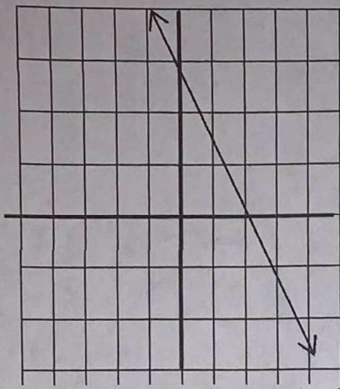
Review Worksheet – Graphing Derivatives

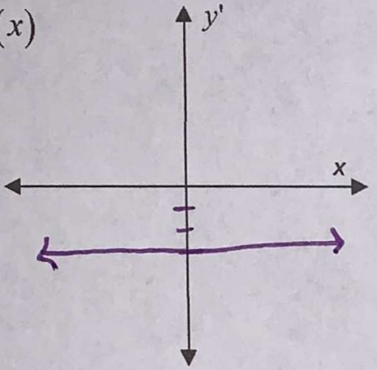
For problems 1-6, sketch a graph of the derivative function of each of the functions.

1.



f'(x) – – – – – m=-3

f'(x)



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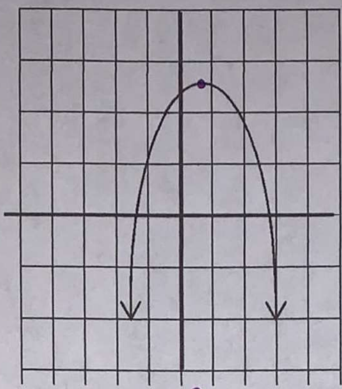
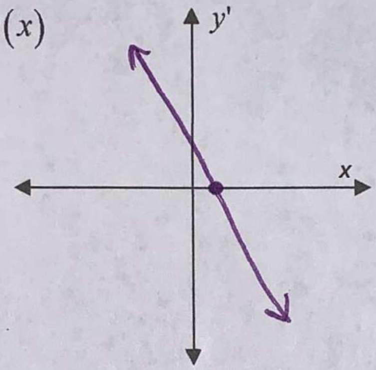


f'(x) + + + 0 – – –

f'(x)



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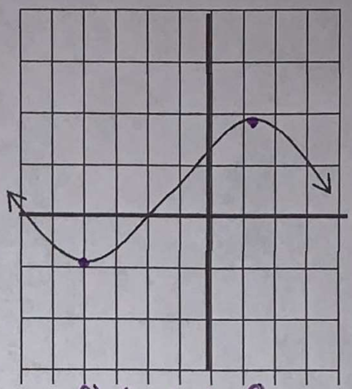
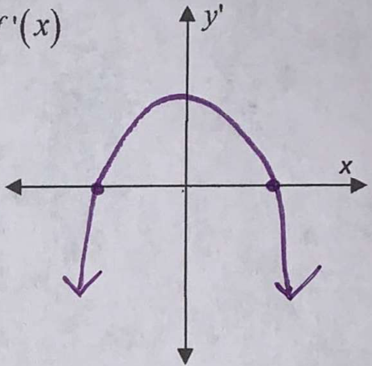


f'(x) – 0 + + + 0 – –

f'(x)



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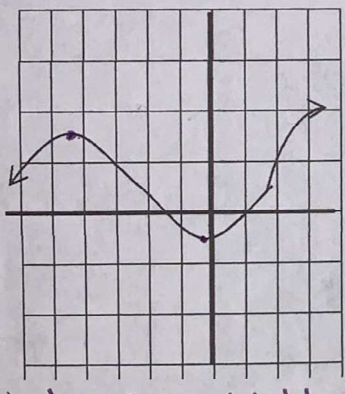
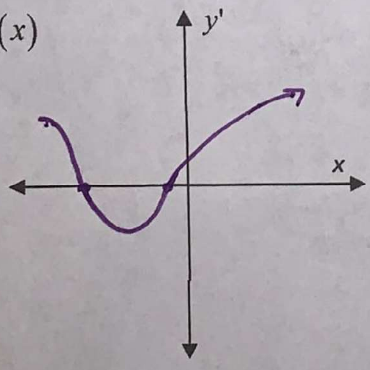


f'(x) + – – – – + + + +

f'(x)



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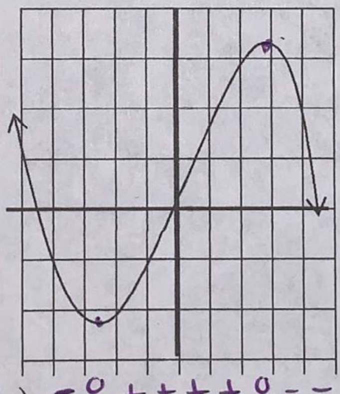
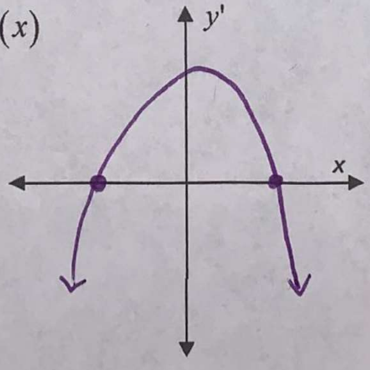


f'(x) – 0 + + + + 0 – –

f'(x)



