Derivative of the Inverse

1. Which inverse trigonometric function $g(x)$ has the derivative $g'(x) = \frac{1}{x^2+1}$?

$g(x) = \tan^{-1} x$

omit

2. If $g(x) = \sqrt[3]{x-1}$ and f is the inverse function of g, then $f'(x) = 3x^2$

$f(x) = g^{-1}(x)$

$f'(x) = \frac{1}{g'(g^{-1}(x))}$

$f'(x) = \frac{1}{g'(x^3+1)}$

Find $g^{-1}(x)$

$y = \sqrt[3]{x-1}$

$x = \sqrt[3]{y-1}$

$g^{-1}(x) = x^3+1$

$g'(x) = \frac{1}{3}(x-1)^{-2/3}$

$g'(g^{-1}(x)) = \frac{1}{3}(x^3+1-1)^{-2/3}$

$g'(x^3+1) = \frac{1}{3\sqrt[3]{x^6}} = \frac{1}{3x^2}$

$= \frac{1}{\frac{1}{3x^2}} = 3x^2$

3. Let $f(x) = x^2 - 3x,\ x > 0$ Find: $f^{-1}(4) = 4$ $(f^{-1})'(4) =$

means $y=4$

$4 = x^2 - 3x$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4 \quad \cancel{x = -1}$

$f^{-1}(4) = 4$

$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))}$

$= \frac{1}{f'(4)}$

$= \frac{1}{5}$

$f'(4) = 2x - 3$

$= 2(4) - 3$

$= 5$

4. Let $f(x) = x^2 - 13,\ x > 0$ Find: $f^{-1}(3) = 4$ $(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$ write 1st

① $3 = x^2 - 13$

$0 = x^2 - 16$

$x = 4$ plug into formula on right

② $f'(x) = 2x$

$f'(4) = 2(4)$

$f'(4) = 8$

plug into formula on right

$= \frac{1}{f'(4)}$

$= \frac{1}{8}$

5. Let $g(x)$ be the inverse of $f(x) = x^3 + 2x + 4$. Calculate $g(7) = 1$ $g'(7) = \frac{1}{f'(f^{-1}(7))}$

$g(x) = f^{-1}(x)$

$g(7) = f^{-1}(7)$

$7 = x^3 + 2x + 4$

$0 = x^3 + 2x - 3$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -3 \\ & & 1 & 1 & 3 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

$(x-1)(x^2+x+3) = 0$

$x = 1 \quad x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2}$

imaginary

$f'(x) = 3x^2 + 2$

$f'(1) = 3(1)^2 + 2$

$f'(1) = 5$

$= \frac{1}{f'(1)}$

$= \frac{1}{5}$

6. Find $g'\left(-\frac{1}{2}\right)$ where $g(x)$ is the inverse of $f(x) = \frac{x^3}{x^2+1}$

$g(x) = f^{-1}(x)$

$g'(-\frac{1}{2}) = \frac{1}{f'(f^{-1}(-\frac{1}{2}))}$

$g'(-\frac{1}{2}) = \frac{1}{f'(-1)}$

$g'(-\frac{1}{2}) = \frac{1}{1} = 1 = g'(-\frac{1}{2})$

$-\frac{1}{2} = \frac{x^3}{x^2+1}$

$2x^3 = -x^2 - 1$

$2x^3 + x^2 + 1 = 0$

$$\begin{array}{r|rrrr} -1 & 2 & 1 & 0 & 1 \\ & \downarrow & -2 & 1 & -1 \\ \hline & 2 & -1 & 1 & 0 \end{array}$$

$(x+1)(2x^2 - x + 1)$

$f^{-1}(-\frac{1}{2}) = -1$

$f'(x) = \frac{(x^2+1)(3x^2) - x^3(2x)}{(x^2+1)^2}$

$f'(-1) = \frac{(1+1)(3) - (-1)(-2)}{4}$

$= \frac{6-2}{4} = \frac{4}{4} = 1$

7. Let $f(x) = \frac{\ln e^{2x}}{x-1}$ for $x > 1$. If g is the inverse of f, then $g'(3) =$

$f(x) = \frac{2x}{x-1}$

$g(x) = f^{-1}(x)$

$g'(3) = \frac{1}{f'(f^{-1}(3))}$

$g'(3) = \frac{1}{f'(3)} = \frac{1}{-1/2} = -2 = g'(3)$

$3 = \frac{2x}{x-1}$

$3x - 3 = 2x$

$x = 3$

$f'(3) = \frac{(x-1)(2) - 2x}{(x-1)^2}$

$= \frac{(3-1)(2) - 2(3)}{(3-1)^2}$

$= \frac{4-6}{4} = -\frac{2}{4} = -\frac{1}{2}$

For questions 8-10, calculate $g(b)$ and $g'(b)$, where g is the inverse of f.

