+C$

25. $\int x(2x+5)^8\,dx$

$u=2x+5$

$du=2\,dx$

$\frac{du}{2}=dx$

$x=\frac{u-5}{2}$

$\frac{1}{2}\int\left(\frac{u-5}{2}\right)u^8\,du$

$\frac{1}{4}\int u^9-5u^8\,du$

$\frac{1}{4}\cdot\frac{u^{10}}{10}-\frac{1}{4}\cdot\frac{5u^9}{9}+C$

$\frac{(2x+5)^{10}}{40}-\frac{5(2x+5)^9}{36}+C$

26. $\int x\sqrt[4]{x+1}\,dx$

$u=x+1$

$du=dx$

$x=1-u$

$\int(1-u)u^{1/4}\,du$

$\int u^{1/4}-u^{5/4}\,du$

$\frac{4u^{5/4}}{5}-\frac{4u^{9/4}}{9}+C$

$\frac{4}{5}(x+1)^{5/4}-\frac{4}{9}(x+1)^{9/4}+C$

Arc Trig Integration

Evaluate the following integrals

1. $\int \frac{1}{\sqrt{1-4x^2}}\,dx$

$\int \frac{1}{\sqrt{1-(2x)^2}}dx$

$\frac{1}{2}\sin^{-1}(2x)+C$

2. $\int \frac{1}{x^2+25}\,dx$

$\int \frac{1}{25(\frac{x^2}{25}+1)}dx$

$\frac{1}{25}\int \frac{1}{(\frac{x}{5})^2+1}dx \qquad u=\frac{x}{5}$

$du=\frac{1}{5}dx$

$\frac{1}{5}\int \frac{1}{u^2+1}du \qquad 5du=dx$

$\frac{1}{5}\tan^{-1}u+C$

$\frac{1}{5}\tan^{-1}(\frac{x}{5})+C$

3. $\int \frac{x}{x^4+16}\,dx$

$\int \frac{x}{16(\frac{x^4}{16}+1)}dx$

$\frac{1}{16}\int \frac{x}{(\frac{x^2}{4})^2+1}dx \qquad u=\frac{x^2}{4}$

$du=\frac{1}{2}x\,dx$

$\frac{1}{8}\int \frac{1}{u^2+1}du \qquad 2du=x\,dx$

$\frac{1}{8}\tan^{-1}u+C$

$\frac{1}{8}\tan^{-1}(\frac{x^2}{4})+C$

4. $\int \frac{1}{\sqrt{2-5x^2}}\,dx$

$\int \frac{1}{\sqrt{2(1-\frac{5x^2}{2})}}$

$\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-(\sqrt{\frac{5}{2}}x)^2}}dx \qquad u=\sqrt{\frac{5}{2}}x$

$du=\sqrt{\frac{5}{2}}dx$

$\frac{\sqrt{2}}{\sqrt{5}}\cdot\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-u^2}}du \qquad \sqrt{\frac{2}{5}}du=dx$

$\frac{1}{\sqrt{5}}\sin^{-1}u+C$

$\frac{1}{\sqrt{5}}\sin^{-1}(\sqrt{\frac{5}{2}}x)+C$

5. $\int \frac{3}{x\sqrt{x^2-9}}\,dx$

$3\int \frac{1}{x\sqrt{9(\frac{x^2}{9}-1)}}dx$

$\frac{\cancel{3}}{\cancel{\sqrt{9}}}\int \frac{1}{x\sqrt{(\frac{x}{3})^2-1}}dx \qquad u=\frac{x}{3}$

$du=\frac{1}{3}dx$

$3\int \frac{1}{3u\sqrt{u^2-1}}du \qquad 3du=dx$

$x=3u$

$\int \frac{1}{u\sqrt{u^2-1}}du$

$\text{arcsec}\,u+C$

$\text{arcsec}\,(\frac{x}{3})+C$

6. $\int \frac{x}{\sqrt{16-9x^4}}\,dx$

$\int \frac{x}{\sqrt{16(1-\frac{9}{16}x^4}}dx$

$\frac{1}{4}\int \frac{x}{\sqrt{1-(\frac{3}{4}x^2)^2}}dx \qquad u=\frac{3}{4}x^2$

$du=\frac{3}{2}x\,dx$

$\frac{2}{3}\cdot\frac{1}{4}\int \frac{1}{\sqrt{1-u^2}}du \qquad \frac{2}{3}du=x\,dx$

$\frac{1}{6}\arcsin u+C$

$\frac{1}{6}\arcsin(\frac{3}{4}x^2)+C$

7. $\int \frac{1}{x\sqrt{16x^2-9}}\,dx$

$$\int \frac{1}{x\sqrt{9\left(\frac{16x^2}{9}-1\right)}}\,dx$$

$$\frac{1}{3}\int \frac{1}{x\sqrt{\left(\frac{4}{3}x\right)^2-1}}\,dx$$

$$\frac{\cancel{3}}{4}\cdot\frac{1}{\cancel{3}}\int \frac{1}{\frac{3}{4}u\sqrt{u^2-1}}\,du$$

$$\frac{4}{3}\cdot\frac{1}{4}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$\frac{1}{3}\sec^{-1}\left(\frac{4}{3}x\right)+C$$

$u=\frac{4}{3}x$

$du=\frac{4}{3}dx$

$\frac{3}{4}du=dx$

$x=\frac{3}{4}u$

8. $\int \frac{e^x}{7+e^{2x}}\,dx$

$$\int \frac{e^x}{7\left(1+\frac{e^{2x}}{7}\right)}\,dx$$

$$\frac{1}{7}\int \frac{e^x}{1+\left(\frac{e^x}{\sqrt{7}}\right)^2}\,dx$$

$$\frac{\sqrt{7}}{7}\int \frac{1}{1+u^2}\,du$$

$$\frac{\sqrt{7}}{7}\arctan\left(\frac{e^x}{\sqrt{7}}\right)+C$$

$u=\frac{e^x}{\sqrt{7}}$

$du=\frac{e^x}{\sqrt{7}}dx$

$\sqrt{7}\,du=e^x dx$

9. $\int \frac{\sin x}{\sqrt{2-\cos^2 x}}\,dx$

$$\int \frac{\sin x}{\sqrt{2(1-\left(\frac{\cos x}{\sqrt{2}}\right)^2}}\,dx$$

$$-\sqrt{2}\cdot\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$-1\cdot\sin^{-1}u+C$$

$$-\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)+C$$

$u=\frac{\cos x}{\sqrt{2}}$

$du=\frac{-\sin x}{\sqrt{2}}dx$

$-\sqrt{2}\,du=\sin x\,dx$

10. $\int \frac{1}{\sqrt{x}(1+x)}\,dx$

$$\int \frac{1}{\sqrt{x}(1+(\sqrt{x})^2}\,dx$$

$$\frac{1}{2}\int \frac{1}{1+u^2}\,du$$

$$\frac{1}{2}\arctan u+C$$

$$\frac{1}{2}\arctan\sqrt{x}+C$$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