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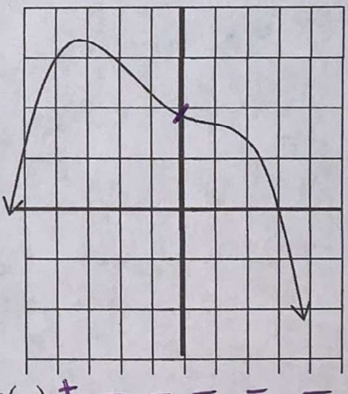
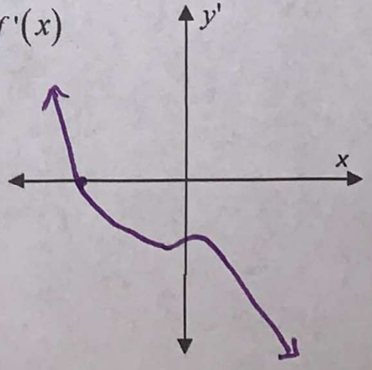
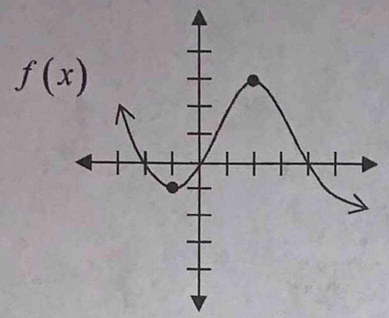


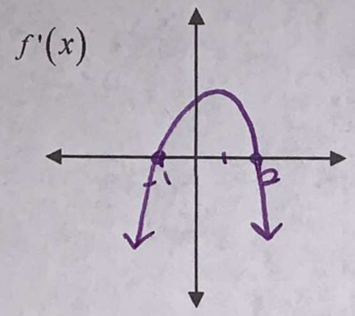
f'(x) + – – – – – –

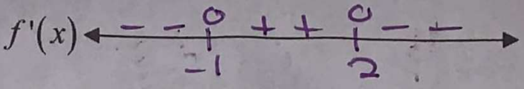
f'(x)



7. The graph of f is shown below.







a. Where does f have critical numbers?

$x = -1, 2$

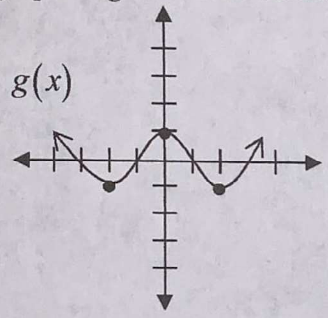
b. On what intervals is f' negative? positive?

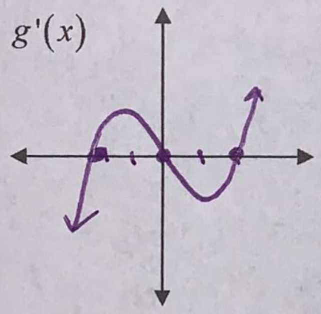
neg: $(-\infty,-1) \cup (2,\infty)$

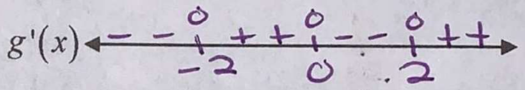
pos: $(-1,2)$

c. Sketch the graph of f'.

8. The graph of g is shown below.







a. Where does g have critical numbers?

$x = -2, 0, 2$

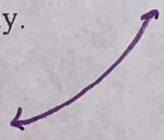
b. On what intervals is g' negative? positive?

neg: $(-\infty,-2) \cup (0,2)$

pos: $(-2,0) \cup (2,\infty)$

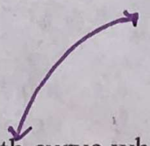
c. Sketch the graph of g'.

9. a. Sketch a smooth curve whose slope is everywhere positive and increasing gradually.



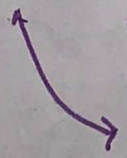
Increase
Concave Up

b. Sketch a smooth curve whose slope is everywhere positive and decreasing gradually.



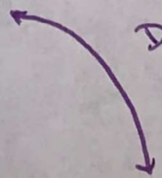
Increase
Concave Down

c. Sketch a smooth curve whose slope is everywhere negative and increasing gradually (becoming less and less negative).



Decrease
Concave Up

d. Sketch a smooth curve whose slope is everywhere negative and decreasing gradually (becoming more and more negative).



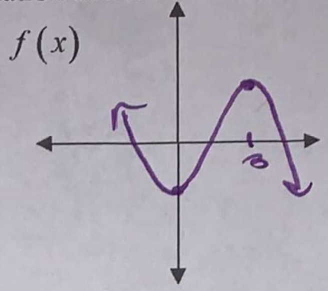
Decrease
Concave Down

10. Draw a possible graph of $f(x)$ given the following information about its derivative:

$f'(x) > 0$ for $1 < x < 3$,

$f'(x) < 0$ for $x < 1$ and $x > 3$,

$f'(x) = 0$ at $x = 1$ and $x = 3$.



$f'(x)$ – – 0 + + 0 – –

1 3

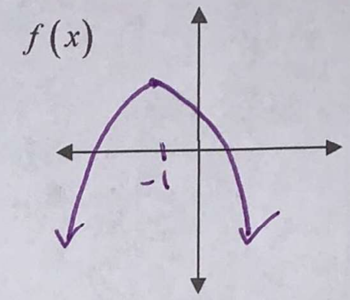
↘ min ↗ max ↘

11. Draw a possible graph of $f(x)$ given the following information about its derivative:

$f'(x) > 0$ for $x < -1$,

$f'(x) < 0$ for $x > -1$,

$f'(x) = 0$ at $x = -1$.