8. $f(x) = x + \cos x, \qquad b = 1$

$g(b) = f^{-1}(b)$

$g(1) = f^{-1}(1)$

$g'(1) = \frac{1}{f'(f^{-1}(1))}$

$g'(1) = \frac{1}{f'(0)}$

$1 = x + \cos x$

$x = 0$ (mental math)

$f^{-1}(1) = 0$

So $g(1) = 0$

$f'(x) = 1 - \sin x$

$f'(0) = 1 - \sin 0$

$f'(0) = 1$

$g'(1) = \frac{1}{1}$

$g'(1) = 1$

9. $f(x) = 4x^3 - 2x, \qquad b = -2$

$g(b) = f^{-1}(b)$

$g(-2) = f^{-1}(-2)$

$g(-2) = \frac{1}{f'(f^{-1}(-2))}$

$g(-2) = \frac{1}{f'(1)}$

$g(-2) = \frac{1}{10}$

$-2 = 4x^3 - 2x$

$0 = 4x^3 - 2x + 2$

$0 = 2x^3 - x + 1$

$$\begin{array}{r|rrrr} -1 & 2 & 0 & -1 & 1 \\ & \downarrow & -2 & 2 & -1 \\ \hline & 2 & -2 & 1 & 0 \end{array}$$

$(x-1)(2x^2 - 2x + 1)$

$x = 1 \quad f^{-1}(-2) = 1$

$g(-2) = 1$

$f'(x) = 12x^2 - 2$

$f'(1) = 12(1)^2 - 2$

$f'(1) = 10$

10. $f(x) = \sqrt{x^2 + 6x}$ for $x \geq 0 \qquad b = 4$

$g(b) = f^{-1}(b)$

$g(4) = f^{-1}(4)$

$g'(4) = \frac{1}{f'(f^{-1}(4))}$

$g'(4) = \frac{1}{f'(2)}$

$g'(4) = \frac{1}{5/4}$

$g'(4) = 4/5$

$4 = \sqrt{x^2 + 6x}$

$16 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$\cancel{x = -8} \quad x = 2$

$g(4) = 2$

$f'(x) = \frac{1}{2}(x^2 + 6x)^{-1/2}(2x + 6)$

$f'(x) = \frac{x+3}{\sqrt{x^2+6x}}$

$f'(2) = \frac{2+3}{\sqrt{(2)^2 + 6(2)}} = \frac{5}{4}$

$f'(2) = \frac{5}{4}$

Derivatives of Inverse Trig Functions

Find the derivative of the following.