Trigonometric Integrals

Integrate

1. $\int \tan^3 x \sec^2 x \, dx$

$u=\tan x$

$du=\sec^2 x\,dx$

$\int u^3\,du$

$\frac{1}{4}u^4+C$

$\frac{\tan^4 x}{4}+C$

2. $\int \frac{\sec^4(2t)}{\tan^9(2t)}\,dt$

$u=\tan(2t)$

$du=2\sec^2 2t\,dt$

$\frac{du}{2}=\sec^2 2t\,dt$

$\int \frac{\sec^2(2t)\sec^2(2t)}{\tan^9(2t)}\,dt$

$\int \frac{\sec^2(2t)\left(1+\tan^2(2t)\right)}{\tan^9(2t)}\,dt$

$\frac{1}{2}\int \frac{1+u^2}{u^9}\,du$

$\frac{1}{2}\int u^{-9}+u^{-7}\,du$

$\frac{1}{2}\cdot\frac{u^{-8}}{-8}+\frac{1}{2}\cdot\frac{u^{-6}}{-6}+C$

$\frac{-1}{16\tan^8 2t}-\frac{1}{12\tan^6 2t}+C$

3. $\int [9\sin^5(3x) - 2\cos^3(3x)]\csc^4(3x)\,dx$

$\int \left[9\sin^5(3x)-2\cos^3(3x)\right]\cdot\frac{1}{\sin^4 3x}\,dx$

$\int 9\sin 3x-\frac{2\cos^3(3x)}{\sin^4(3x)}\,dx$

$\int 9\sin 3x-\frac{2\cos(3x)\left(1-\sin^2(3x)\right)}{\sin^4(3x)}\,dx$

$u=\sin 3x$

$du=3\cos 3x\,dx$

$\frac{du}{3}=\cos 3x\,dx$

$9\int \sin 3x\,dx-2\cdot\frac{1}{3}\int\frac{1-u^2}{u^4}\,du$

$-9\cdot\frac{1}{3}\cos 3x-\frac{2}{3}\int u^{-4}-u^{-2}\,du$

$-3\cos 3x-\frac{2}{3}\cdot\frac{u^{-3}}{-3}+\frac{2}{3}\cdot\frac{u^{-1}}{-1}+C$