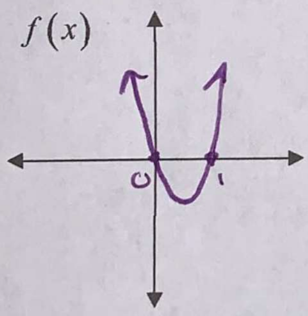
max

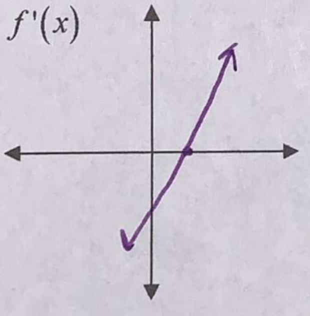
$f'(x)$ + + + 0 – – –

↗ -1 ↘

12. Sketch the graph of $f(x)$ and use the graph to sketch the graph of $f'(x)$.

a. $f(x) = x(x-1)$

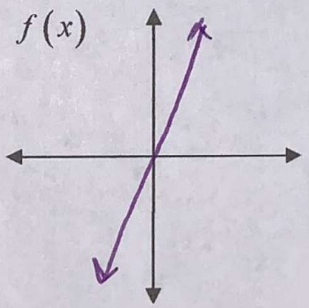


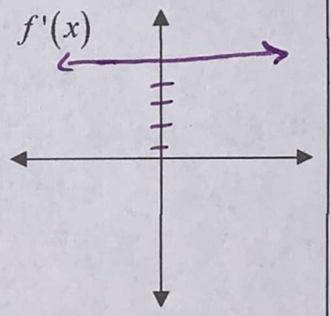


$f'(x)$ – – + + + +

1/2

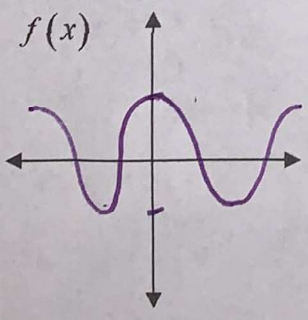
b. $f(x) = 5x$

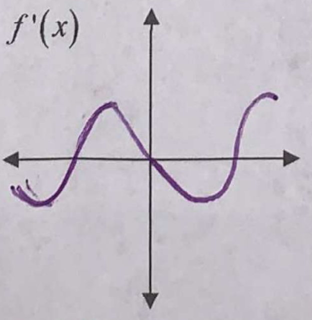




$f'(x)$ m=5 everywhere

c. $f(x) = \cos x$

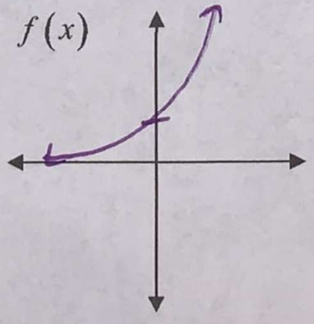


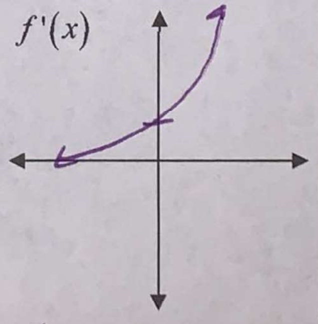


$f'(x)$ – – ++ – – ++

-π 0 π

d. $f(x) = e^x$

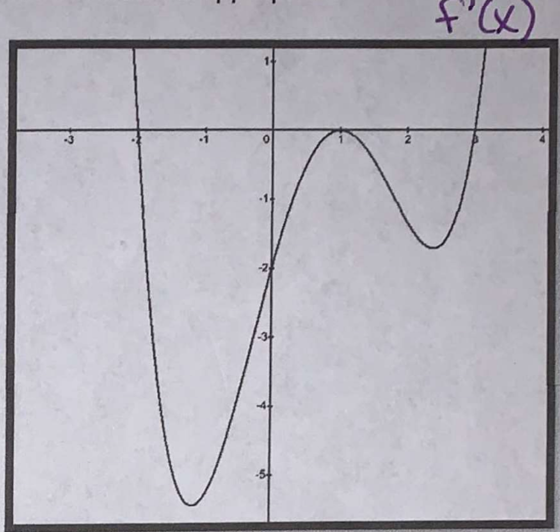




$f'(x)$ + + + + +

Curve Sketching Practice

1. Given the graph of $f'(x)$ answer the following questions and explain your answer.
If $f(x)$ is a continuous function indicate the interval or the point whichever is appropriate.



a. Where is $f(x)$ increasing? $(-\infty,-2)\cup(3,\infty)$

b. Where is $f(x)$ decreasing? $(-2,1)\cup(1,3)$

c. Where does $f(x)$ have a horizontal tangent
$x=-2,1,3$

d. Where is $f(x)$ concave up? $(-1,1)\cup(2.5,\infty)$

e. Where is $f(x)$ concave down? $(-\infty,-1)\cup(1,2.5)$

f. Where does $f(x)$ have a relative (local) min? $x=3$

g. Where does $f(x)$ have a relative (local) max? $x=-2$

h. Where does $f(x)$ have points of inflection? $x=-1,1,2.5$

2. Consider the function f, whose formula and derivatives are given by

$$f(x)=\frac{x^2-4}{(x-1)^2},\quad f'(x)=\frac{-2x+8}{(x-1)^3},\quad f''(x)=\frac{4x-22}{(x-1)^4}$$

a. Find and describe all of the vertical and horizontal asymptotes of this function, if any. Justify.

VA: $x=1$ since $\lim_{x\to1^+}f(x)=-\infty$ and $\lim_{x\to1^-}f(x)=-\infty$ and $f(1)$ DNE

HA: $y=1$ since $\lim_{x\to\pm\infty}f(x)=1$

b. Find all of the roots of this function, if any.