1. $y = \tan^{-1}(2x)$

$y' = \frac{1}{1+(2x)^2} \cdot 2$

$y' = \frac{2}{1+4x^2}$

2. $y = \sin^{-1}(x^2)$

$y' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$

$y' = \frac{2x}{\sqrt{1-x^4}}$

3. $y = \sec^{-1}(x^3)$

$y' = \frac{1}{|x^3|\sqrt{(x^3)^2-1}} \cdot 3x^2$

$y' = \frac{3x^2}{|x^3|\sqrt{x^6-1}} = \frac{3}{|x|\sqrt{x^6-1}}$

4. $y = \arctan(x^2+1)$

$y' = \frac{1}{1+(x^2+1)^2} \cdot 2x$

$y' = \frac{2x}{1+(x^2+1)^2}$

5. $y = \arcsin(5x)$

$y' = \frac{1}{\sqrt{1-(5x)^2}} \cdot 5$

$y' = \frac{5}{\sqrt{1-25x^2}}$

6. $y = \operatorname{arcsec}(5x)$

$y' = \frac{1}{|5x|\sqrt{(5x)^2-1}} \cdot 5$

$y' = \frac{5}{|5x|\sqrt{25x^2-1}}$

$y' = \frac{1}{|x|\sqrt{25x^2-1}}$

7. $y = \arctan(\sqrt{x})$

$y' = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2\sqrt{x}(1+x)}$

8. $y = \sin^{-1}(\sqrt{x})$

$y' = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2\sqrt{x}\sqrt{1-x}}$

$y' = \frac{1}{2\sqrt{x-x^2}}$

9. $y = \tan^{-1}(x^2 + 2x)$

$$y' = \frac{1}{1+(x^2+2x)^2} \cdot 2x+2$$

$$y' = \frac{2x+2}{1+(x^2+2x)^2}$$

10. $y = \tan^{-1}(e^x)$

$$y' = \frac{1}{1+(e^x)^2} \cdot e^x$$

$$y' = \frac{e^x}{1+e^{2x}}$$

11. $y = x\tan^{-1}(x)$

$$y' = x \cdot \frac{1}{1+x^2} + \tan^{-1}(x)$$

$$y' = \frac{x}{1+x^2} + \tan^{-1}(x)$$

12. $y = x^2 \sin^{-1}(x)$

$$y' = x^2 \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}(x) \cdot 2x$$

$$y' = \frac{x^2}{\sqrt{1-x^2}} + 2x\sin^{-1}(x)$$

13. $y = e^x \sin^{-1}(x)$

$$y' = e^x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}(x)e^x$$

$$y' = \frac{e^x}{\sqrt{1-x^2}} + e^x\sin^{-1}(x)$$

14. $y = \ln(x)\arctan(x)$

$$y' = \ln x \cdot \frac{1}{1+x^2} + \tan^{-1}(x) \cdot \frac{1}{x}$$

$$y' = \frac{\ln x}{1+x^2} + \frac{\tan^{-1}(x)}{x}$$

Limits and L'Hopitals Rule

Evaluate each Limit. Use L'Hopitals Rule when possible.

1. $\lim_{x\to 2} \frac{x^3-x-2}{x-2}$

$\lim_{x\to 2} x^3-x-2 = 8-2-2=4$

$\lim_{x\to 2} x-2=0$ $\quad \frac{4}{0}$

$\lim_{x\to 2^-} \frac{x^3-x-2}{x-2} = \frac{+\#}{-\#} = -\#$

$\lim_{x\to 2^+} \frac{x^3+x-2}{x-2} = \frac{\#}{\#} = +\#$

DNE

2. $\lim_{x\to\infty} \frac{(\ln x)^3}{x}$

$\lim_{x\to\infty} (\ln x)^3 = \infty$

$\lim_{x\to\infty} x = \infty$

LH: $\lim_{x\to\infty} \frac{3(\ln x)^2}{x}$

$\lim_{x\to\infty} 3\ln^2 x = \infty$

$\lim_{x\to\infty} x = \infty$

LH: $\lim_{x\to\infty} \frac{6\ln x}{x}$

$\lim_{x\to\infty} 6\ln x = \infty$

$\lim_{x\to\infty} x = \infty$

LH: $\lim_{x\to\infty} \frac{6}{x}$

$= \frac{6}{\infty}$

$= 0$

3. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x}$

$\lim_{x\to 0} \sqrt{4-x^2}-2=0$

$\lim_{x\to 0} x=0$

LH: $\lim_{x\to 0} \frac{1}{2}(4-x^2)^{-1/2}\cdot -2x$

$\lim_{x\to 0} \frac{-x}{2\sqrt{4-x^2}} = \frac{0}{2\sqrt{4-0^2}}$

$=0$

4. $\lim_{x\to 0} \frac{e^x-(1-x)}{x}$

$\lim_{x\to 0} e^x-(1-x) = 1-1+0=0$

$\lim_{x\to 0} x=0$

LH: $\lim_{x\to 0} e^x+1 = e^0+1$

$=1+1$

$=2$

5. $\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$

$\lim_{x\to 0} \sin(2x) = \sin 0 = 0$

$\lim_{x\to 0} \sin(3x) = \sin 0 = 0$

LH: $\lim_{x\to 0} \frac{2\cos(2x)}{3\cos(3x)} = \frac{2\cos(0)}{3\cos(0)}$

$= \frac{2(1)}{3(1)}$

$= 2/3$

6. $\lim_{x\to 0} \frac{x+\sin 3x}{x-\sin 3x}$

$\lim_{x\to 0} x+\sin 3x = 0$

$\lim_{x\to 0} x-\sin 3x = 0$

LH: $\lim_{x\to 0} \frac{1+3\cos 3x}{1-3\cos 3x} = \frac{1+3(1)}{1-3(1)} = \frac{1+3}{1-3}$

$= \frac{4}{-2} = -2$

7. $\lim_{x\to\infty} \frac{x^2+2x+1}{x-1}$ you can figure this out based on end behavior too