$-3\cos 3x+\frac{2}{9\sin^2 3x}-\frac{2}{3\sin 3x}+C$

4. $\int \cos^3(2x)\sin(2x)\,dx$

$u=\cos 2x$

$du=-2\sin 2x\,dx$

$\frac{du}{-2}=\sin 2x\,dx$

$-\frac{1}{2}\int u^3\,du$

$-\frac{1}{2}\cdot\frac{u^4}{4}+C$

$-\frac{\cos^4 2x}{8}+C$

5. $\int \cos^4(3x)dx$

$\int \cos^2(3x)\cos^2(3x)dx$

$\int \frac{1}{2}(1+\cos(6x))\cdot\frac{1}{2}(1+\cos 6x)dx$

$\frac{1}{4}\int 1+2\cos 6x+\cos^2 6x\,dx$

$\frac{1}{4}\int 1+2\cos 6x+\frac{1}{2}+\frac{1}{2}\cos 12x\,dx$

$\frac{1}{4}(x+\frac{1}{3}\sin 6x+\frac{1}{2}x+\frac{1}{24}\sin 12x)+C$

$\frac{1}{4}x+\frac{1}{12}\sin 6x+\frac{1}{8}x+\frac{1}{96}\sin 12x+C$

$\frac{3}{8}x+\frac{1}{12}\sin 6x+\frac{1}{96}\sin 12x+C$

6. $\int_0^{\frac{\pi}{3}} \sin^3 x\; dx$

$\int_0^{\pi/3} \sin x\cdot\sin^2 x\,dx$

$\int_0^{\pi/3} \sin x(1-\cos^2 x)dx$ $u=\cos x$ $du=-\sin x\,dx$

$-\int_0^{\pi/3} 1-u^2\,du$

$-(u-\frac{u^3}{3})\Big|_1^{1/2} = -u+\frac{u^3}{3}\Big|_1^{1/2}$

$(-\frac{1}{2}+\frac{1}{24})-(-1+\frac{1}{3})$

$-\frac{11}{24}+\frac{2}{3}$

$\frac{5}{24}$

7. $\int \tan^2(3x)\sec^6(3x)\;dx$

$\int \tan^2(3x)\sec^2(3x)\sec^4(3x)dx$

$\int \tan^2(3x)\sec^2(3x)\sec^2(3x)\sec^2 3x\,dx$ $u=\tan 3x$

$\int \tan^2(3x)\sec^2(3x)(\tan^2 3x-1)(\tan^2 3x-1)dx$ $du=3\sec^2 3x\,dx$

$\frac{du}{3}=\sec^2 3x\,dx$

$\frac{1}{3}\int u^2(u^2-1)(u^2-1)\,du$

$\frac{1}{3}\int u^6-2u^4+u^2\,du$

$\frac{1}{3}(\frac{u^7}{7}-\frac{2u^5}{5}+\frac{u^3}{3})+C$

$\frac{\tan^7(3x)}{21}-\frac{2\tan^5(3x)}{15}+\frac{\tan^3(3x)}{9}+C$

8. $\int \sin^3\left(\frac{2}{3}x\right)\cos^4\left(\frac{2}{3}x\right)\;dx$

$u=\cos(\frac{2}{3}x)$

$du=-\frac{2}{3}\sin(\frac{2}{3}x)dx$

$-\frac{3}{2}du=\sin(\frac{2}{3}x)dx$

$\int \sin(\frac{2}{3}x)\sin^2(\frac{2}{3}x)\cos^4(\frac{2}{3}x)dx$

$\int \sin(\frac{2}{3}x)(1-\cos^2(\frac{2}{3}x))\cos^4(\frac{2}{3}x)dx$

$-\frac{3}{2}\int (1-u^2)u^4\,du$

$-\frac{3}{2}\int u^4-u^6\,du$

$-\frac{3}{2}(\frac{u^5}{5}-\frac{u^7}{7})+C$

$\frac{-3\cos^5(\frac{2}{3}x)}{10}+\frac{3.\cos^7(\frac{2}{3}x)}{14}+C$

Integration by Parts

Find the Antiderivative

1. $\int \overset{A}{t}\,\overset{E}{e^{5t}}\,dt$

u	dV
$+\ t$	e^{5t}
$-\ 1$	$\frac{1}{5}e^{5t}$
$-\ 0$	$\frac{1}{25}e^{5t}$

$\frac{1}{5}te^{5t}-\frac{1}{25}e^{5t}+C$

2. $\int \overset{A}{t^2}\,\overset{E}{e^{5t}}\,dt$

u	dV
$+\ t^2$	e^{5t}
$-\ 2t$	$\frac{1}{5}e^{5t}$
$+\ 2$	$\frac{1}{25}e^{5t}$
0	$\frac{1}{125}e^{5t}$

$\frac{t^2e^{5t}}{5}-\frac{2te^{5t}}{25}+\frac{2e^{5t}}{125}+C$

3. $\int \overset{A}{p}\,\overset{E}{e^{-0.1p}}\,dp$

u	dV
$+\ P$	$e^{-.1p}$
$-\ 1$	$-10e^{-.1p}$
$+\ 0$	$100e^{-.1p}$

$-10pe^{-.1p}-100e^{-.1p}+C$

4. $\int \overset{A}{t}\,\overset{T}{\sin t}\,dt$

u	dV
$+\ t$	$\sin t$
$-\ 1$	$-\cos t$
$+\ 0$	$-\sin t$

$-t\cos t+\sin t+C$

5. $\int \overset{A}{y}\,\overset{L}{\ln y}\,dy$

$u=\ln y \quad dV=y$

$du=\frac{1}{y}dy \quad V=\frac{y^2}{2}$

$\ln y\cdot\frac{y^2}{2}-\int\frac{y^2}{2}\cdot\frac{1}{y}dy$

$\frac{y^2\ln y}{2}-\frac{1}{2}\int y\,dy$

$\frac{y^2\ln y}{2}-\frac{y^2}{4}+C$

6. $\int \overset{A}{x^3}\,\overset{L}{\ln x}\,dx$

$u=\ln x \quad dV=x^3$

$du=\frac{1}{x}dx \quad V=\frac{x^4}{4}$

$\ln x\cdot\frac{x^4}{4}-\int\frac{x^4}{4}\cdot\frac{1}{x}dx$

$\frac{1}{4}x^4\ln x-\frac{1}{4}\int x^3dx$

$\frac{1}{4}x^4\ln x-\frac{x^4}{16}+C$

7. $\int (\overset{A}{z+1})\,\overset{E}{e^{2z}}\,dz$

u	dV
$+\ Z+1$	e^{2Z}
$-\ 1$	$\frac{1}{2}e^{2Z}$
$+\ 0$	$\frac{1}{4}e^{2Z}$

$\frac{1}{2}(Z+1)e^{2Z}-\frac{1}{4}e^{2Z}+C$

8. $\int \frac{z}{e^z}\,dz$

$\int \overset{A}{Z}\cdot\overset{E}{e^{-Z}}\,dZ$

u	dV
$+\ Z$	e^{-Z}
$-\ 1$	$-e^{-Z}$
$+\ 0$	e^{-Z}

$-Ze^{-Z}-e^{-Z}+C$ or $\frac{-Z}{e^Z}-\frac{1}{e^Z}+C$

9. $\int \overset{A}{t^2} \overset{T}{\sin t}\, dt$

	u	dv
+	t^2	$\sin t$
-	$2t$	$-\cos t$
+	2	$-\sin t$
-	0	$\cos t$

$-t^2\cos t + 2t\sin t + 2\cos t + C$

10. $\int \overset{A}{\theta^2} \cdot \overset{T}{\cos 3\theta}\, d\theta$

	u	dV
+	θ^2	$\cos 3\theta$
-	2θ	$\frac{1}{3}\sin 3\theta$
+	2	$-\frac{1}{9}\cos 3\theta$
-	0	$-\frac{1}{27}\sin 3\theta$

$\frac{1}{3}\theta^2\sin 3\theta + \frac{2}{9}\theta\cos 3\theta - \frac{2}{27}\sin 3\theta + C$

11. $\int \sin^2\theta\, d\theta$

$\frac{1}{2}\int 1-\cos 2\theta\, d\theta$

$\frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$

$\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$

12. $\int \overset{A}{(\theta+1)} \overset{T}{\sin(\theta+1)}\, d\theta$

	u	dV
+	$\theta+1$	$\sin(\theta+1)$
-	1	$-\cos(\theta+1)$
+	0	$-\sin(\theta+1)$

$-(\theta+1)\cos(\theta+1) + \sin(\theta+1) + C$

13. $\int \cos^2(3x+1)dx$

$\int \frac{1}{2}\left(1+\cos(6x+2)\right)dx$

$\frac{1}{2}\left(x + \frac{\sin(6x+2)}{6}\right) + C$

$\frac{1}{2}x + \frac{\sin(6x+2)}{12} + C$

14. $\int \frac{\ln x}{x^2} dx = \int \overset{L}{\ln x} \cdot \overset{A}{x^{-2}} dx$