$0=x^2-4$

$x=\pm2$

c. Find and classify all of the local extrema of this function, if any. Show justification.

$f'(x)=0$

$-2x+8=0$

$x=4$

$\overset{+}{} \overset{\text{DNE}}{\underset{1}{|}} \overset{+}{} \overset{0}{\underset{4}{|}} \overset{-}{}$

↗ ↘

local max at $x=4$ since $f'(x)$ changes from + to −

$f(4)=\frac{4^2-4}{(4-1)^2}=\frac{12}{9}=\frac{4}{3}$ Local max $(4,\frac{4}{3})$

d. Find all of the inflection points of this function, if any. Show justification.

$f''(x)=0$

$4x-22=0$

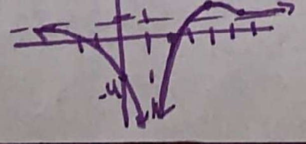
$x=11/2$

$f''\ \overset{-}{} \overset{\text{DNE}}{\underset{1}{|}} \overset{-}{} \overset{0}{\underset{11/2}{|}} \overset{+}{}$

POI at $x=11/2$ since $f''(x)$ changes from − to +

$f(\frac{11}{2})=\frac{(\frac{11}{2})^2-4}{(\frac{11}{2}-1)^2}=\frac{35}{27}$ POI: $(\frac{11}{2},\frac{35}{27})$

e. Sketch the function and include all of the features above.



3. Given $f(x) = 5x^3 - 3x^2 - 32x - 12$, find the following and sketch a graph of $f(x), f'(x), f''(x)$. Show all scratch work – organized neatly! You may use graphs for information but be sure to show documentation.

a. $f'(x) =$

$f'(x) = 15x^2 - 6x - 32$

b. $f''(x) =$

$f''(x) = 30x - 6$

c. Roots of $f(x)$

P/q: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm 1/5, \pm 2/5, \pm 3/5,$
$\pm 4/5, \pm 6/5, \pm 12/5$

$$\begin{array}{r|rrrr} 3 & 5 & -3 & -32 & -12 \\ & \downarrow & 15 & 36 & 12 \\ \hline -2 & 5 & 12 & 4 & :0 \\ & \downarrow & -10 & -4 & \\ \hline & 5 & 2 & 0 & \end{array}$$

$x = 3, -2, -\frac{2}{5}$

$5x + 2 = 0$
$x = -2/5$

d. y intercept of $f(x)$

$f(0) = 5(0)^3 - 3(0)^2 - 32(0) - 12$
$f(0) = -12$
$(0, -12)$

e. Vertical and horizontal asymptote

none bc f(x) is a polynomial

f. Critical values

$0 = 15x^2 - 6x - 32$
$\frac{6 \pm \sqrt{36 - 4(15)(-32)}}{2(15)} = \frac{6 \pm 2\sqrt{489}}{30}$
$x = \frac{3 \pm \sqrt{489}}{15}$

g. Where is $f(x)$ increasing?

$+ \quad - \quad +$
$\frac{3-\sqrt{489}}{15} \quad \frac{3+\sqrt{489}}{15}$

$\left(-\infty, \frac{3-\sqrt{489}}{15}\right) \cup \left(\frac{3+\sqrt{489}}{15}, \infty\right)$

h. Where is $f(x)$ decreasing?

$\left(\frac{3-\sqrt{489}}{15}, \frac{3+\sqrt{489}}{15}\right)$

i. Where does $f(x)$ have a horizontal tangent?

$x = \frac{3 \pm \sqrt{489}}{15}$

J. Where is $f(x)$ concave up?

$0 = 30x - 6$
$x = 1/5$

$f'' \quad - \quad +$
$1/5$

$\left(\frac{1}{5}, \infty\right)$

k. Where is $f(x)$ concave down?

$\left(-\infty, \frac{1}{5}\right)$

l. Where does $f(x)$ have a relative (local) min?

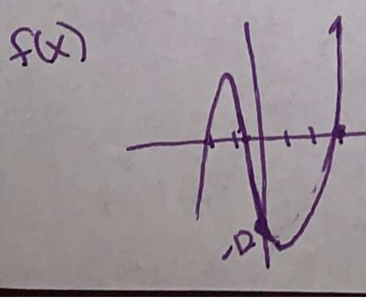
$x = \frac{3+\sqrt{489}}{15}$

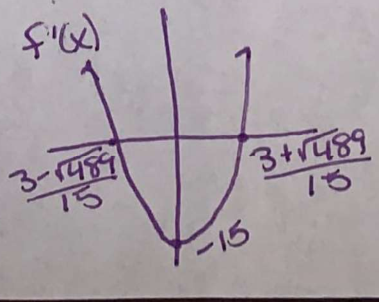
m. Where does $f(x)$ have a relative (local) max?

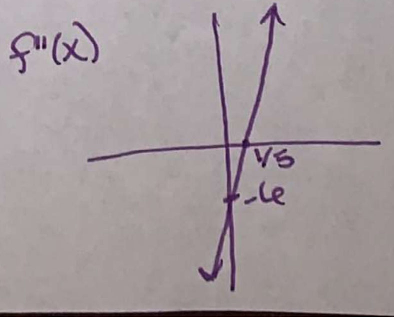
$x = \frac{3-\sqrt{489}}{15}$

n. Where does $f(x)$ have points of inflection?

$x = 1/5$







4. Given $f(x) = \frac{x^2-3}{(x+2)^2}$, $f'(x) = \frac{2(2x+3)}{(x+2)^3}$, $f''(x) = \frac{-2(4x+5)}{(x+2)^4}$, find the following and sketch a graph of $f(x), f'(x), f''(x)$. Attach all scratch work – organized neatly! You may use your calculator for calculations only. Sketch the graph using your information.

a. Roots of $f(x)$

$0 = x^2 - 3$

$x = \pm\sqrt{3}$

b. y intercept of $f(x)$

$f(0) = \frac{(0)^2-3}{(0+2)^2} = \frac{-3}{4}$

$(0, -3/4)$

c. Vertical and horizontal asymptote

VA: $(x+2)^2 \neq 0$

$x = -2$

HA: $y = 1$

d. Critical values

$2(2x+3) = 0$

$x = -3/2$

$x = -2, -3/2$

e. Where is $f(x)$ increasing?

$+ \quad \text{DNE} \quad - \quad 0 \quad +$

$-2 \quad -3/2$

$(-\infty, -2) \cup (-3/2, \infty)$

f. Where is $f(x)$ decreasing?

$(-2, -3/2)$

g. Where does $f(x)$ have a horizontal tangent?

$x = -3/2$

h. Where is $f(x)$ concave up?

$-2(4x+5) = 0$

$x = -5/4$

$+ \quad \text{DNE} \quad + \quad 0 \quad -$

$-2 \quad -5/4$

$(-\infty, -2) \cup (-2, -5/4)$

i. Where is $f(x)$ concave down?

$(-5/4, \infty)$

j. Where does $f(x)$ have a relative (local) min?

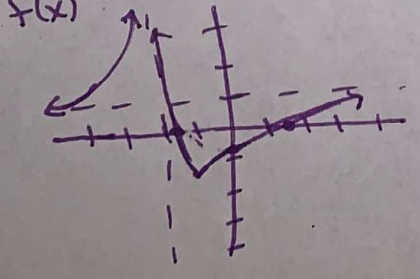
$x = -3/2$

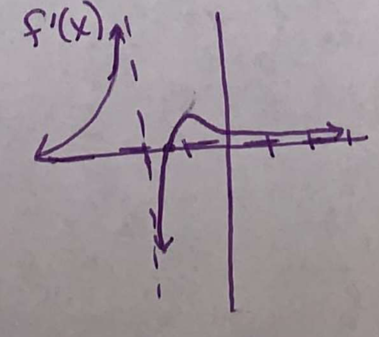
k. Where does $f(x)$ have a relative (local) max?

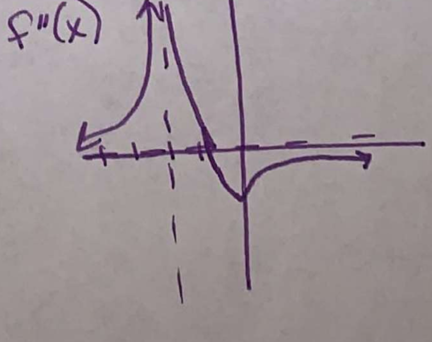
none

l. Where does $f(x)$ have points of inflection?

$x = -5/4$

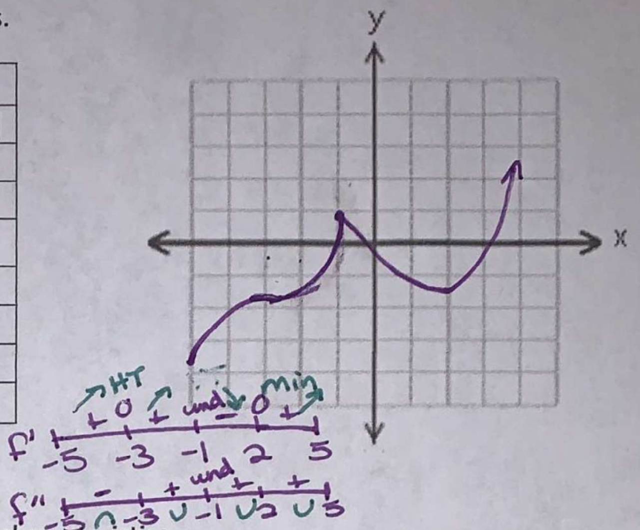






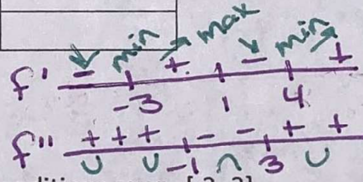
5. Sketch the graph of F(x) over [-5, 5]. F(x) is continuous.

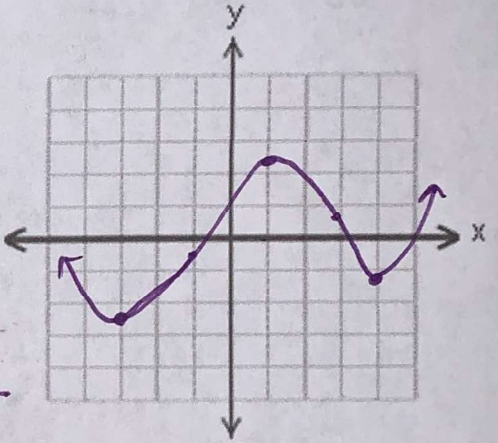
	Interval
$F' > 0$	$[-5,-3), (-3,-1), and\ (2,5]$
$F' < 0$	$(-1,2)$
$F' = 0$	$x = -3, 2$
F' is undefined	$x = -1$
$F'' > 0$	$(-3,-1)\ and\ (-1,5)$
$F'' < 0$	$(-5,-3)$
$F'' = 0$	$x = -3$
F'' is undefined	$x = -1$