$u = \ln x$ $\quad dV = x^{-2}$

$du = \frac{1}{x}dx$ $\quad V = -\frac{1}{x}$

$\ln x \cdot \frac{-1}{x} - \int \frac{-1}{x} \cdot \frac{1}{x} dx$

$-\frac{\ln x}{x} + \int x^{-2} dx$

$-\frac{\ln x}{x} - \frac{1}{x} + C$

15. $\int \overset{A}{q^5} \overset{L}{\ln 5q}\, dq$

$u = \ln 5q$ $\quad dV = q^5$

$du = \frac{1}{q}dq$ $\quad V = \frac{q^6}{6}$

$\ln 5q \cdot \frac{q^6}{6} - \int \frac{q^6}{6} \cdot \frac{1}{q} dq$

$\frac{q^6}{6}\ln 5 - \frac{1}{6}\int q^5\, dq$

$\frac{q^6}{6}\ln 5 - \frac{1}{6} \cdot \frac{q^6}{6} + C$

$\frac{q}{6}\ln^5 - \frac{q^6}{36} + C$

16. $\int y\sqrt{y+3}\, dy$

$u = y+3$ $\quad y = u-3$

$du = dy$

$\int (u-3)u^{1/2} du$

$\int u^{3/2} - 3u^{1/2} du$

$\frac{2}{5}u^{5/2} - 3 \cdot \frac{2}{3}u^{3/2} + C$

$\frac{2}{5}(y+3)^{5/2} - 2(y+3)^{3/2} + C$

17. $\int x^3 e^{x^2} dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int u e^u du$

u	dV
$+u$	e^u
-1	e^u
$+0$	e^u

$\frac{1}{2}(ue^u - e^u) + C$

$\frac{1}{2}(x^2e^{x^2} - e^{x^2}) + C$

18. $\int x^5 \cos x^3\ dx$

$u = x^3$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$\frac{1}{3}\int u \cos u\, du$

u	dv
$+u$	$\cos u$
-1	$\sin u$
$+0$	$-\cos u$

$\frac{1}{3}(u \sin u + \cos u) + C$

$\frac{1}{3}x^3 \sin x^3 + \frac{1}{3}\cos x^3 + C$

19. $\int \overset{E}{e^{2x}} \overset{T}{\cos x}\ dx$

$u = \cos x \quad dv = e^{2x}$

$du = -\sin x\,dx \quad v = \frac{e^{2x}}{2}$

$\cos x \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \sin x\,dx$

$\frac{1}{2}\cos x e^{2x} + \frac{1}{2}\int e^{2x} \sin x\,dx$

$u = \sin x$

$du = \cos dx$

$dv = e^{2x}$

$v = \frac{e^{2x}}{2}$

$\frac{1}{2}\cos x e^{2x} + \frac{1}{2}\left(\sin x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\cos x\,dx\right)$

$\frac{1}{2}\cos x e^{2x} + \frac{1}{4}\sin x e^{2x} - \frac{1}{4}\int e^{2x}\cos x\,dx$

$\int e^{2x}\cos x\,dx = \frac{1}{2}\cos x e^{2x} + \frac{1}{4}\sin x \cdot e^{2x} - \frac{1}{4}\int e^{2x}\cos x\,dx$

$\frac{5}{4}\int e^{2x}\cos x\,dx = \frac{1}{2}\cos x e^{2x} + \frac{1}{4}\sin x e^{2x}$

$\int e^{2x}\cos x\,dx = \frac{2}{5}\cos x e^{2x} + \frac{1}{5}\sin x \cdot e^{2x} + C$

20. $\int e^{-x} \cos 2x\ dx$

$u = \cos 2x \quad dv = e^{-x} dx$

$du = -2\sin 2x\,dx \quad v = -e^{-x}$

$-e^{-x}\cos 2x - 2\int e^{-x} \sin 2x\,dx$

$u = \sin 2x \quad dv = e^{-x} dx$

$du = 2\cos 2x\,dx \quad v = -e^{-x}$

$-e^{-x}\cos 2x - 2\left(-e^{-x}\sin 2x + 2\int e^{-x}\cos 2x\,dx\right)$

$\int e^{-x}\cos 2x = -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\,dx$

$5\int e^{-x}\cos 2x = -e^{-x}\cos 2x + 2e^{-x}\sin 2x$

$= \frac{-e^{-x}\cos 2x + 2e^{-x}\sin 2x}{5} + C$

21. $\int e^{3x} \sin(-x)\ dx$

$u = \sin(-x) \quad dv = e^{3x}dx$

$du = -\cos(-x)dx \quad v = \frac{e^{3x}}{3}$

$\frac{\sin(-x)e^{3x}}{3} - \int \frac{-\cos(-x)e^{3x}}{3} dx$

$u = \cos(-x) \quad dv = e^{3x}dx$

$du = \sin(-x) \quad v = \frac{e^{3x}}{3}$

$\frac{\sin(-x)e^{3x}}{3} + \frac{1}{3}\int \cos(-x)e^{3x}dx$

$\frac{\sin(-x)e^{3x}}{3} + \frac{1}{3}\left(\frac{e^{3x}\cos(-x)}{3} - \frac{1}{3}\int e^{3x}\sin(-x)\,dx\right)$

$\int e^{3x}\sin(-x) = \frac{e^{3x}\sin(-x)}{3} + \frac{e^{3x}\cos(-x)}{9} - \frac{1}{9}\int e^{3x}\sin(-x)dx$

$\frac{9}{10}\cdot\left(\frac{10}{9}\int e^{3x}\sin(-x) = \frac{e^{3x}\sin(-x)}{3} + \frac{e^{3x}\cos(-x)}{9}\right)$

$= \frac{3e^{3x}\sin(-x)}{10} + \frac{e^{3x}\cos(-x)}{10} + C$

or $-\frac{3e^{3x}\sin x}{10} + \frac{e^{3x}\cos x}{10} + C$

(Even/odd trig identity)

22. $\int 2 \cdot e^{3x} \cos(-3x)\ dx$

$u = \cos(-3x) \quad dv = e^{3x}dx$

$du = 3\sin(-3x)dx \quad v = \frac{e^{3x}}{3}$

$2\int e^{3x}\cos(-3x)\,dx$

$2(\frac{1}{3}e^{3x}\cos(-3x) - \int e^{3x}\sin(-3x)dx$

$u = \sin(-3x)$

$du = -3\cos(-3x)dx$

$dv = e^{3x}dx$

$v = \frac{e^{3x}}{3}$

$\frac{2}{3}e^{3x}\cos(-3x) - 2\int e^{3x}\sin(-3x)dx$

$\frac{2}{3}e^{3x}\cos(-3x) - 2\left(\frac{e^{3x}\sin(-3x)}{3} + \int e^{3x}\cos(-3x)dx\right)$