6. Sketch a continuous graph of f(x) having the following characteristics.

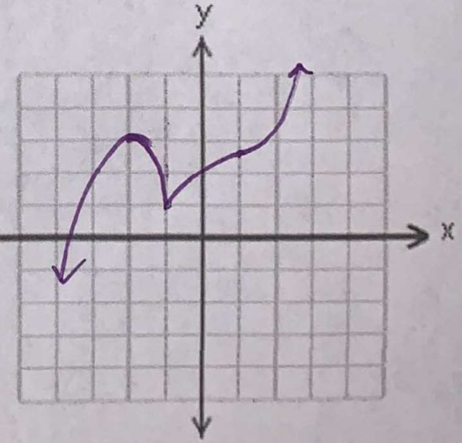
$f'(x) < 0$	$x < -3$	$1 < x < 4$
$f'(x) > 0$	$-3 < x < 1$	$x > 4$
$f'(x) = 0$	$x = -3, 1, 4$	
$f''(x) = 0$	$x = -1, 3, 5$	
$f''(x) > 0$	$x > 3$	$x < -1$
$f''(x) < 0$	$-1 < x < 3$	

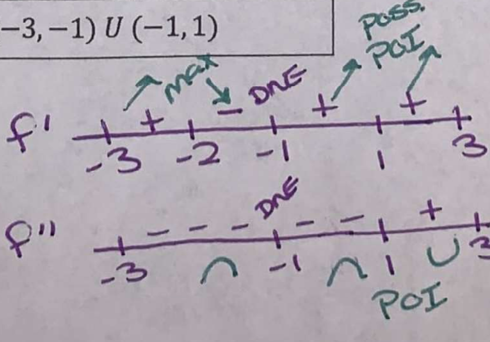




7. Sketch the graph with the following conditions over [-3, 3].

$f(x)$ is continuous	
$f'(x) > 0$	$(-3,-2)\ U\ (-1,1)\ U\ (1,3)$
$f'(x) < 0$	$(-1,2)$
$f'(x) = 0$	$x = -2, 1$
$f'(x)$ is undef	$x = -1$
$f''(x) > 0$	$(1,3)$
$f''(x) < 0$	$(-3,-1)\ U\ (-1,1)$

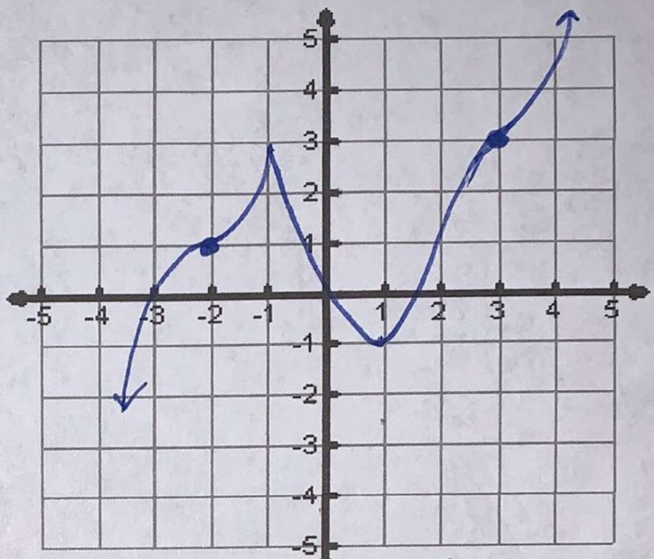


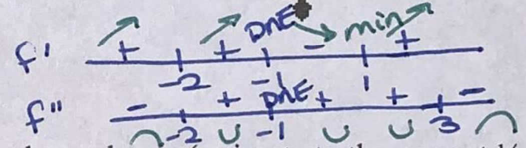


Derivative Applications Practice Test

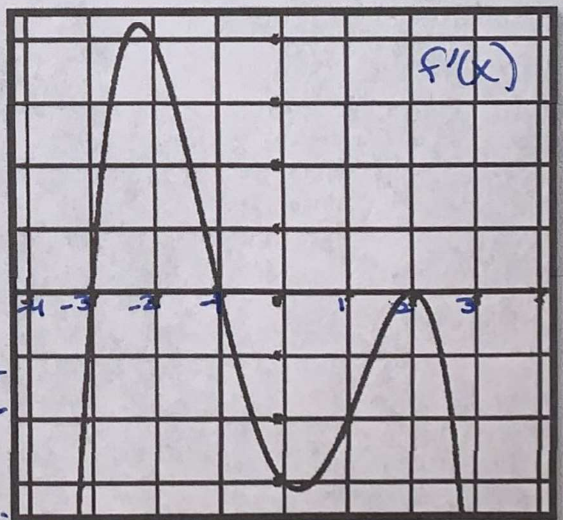
Sketch the graph of f(x) with the following conditions.
The graph of $f(x)$ is continuous. Use the information below to sketch the graph.

$f(-2)=1, f(3)=3$	
$f'(x)=0$	$x=-2$ and $x=1$
$f'(x)$ undefined	$x=-1$
$f'(x)>0$	$(-\infty,-2)\cup(-2,-1)\cup(1,\infty)$
$f'(x)<0$	$(-1,1)$
$f''(x)=0$	$x=-2$ and $x=3$
$f''(x)$ undefined	$x=-1$
$f''(x)>0$	$(-2,-1)\cup(-1,3)$
$f''(x)<0$	$(-\infty,-2)\cup(3,\infty)$





Given the graph of f '(x), find the following intervals or x values where: (estimate to the nearest ¼ unit)



a) f(x) in increasing. Justify.

(-3, -1) because f'(x) is positive

b) f(x) has horizontal tangents. Justify.

x = -3, -1, 2 bc f'(x) = 0

c) f(x) has a local extrema. Justify. Be sure and identify the local extrema as a local max or local min.

x = -3 local min bc f'(x) changes from − to +
x = -1 local max bc f'(x) changes from + to −

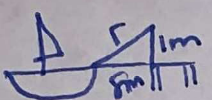
d) f(x) is concave down. Justify.

(-2.5, .25) ∪ (2, ∞) bc f'(x) is decreasing

e) f(x) has a point of inflection. Justify.

x = -2.5, 0.25, 2 bc f'(x) has local extrema at these values.

3. A boat is pulled by a rope, attached to the bow of the boat, and passing through a pulley on a dock that is 1 meter higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/sec, how fast is the boat approaching the dock when it is 8 meters from the dock?



$K: \frac{dr}{dt} = -1 m/sec$

$F: \frac{dx}{dt}$

$W: x = 8m,\ y = 1m,\ r = \sqrt{65}$
constant

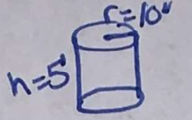
$x^2 + y^2 = r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$2(8)\frac{dx}{dt} + 2(1)(0) = 2\sqrt{65}(-1)$

$\frac{dx}{dt} = \frac{-2\sqrt{65}}{16} = -\frac{\sqrt{65}}{8} m/sec$

4. A cylinder with a height of 5 ft and a base radius of 10 in is filled with water. The water is being drained out at a rate of 3 cubic inches per minute. How fast is the water level decreasing?



$K: \frac{dV}{dt} = -3 in^3/min$

$F: \frac{dh}{dt}$

$W: r = 10'',\ h = 60''$ ← constant

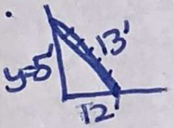
$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$-3 = \pi(10)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-3}{100\pi}$ in/min.

5. A 13-foot ladder propped up against a wall is sliding downward such that the rate at which the top of the ladder is falling to the floor is 7 ft/sec. Find the rate at which the distance between the bottom of the ladder and the base of the wall is increasing when the top of the ladder is 5 ft from the base of the wall.



$K: \frac{dy}{dt} = -7 ft/sec$

$F: \frac{dx}{dt}$

constant
$W: x = 12ft,\ y = 5ft,\ L = 13ft$

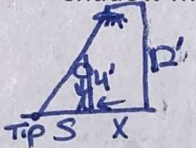
$x^2 + y^2 = L^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$12\frac{dx}{dt} + 5(-7) = 0$

$\frac{dx}{dt} = \frac{35}{12}$ ft/sec.

6. A street light is mounted at the top of a 12 ft pole. A 4 ft child walks away from the pole at a speed of 3 ft/sec. How fast is the tip of her shadow moving?



$K: \frac{dx}{dt} = 3ft/sec$

$F: \frac{dS}{dt}, \frac{dtip}{dt}$

$\frac{12}{x+S} = \frac{4}{S}$

$4x + 4S = 12S$

$4x = 8S$

$4\frac{dx}{dt} = 8\frac{dS}{dt}$

$\frac{4(3)}{8} = \frac{dS}{dt}$

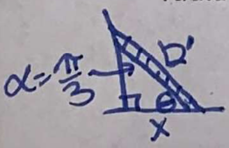
$\frac{dS}{dt} = \frac{3}{2} ft/sec$

$\frac{dtip}{dt} = \frac{dS}{dt} + \frac{dx}{dt}$

$\frac{dtip}{dt} = \frac{3}{2} + 3$

$\frac{dtip}{dt} = \frac{9}{2} ft/sec$

7. A 12-foot ladder is propped up against a wall. If the bottom of the ladder slides away from the wall at a rate of 3 ft/sec, how fast is the measure of the angle between the bottom of the ladder and the floor changing when the angle between the top of the ladder and the wall measures π/3 radians?



$K: \frac{dx}{dt} = 3ft/sec$

$F: \frac{d\theta}{dt}$

$W: \alpha = \frac{\pi}{3}$

$\theta = \pi - \frac{\pi}{2} - \frac{\pi}{3}$

$\frac{6\pi}{6} - \frac{3\pi}{6} - \frac{2\pi}{6}$

$\theta = \frac{\pi}{6}$

$\sin\theta = \sin\frac{\pi}{6}$

$= \frac{1}{2}$

$\cos\theta = \frac{x}{12}$

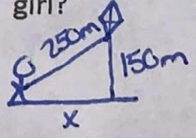
$12\cos\theta = x$

$-12\sin\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

$-12\left(\frac{1}{2}\right)\frac{d\theta}{dt} = 3$

$\frac{d\theta}{dt} = -\frac{1}{2}$ rad/sec

8. A girl is flying a kite at a height of 150 meters. If the kite moves horizontally away from the girl at the rate of 20 m/s, how fast is the string being released when the kite is 250 meters from the girl?