$2\int e^{3x}\cos(-3x) = \frac{2}{3}e^{3x}\cos(-3x) - \frac{2}{3}e^{3x}\sin(-3x) - 2\int e^{3x}\cos(3$

$4\int e^{3x}\cos(-3x) = \frac{2}{3}e^{3x}\cos(-3x) - \frac{2}{3}e^{3x}\sin(-3x)$

$= \frac{1}{6}e^{3x}\cos(-3x) - \frac{1}{6}\sin(-3x) + C$

$\frac{1}{6}e^{3x}\cos(3x) + \frac{1}{6}\sin(3x) + C$

Integration Using Partial Fractions

1. $\int \frac{1}{2x^3+x^2-x}\,dx$

$x(2x-1)(x+1)$

$\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{x+1}$

$A(2x^2+x-1)+B(x^2+x)+C(2x^2-x)$

$2Ax^2+Ax-A+Bx^2+Bx+2Cx^2-Cx=1$

$2A+B+2C=0$

$A+B-C=0$

$-A=1$ so $A=-1$

$-2+B+2C=0$

$\underline{-(-1+B-C=0)}$

$-1+3C=0$

$C=1/3$

$A+B-C=0$

$-1+B-1/3=0$

$B=4/3$

$\int \frac{-1}{x}+\frac{4/3}{2x-1}+\frac{1/3}{x+1}\,dx$

$-\ln|x|+\frac{2}{3}\ln|2x-1|+\ln|x+1|+C$

2. $\int \frac{3x^3-5x^2-11x+9}{x^2-2x-3}\,dx$

$3x+1+\frac{12}{x^2-2x-3}$

$x^2-2x-3\,\overline{)\,3x^3-5x^2-11x+9}$

$\underline{-3x^3+6x^2+9x}$

x^2-2x+9

$\underline{-x^2+2x+3}$

12

$\int 3x+1+\frac{12}{x^2-2x-3}\,dx$

$(x-3)(x+1)$

$\frac{A}{x-3}+\frac{B}{x+1}$

$Ax+A+Bx-3B=12$

$-(A+B=0)$

$\underline{A-3B=12}$

$-4B=12 \therefore B=-3, A=3$

$\int 3x+1+\frac{3}{x-3}+\frac{-3}{x+1}\,dx$

$\frac{3x^2}{2}+x+3\ln|x-3|-3\ln|x+1|+C$

3. $\int \frac{x^2+12x-5}{(x+1)^2}\,dx$

$\int \frac{x^2+12x-5}{x^2+2x+1}\,dx$

$1+\frac{10x-6}{(x+1)^2}$

$x^2+2x+1\,\overline{)\,x^2+12x-5}$

$\underline{-x^2-2x-1}$

$10x-6$

$\int 1+\frac{10x-6}{(x+1)^2}\,dx$

$\frac{A}{x+1}+\frac{B}{(x+1)^2}$

$Ax+A+B=10x-6$

$A=10$

$A+B=-6$

$B=-16$

$\int -16(x+1)^{-2}\,dx$

$\int 1+\frac{10}{x+1}-\frac{16}{(x+1)^2}\,dx$

$x+10\ln|x+1|+\frac{16}{x+1}+C$

4. $\int \frac{8x^2-3x-4}{(4x-1)(x^2+1)}\,dx$

$\frac{A}{4x-1}+\frac{Bx+C}{x^2+1}$ $(4x-1)$

$Ax^2+A+4Bx^2-Bx+4Cx-C=8x^2-3x-4$

$A+4B=8$

$-B+4C=-3$

$A-C=-4$

$A=C-4$

$A=0-4$

$A=-4$

$C-4+4B=8$

$4B-C=12$

$4(-B+4C=-3) \rightarrow$

$4B-C=12$

$\underline{-4B+16C=-12}$

$15C=0$

$C=0$

$-B+4(0)=-3$

$B=3$

$\int \frac{-4}{4x-1}+\frac{3x}{x^2+1}\,dx$

$-\ln|4x-1|+\frac{3}{2}\ln(x^2+1)+C$

$\int \frac{3x}{x^2+1}\,dx$ $u=x^2+1$

$\frac{3}{2}\int \frac{1}{u}\,du$ $du=2x\,dx$

$\frac{du}{2}=x\,dx$

$\frac{3}{2}\ln(x^2+1)+C$

5. $\int \frac{4x^3+2x^2+1}{(4x^3-x)}\,dx$

$$1+\frac{2x^2+x+1}{4x^3-x}$$

$$4x^3+0x^2-x+0\overline{)4x^3+2x^2+0x+1}$$

$$\underline{-4x^3-0x^2+1x-0}$$

$$2x^2+x+1$$

$$\int 1+\frac{2x^2+x+1}{x(2x-1)(2x+1)}\,dx$$

$$\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{2x+1}$$

$$4Ax^2-A+2Bx^2+Bx+2Cx^2-Cx=2x^2+x+1$$

$4A+2B+2C=2$

$B-C=1$

$-A=1$

$\nearrow A=-1$

$-4+2B+2C=2$

$2B+2C=6$

$\underline{2B-2C=2}$

$4B=8$

$B=2$

$B-C=1$

$2-C=1$

$-C=-1$

$C=1$

$$\int 1+\frac{-1}{x}+\frac{2}{2x-1}+\frac{1}{2x+1}\,dx$$