$K: \frac{dx}{dt} = 20m/s$

$F: \frac{ds}{dt}$

constant
$W: S = 250m,\ y = 150m$

$x^2 + 150^2 = 250^2$

$x = \sqrt{250^2 - 150^2}$

$x = 200m$

$x\frac{dx}{dt} + y\frac{dy}{dt} = s\frac{ds}{dt}$

$200(20) + 150(0) = 250\frac{ds}{dt}$

$\frac{ds}{dt} = \frac{4000}{250} = 16m/s$

9. Find the absolute maximum and absolute minimum **values** guaranteed by the Extreme Value Theorem of $f(x) = 2x^3 - 15x^2 + 24x + 7$ on the interval [- 3, 5].

f(x) is continuous on [-3,5]

$f'(x) = 6x^2 - 30x + 24$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x = 1, 4$

$f(-3) = 2(-3)^3 - 15(-3)^2 + 24(-3) + 7 = -254$

$f(1) = 2(1)^3 - 15(1)^2 + 24(1) + 7 = 18$

$f(4) = 2(4)^3 - 15(4)^2 + 24(4) + 7 = -9$

$f(5) = 2(5)^3 - 15(5)^2 + 24(5) + 7 = 2$

Abs. max. = -254

Abs. min. = -9

10. Let $f(x) = \dfrac{x}{x+1}$. Find the value of c that satisfies the conclusion of the Mean Value Theorem on the interval [2, 3].

f(x) is cont. on [2,3]

f'(x) is diff. on (2,3)

$m_{sec} = \dfrac{f(3) - f(2)}{3-2} = \dfrac{\frac{3}{4} - \frac{2}{3}}{1} = \dfrac{1}{12}$

$f'(x) = \dfrac{(x+1) - x}{(x+1)^2}$

$f'(x) = \dfrac{1}{(x+1)^2}$

$\dfrac{1}{12} = \dfrac{1}{(x+1)^2}$

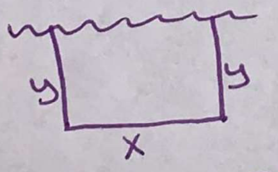
$(x+1)^2 = 12$

$x + 1 = \pm 2\sqrt{3}$

$x = -1 \pm 2\sqrt{3}$

$c = -1 + 2\sqrt{3}$

11. A farmer has 800 ft of fencing and wants to fence off a rectangular field that borders a relatively straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?



$800 = x + 2y$

$x = 800 - 2y$

$A = x \cdot y$

$A = (800 - 2y)y$

$A = 800y - 2y^2$

$A' = 800 - 4y$

$0 = 800 - 4y$

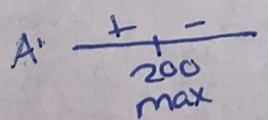
$4y = 800$

$y = 200\text{ft}$

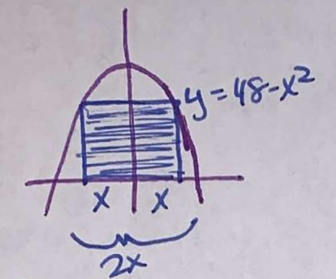
$x = 800 - 2(200)$

$x = 400\text{ft}$

400 ft × 200 ft



12. Find the dimensions of the rectangle with maximum area that has its base on the x-axis and its other two vertices above the x axis and lying on the parabola y = 48 - x^2.



$b=2x$
$h=48-x^2$

$A=bh$
$A=2x(48-x^2)$
$A=96x-2x^3$
$A'=96-6x^2$
$0=96-6x^2$
$6x^2=96$
$x^2=16$
$x=4$

$b=2(4)=8$
$h=48-(4)^2=32$

8×32 units

13. Find the following limits.

a) $\lim_{x\to\infty}\frac{x^2}{e^{5x}}$ $\lim_{x\to\infty} x^2=\infty$ $\lim_{x\to\infty} e^{5x}=\infty$

LH: $\lim_{x\to\infty}\frac{2x}{5e^{5x}}$

$\lim_{x\to\infty} 2x=\infty$
$\lim_{x\to\infty} 5e^{5x}=\infty$

LH: $\lim_{x\to\infty}\frac{2}{25e^{5x}}=0$

(b) $\lim_{x\to 0}\frac{2x+\sin(5x)}{\sin(3x)}$ $\lim_{x\to 0} 2x+\sin(5x)=0$ $\lim_{x\to 0}\sin(3x)=0$

LH: $\lim_{x\to 0}\frac{2+5\cos(5x)}{3\cos(3x)}$

$=\frac{2+5(1)}{3(1)}$

$=\frac{7}{3}$

c) $\lim_{x\to\frac{3\pi}{2}}\frac{\sin x}{\pi-x}$

$=\frac{\sin\frac{3\pi}{2}}{\pi-\frac{3\pi}{2}}$

$=\frac{-1}{\frac{-\pi}{2}}=\frac{2}{\pi}$

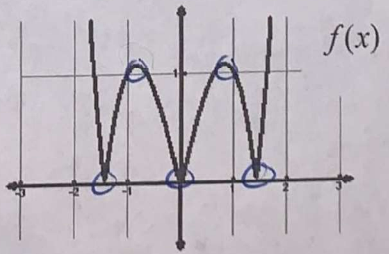
Multiple Choice:
You must show reasoning and all work for the multiple choice questions.

D ⑭ A function f is continuous for all x and has a local maximum at (3,5). Which statement **must** be true?

f ↗ ↘
f' $\overset{++}{}\underset{3}{|}\overset{--}{}$

a. f ' (3) = 0 no bc it could a corner/cusp $\wedge$
b. the graph of f is concave up at x = 3
c. f ' (x) exists at x = 3 $\wedge$ $\wedge$
d f ' (x) is positive if x < 3 and f ' (x) is negative if x > 3
e. f ' (x) is negative if x < 3 and f ' (x) is positive if x > 3

C 15. f(x) is graphed to the right.
How many critical numbers does $f(x)$ have?
a. 3 b. 4 c. 5 d. 6 e. infinitely many



$f'(x)=0$ or undefined