$$x-\ln|x|+\ln|2x-1|+\frac{1}{2}\ln|2x+1|+C$$

6. $\int \frac{3x-2}{x^3+x^2-x-1}\,dx$

$(x+1)(x^2-1)$

$(x+1)(x+1)(x-1)=(x-1)(x+1)^2$

$$\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

$$Ax^2+2Ax+A+Bx^2-B+Cx-C=3x-2$$

$A+B=0\rightarrow A=-B$

$2A+C=3$

$A-B-C=-2$

$A+A-C=-2$

$2A-C=-2$

$2A+C=3$

$\underline{+2A-C=-2}$

$4A=1$

$A=1/4$

$B=-1/4$

$2(\frac{1}{4})+C=3$

$C=\frac{5}{2}$

$$\int \frac{1/4}{x-1}+\frac{-1/4}{x+1}+\frac{5/2}{(x+1)^2}\,dx$$

$$\frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|-\frac{5}{2(x+1)}+C$$

7. $\int \frac{6x^2-x-1}{3x-1}\,dx$

$$2x+\frac{1}{3}-\frac{2}{3(3x-1)}$$

$$3x-1\overline{)6x^2-x-1}$$

$$\underline{-6x^2+2x}$$

$$x-1$$

$$\underline{-x+1/3}$$

$$-2/3$$

$$\int 2x+\frac{1}{3}-\frac{2/3}{3x-1}\,dx$$

$$x^2+\frac{1}{3}x-\frac{2}{9}\ln|3x-1|+C$$

8. $\int \frac{3x+5}{x^2+4x-32}\,dx$

$(x+8)(x-4)$

$$\frac{A}{x+8}+\frac{B}{x-4}$$

$$Ax-4A+Bx+8B=3x+5$$

$A+B=3\rightarrow 4A+4B=12$

$-4A+8B=5$

$\underline{-4A+8B=5}$

$12B=17$

$B=17/12$

$\frac{17}{12}+B=3$

$B=19/12$

$$\int \frac{19/12}{x+8}+\frac{17/12}{x-4}\,dx$$

$$\frac{19}{12}\ln|x+8|+\frac{17}{12}\ln|x-4|+C$$

9. $\int \frac{1}{x^2-4}\,dx$

$(x+2)(x-2)$

$$\frac{A}{x+2}+\frac{B}{x-2}$$

$$Ax-2A+Bx+2B=1$$

$A+B=0\therefore A=-B$

$-2A+2B=1$

$-2(-B)+2B=1$

$B=1/4$ $A=-1/4$

$$\int \frac{-1/4}{x+2}+\frac{1/4}{x-2}$$

$$-\frac{1}{4}\ln|x+2|+\frac{1}{4}|x-2|+C$$

10. $\int \frac{2x+3}{x^2-9}\,dx$

$(x+3)(x-3)$

$$\frac{A}{x+3}+\frac{B}{x-3}$$

$$Ax-3A+Bx+3B=2x+3$$

$3(A+B=2)$

$\underline{-3A+3B=3}$

$6B=9$

$B=3/2$

$A+3/2=2$

$A=1/2$

$$\int \frac{1/2}{x+3}+\frac{3/2}{x-3}\,dx$$

$$\frac{1}{2}\ln|x+3|+\frac{3}{2}\ln|x-3|+C$$

Integration Rules – Practice Test

Multiple Choice.

1. $\int \sec^2 x \; dx =$

[✓ a.] $\tan x + c$

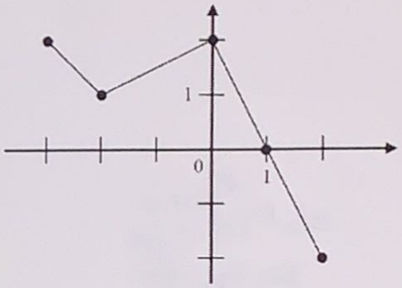
b. $\csc^2 x + c$

c. $\cos^2 x + c$

d. $\frac{\sec^3 x}{3} + c$

e. $2\sec^2 x \tan x + c$

2. The graph of the piecewise linear function f is shown in the figure. If $g(x) = \int_{-2}^{x} f(t)dt$, which of the following values is greatest?



Graph of f

a. $g(-3) = -\frac{1}{2}(1+2) = -3/2$

b. $g(-2) = 0$

c. $g(0) = \frac{1}{2}(2)(1+2) = 3$

[✓ d.] $g(1) = 3 + \frac{1}{2}(1)(2) = 4$

e. $g(2) = 4 - \frac{1}{2}(1)(2) = 3$

3. $\int \frac{x}{x^2-4} \, dx =$ $\frac{1}{2}\int \frac{1}{u} du$

$u = x^2 - 4$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

a. $-\frac{1}{4(x^2-4)^2} + c$

b. $\frac{1}{2(x^2-4)^2} + c$

[✓ c.] $\frac{1}{2}\ln|x^2 - 4| + c$

d. $2\ln|x^2 - 4| + c$

e. $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + c$

4. $\int_0^1 \frac{1}{e^{4x}} \, dx$ $\frac{e^{-4x}}{-4}\Big|_0^1 = \frac{-1}{4e^{4x}}\Big|_0^1 = \frac{-1}{4e^4} + \frac{1}{4}$

a. $-\frac{e^{-4}}{4}$

b. $-4e^{-4}$

c. $e^{-4} - 1$

[✓ d.] $\frac{1}{4} - \frac{e^{-4}}{4}$

e. $4 - 4e^{-4}$

5. $\int_0^{\frac{\pi}{4}} \sin x \; dx$

$-\cos x\Big|_0^{\pi/4}$

$-\cos\frac{\pi}{4} + \cos 0$

$-\frac{\sqrt{2}}{2} + 1$

a. $\frac{\sqrt{2}}{2}$

b. $-\frac{\sqrt{2}}{2}$

c. $-\frac{\sqrt{2}}{2} - 1$

[✓ d.] $-\frac{\sqrt{2}}{2} + 1$

e. $\frac{\sqrt{2}}{2} - 1$

6. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

a. $\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{u}\,du$

b. $\frac{1}{2}\int_0^2 \sqrt{u}\,du$

[✓ c.] $\frac{1}{2}\int_1^5 \sqrt{u}\,du$

d. $\int_0^2 \sqrt{u}\,du$

e. $\int_1^5 \sqrt{u}\,du$

$\frac{1}{2}\int_0^5 \sqrt{u}$

$u = 2x+1$ $\frac{du}{2} = 2dx$

$u = 2(2)+1 = 5$ upper

$u = 2(0)+1 = 1$ lower

7. $\int_1^e \frac{x^2-1}{x}\,dx$ $= \int_1^e x - \frac{1}{x}\,dx$

a. $e^2 - 1$

b. $e^2 - 2$

c. $\frac{e^2}{2} + \frac{1}{2}$

d. $\frac{e^2}{2} - \frac{1}{2}$

[✓ e.] $\frac{e^2}{2} - \frac{3}{2}$

$\frac{x^2}{2} - \ln|x|\Big|_1^e$

$\frac{e^2}{2} - \overset{1}{\cancel{\ln e}} - \frac{1}{2} - 0$

8. $\int x^2 \cdot \cos(x^3)dx$

a. $-\frac{1}{3}\sin(x^3) + c$

[✓ b.] $\frac{1}{3}\sin(x^3) + c$

c. $-\frac{x^3}{3}\sin(x^3) + c$

d. $\frac{x^3}{3}\sin(x^3) + c$

e. $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + c$

$u = x^3$

$du = 3x^2dx$

$\frac{du}{3} = x^2dx$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin x^3 + C$

9. $\int x \cdot \sin(2x)dx$

[✓ a.] $-\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + c$

b. $-\frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + c$

c. $\frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + c$

d. $-2x\cos(2x) + \sin(2x) + c$

e. $-2x\cos(2x) - 4\sin(2x) + c$

u	dv
$+x$	$\sin 2x$
-1	$-\frac{1}{2}\cos 2x$
$+0$	$-\frac{1}{4}\sin 2x$

10. $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

[✓ a.] $2\sqrt{x^3+1} + c$

b. $\frac{3}{2}\sqrt{x^3+1} + c$

c. $\sqrt{x^3+1} + c$

d. $\ln\sqrt{x^3+1} + c$

e. $\ln(x^3+1) + c$

$u = x^3+1$

$du = 3x^2dx$

$\int \frac{1}{\sqrt{u}}du = \int u^{-1/2}du$

$2u^{1/2} + C$

$2\sqrt{x^3+1} + C$

11. $\int (x^2+1)^2\,dx =$ $\int x^4 + 2x^2 + 1\,dx$

$\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

a. $\frac{(x^2+1)^3}{3} + c$

b. $\frac{(x^2+1)^3}{6x} + c$

c. $\left(\frac{x^3}{3} + x\right)^2 + c$

d. $\frac{2x(x^2+1)^3}{3} + c$

[✓ e.] $\frac{x^5}{5} + \frac{2x^3}{3} + x + c$

12. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$ $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4(1-(\frac{x}{2})^2}}$

$u=\frac{x}{2}$
$du=\frac{1}{2}dx$
$2du=dx$

$u=\frac{\sqrt{3}}{2}$
$u=\frac{0}{2}$

$2\cdot\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-u^2}}du$

$\sin^{-1}u \Big|_0^{\sqrt{3}/2}$

$\sin^{-1}(\frac{\sqrt{3}}{2})-\sin^{-1}(0) = \frac{\pi}{3}-0=\frac{\pi}{3}$

[✓ a.] $\frac{\pi}{3}$ b. $\frac{\pi}{6}$
c. $2-\sqrt{3}$ d. $\frac{1}{2}\ln 2$
e. $-\ln 2$

13. $\int xf(x)dx =$
a. $xf(x) - \int xf'(x)dx$ [✓ b.] $\frac{x^2}{2}f(x) - \int \frac{x^2}{2}f'(x)dx$
c. $xf(x) - \frac{x^2}{2}f(x) + C$ d. $xf(x) - \int f'(x)dx$
e. $\frac{x^2}{2}\int f(x)dx$

$u=f(x)$ $dv=x\,dx$
$du=f'(x)dx$ $v=\frac{x^2}{2}$
$\frac{x^2}{2}f(x)-\int\frac{x^2}{2}f'(x)\,dx$

14. $\int(\sin(2x)+\cos(2x))dx =$ $-\frac{\cos(2x)}{2}+\frac{\sin(2x)}{2}+C$
a. $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+c$ [✓ b.] $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+c$
c. $\frac{1}{2}\cos(2x)-\frac{1}{2}\sin(2x)+c$ d. $2\cos(2x)-2\sin(2x)+c$
e. $-2\cos(2x)+2\sin(2x)+c$

15. $\int \frac{x}{1+9x^4}dx$ $\int \frac{x}{1+(3x^2)^2}$

$u=3x^2$
$du=6x\,dx$
$\frac{du}{6}=x\,dx$

$\frac{1}{6}\int\frac{1}{1+u^2}$

$\frac{1}{6}\tan^{-1}3x^2+C$

a. $\frac{1}{36x^2}\ln|1+9x^4|+c$ b. $\frac{1}{36x^3}\ln|1+9x^4|+c$
[✓ c.] $\frac{1}{6}\arctan(3x^2)+c$ d. $\frac{1}{18}\arctan(9x^2)+c$
e. $\arctan(3x^2)+c$

Integrate:

16. $\int\frac{\csc^2\sqrt{x}}{\sqrt{x}}dx$

$u=\sqrt{x}$
$du=\frac{1}{2\sqrt{x}}dx$
$2du=\frac{1}{\sqrt{x}}dx$

$2\int\csc^2u\,du$

$-2\cot u+C$

$-2\cot\sqrt{x}+C$

17. $\int\frac{(\ln x)^4}{x}dx$

$u=\ln x$
$du=\frac{1}{x}dx$

$\int u^4du$

$\frac{u^5}{5}+C$

$\frac{(\ln x)^5}{5}+C$

18. $\int \frac{3x^3-39x+20}{x+4}\, dx$

$$\begin{array}{r|rrrr} -4 & 3 & 0 & -39 & 20 \\ & \downarrow & -12 & 48 & -36 \\ \hline & 3 & -12 & 9 & -16 \end{array}$$

$\int 3x^2-12x+9-\frac{16}{x+4}\, dx$

$x^3-6x^2+9x-16\ln|x+4|+C$

19. $\int \sin^3(2x)\, dx$

$\int \sin(2x)\sin^2(2x)\, dx$ $\quad u=\cos 2x$

$\int \sin 2x\,(1-\cos^2 2x)\, dx$ $\quad du=-2\sin 2x\, dx$

$\frac{du}{-2}=\sin 2x\, dx$

$-\frac{1}{2}\int 1-u^2\, du$

$-\frac{1}{2}u+\frac{1}{6}u^3+C$

$\cos 2x+\frac{1}{6}\cos^3 2x+C$

20. $\int \arctan x\, dx$ $\quad u=\arctan x \quad dV=dx$

$x\arctan x-\int\frac{x}{1+x^2}dx$ $\quad du=\frac{1}{1+x^2}dx \quad V=x$

$x\arctan x-\frac{1}{2}\int\frac{1}{u}\, du$ $\quad u=1+x^2$

$du=2x\, dx$

$\frac{du}{2}=x\, dx$

$x\arctan x-\frac{1}{2}\ln(1+x^2)+C$

21. $\int \overset{u}{x^3}\,\overset{dv}{\sin(2x)}\, dx$

$$\begin{array}{l|l} u & dv \\ \hline +\ x^3 & \sin 2x \\ -\ 3x^2 & -\frac{1}{2}\cos 2x \\ +\ 6x & -\frac{1}{4}\sin 2x \\ -\ 6 & \frac{1}{8}\cos 2x \\ +\ 0 & \frac{1}{16}\sin 2x \end{array}$$

$-\frac{1}{2}x^3\cos 2x+\frac{3}{4}x^2\sin 2x+\frac{3}{4}x\cos 2x-\frac{3}{8}\sin 2x+C$

22. $\int \cos(3x)\sin^8(3x)dx$ $\quad u=\sin 3x$

$du=3\cos 3x\, dx$

$\frac{du}{3}=\cos 3x\, dx$

$\frac{1}{3}\int u^8\, du$

$\frac{1}{3}\cdot\frac{u^9}{9}+C$

$\frac{\sin^9 3x}{27}+C$

23. $\int \tan(4x)dx$

$\int\frac{\sin 4x}{\cos 4x}$ $\quad u=\cos 4x$

$du=-4\sin 4x\, dx$

$-\frac{1}{4}\int\frac{1}{u}\, du$ $\quad \frac{du}{-4}=\sin 4x\, dx$

$-\frac{1}{4}\ln|\cos 4x|+C$

24. $\int x(2-x)^{\frac{2}{3}}\, dx$ $\quad u=2-x\ ,\ x=2-u$

$-\int(2-u)\,u^{2/3}\, du$ $\quad du=-dx$

$-\int 2u^{2/3}-u^{5/3}\, du$

$-\frac{6}{5}u^{5/3}+\frac{3u^{8/3}}{8}+C$

$-\frac{6}{5}(2-x)^{5/3}+\frac{3}{8}(2-x)^{8/3}+C$

25. $\int \frac{5x-4}{2x^2+x-1}\, dx$

$(2x-1)(x+1)$

$\frac{A}{2x-1}+\frac{B}{x+1}$

$Ax+A+2Bx-B=5x-4$

$$\begin{array}{r} A+2B=5 \\ -(A-B=-4) \\ \hline 3B=9 \\ B=3 \end{array} \qquad \begin{array}{r} A-3=-4 \\ A=-1 \end{array}$$

$\int\frac{-1}{2x-1}+\frac{3}{x+1}\, dx$

$-\frac{1}{2}\ln|2x-1|+3\ln|x+1|+C$

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26. $\int e^{3x}\sin(2x)dx$

$u = \sin 2x$

$du = 2\cos 2x\,dx$

$dV = e^{3x}dx$

$V = \frac{e^{3x}}{3}$

$\frac{1}{3}e^{3x}\sin 2x - \int \frac{2}{3}e^{3x}\cos 2x\,dx$

$\frac{1}{3}e^{3x}\sin 2x - \frac{2}{3}\left(\frac{1}{3}e^{3x}\cos 2x + \frac{2}{3}\int e^{3x}\sin 2x\right)$

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$dv = e^{3x}dx\,;\ V = \frac{e^{3x}}{3}$

$\int e^{3x}\sin 2x\,dx = \frac{1}{3}e^{3x}\sin 2x + \frac{2}{9}e^{3x}\cos 2x - \frac{4}{9}\int e^{3x}\sin 2x\,dx$

$\frac{13}{9}\int e^{3x}\sin 2x\,dx = \frac{1}{3}e^{3x}\sin 2x - \frac{2}{9}e^{3x}\cos 2x$

$\frac{9}{13}\left(\frac{1}{3}e^{3x}\sin 2x - \frac{2}{9}e^{3x}\cos 2x\right)$

$\frac{3}{13}e^{3x}\sin 2x - \frac{2}{13}e^{3x}\cos 2x + C$

27. $\int \sin^2(5x)dx$ $= \frac{1}{2}\int 1 - \cos 10x\,dx$

$\frac{1}{2}x - \frac{1}{20}\sin 10x + C$

28. $\int \sec^2 x\tan^2 x\ dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^2 du$

$\frac{u^3}{3} + C$

$\frac{\tan^3 x}{3} + C$

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29. $\int r^4 \ln r\ dr$

$u = \ln r$ $dV = r^4$

$du = \frac{1}{r}dr$ $V = \frac{r^5}{5}$

$\frac{r^5}{5}\ln r - \int \frac{r^5}{5}\cdot\frac{1}{r}dr$

$\frac{r^5}{5}\ln r - \frac{1}{5}\int r^4 dr$

$\frac{r^5}{5}\ln r - \frac{r^5}{25} + C$

30. $\int e^{2x}\sec(e^{2x})\tan(e^{2x})$

$u = e^{2x}$

$du = 2e^{2x}dx$

$\frac{du}{2} = e^{2x}dx$

$\frac{1}{2}\int \sec u \tan u$

$\frac{1}{2}\sec u + C$

$\frac{1}{2}\sec(e^{2x}) + C$

31. $\int \frac{e^{x^2}+2x}{e^{x^2}}\ dx$

$\int \frac{e^{x^2}}{e^{x^2}} + \frac{2x}{e^{x^2}}dx$

$\int 1 + 2xe^{-x^2}\ dx$

$x + 2\int xe^{-x^2}dx$

$u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$x - \int e^u du$

$x - e^{-x^2} + C$

$x - \frac{1}{e^{x^2}} + C$