$\lim_{x\to\infty} x^2+2x+1 = \infty$

$\lim_{x\to\infty} x-1 = \infty$

LH: $\lim_{x\to\infty} 2x+2 = \infty$

8. $\lim_{x\to 16} \frac{\sqrt[4]{x}-2}{x-16}$

$\lim_{x\to 16} \sqrt[4]{x}-2 = 0$

$\lim_{x\to 16} x-16 = 0$

LH: $\lim_{x\to 16} \frac{1}{4}x^{-3/4} = \frac{1}{4\sqrt[4]{16^3}} = \frac{1}{32}$

9. $\lim_{x\to\infty} \frac{x^2}{e^{5x}}$

$\lim_{x\to\infty} x^2 = \infty$

$\lim_{x\to\infty} e^{5x} = \infty$

LH: $\lim_{x\to\infty} \frac{2x}{5e^{5x}}$

$\lim_{x\to\infty} 2x = \infty$

$\lim_{x\to\infty} 5e^{5x} = \infty$

LH: $\lim_{x\to\infty} \frac{2}{25e^{5x}} = \frac{2}{\infty} = 0$

10. $\lim_{x\to\infty} \frac{\ln x}{x}$

$\lim_{x\to\infty} \ln x = \infty$

$\lim_{x\to\infty} x = \infty$

LH: $\lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

11. $\lim_{x\to 0} \frac{e^{2x}-1}{e^x}$

$\lim_{x\to 0} e^{2x}-1 = 0$

$\lim_{x\to 0} e^x = 1$

$\therefore \lim_{x\to 0} \frac{e^{2x}-1}{e^x} = \frac{0}{1} = 0$

12. $\lim_{x\to 3} \frac{2x-6}{x^2-9}$

$\lim_{x\to 3} 2x-6 = 0$

$\lim_{x\to 3} x^2-9 = 0$

LH: $\lim_{x\to 3} \frac{2}{2x} = \frac{1}{3}$

AP Calculus Multiple Choice and Free Response Practice

Non-Calculator Section

1. If $y = x \cdot \sin x$, then $\frac{dy}{dx} =$
   a. $\sin x + \cos x$
   b. $\sin x + x \cdot \cos x$
   c. $\sin x - x \cdot \cos x$
   d. $x(\sin x + \cos x)$
   e. $x(\sin x - \cos x)$

$x\cos x + \sin x$

2. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$
   a. 4
   b. 5
   c. 6
   d. 7
   e. 8

$f'(x) = 7 + \frac{1}{x}$

$f'(1) = 7 + 1 = 8$

3. If y= $(x^3 - \cos x)^5$, then $y' =$
   a. $5(x^3 - \cos x)^4$
   b. $5(3x^2 + \sin x)^4$
   c. $5(3x^2 + \sin x)$
   d. $5(3x^2 + \sin x)^4 \cdot (6x + \cos x)$
   e. $5(x^3 - \cos x)^4 \cdot (3x^2 + \sin x)$

$y' = 5(x^3 - \cos x)^4(3x^2 + \sin x)$

$f'(x) = \frac{1}{2}(x^2-4)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2-4}}$ $\quad f'(7) = \frac{7}{\sqrt{45}}$

4. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is
   a. $\frac{7}{\sqrt{5}}$
   b. $\frac{14}{\sqrt{5}}$
   c. $\frac{18}{\sqrt{5}}$
   d. $\frac{15}{\sqrt{21}}$
   e. $\frac{30}{\sqrt{21}}$

$g(3) = 7$

$g'(x) = 3$

$f(g(3)) = f'(g(3)) \cdot g'(3)$

$= f'(7) \cdot g'(3)$

$= \frac{7}{3\sqrt{5}} \cdot 3$

$= \frac{7}{\sqrt{5}}$

5. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?
   a. (0,0) only
   b. $\left(\frac{1}{2}, \frac{1}{5}\right)$ only
   c. (0,0) and (−4,2)
   d. (0,0) and $\left(4, \frac{2}{3}\right)$
   e. There are no such points

$f'(x) = \frac{(x+2) - x}{(x+2)^2}$ $\quad f(0) = \frac{0}{0+2} = 0$

$f(-4) = \frac{-4}{-4+2} = 2$

$f'(x) = \frac{2}{(x+2)^2}$

$\frac{1}{2} = \frac{2}{(x+2)^2}$

$4 = x^2 + 4x + 4$ $\quad 0 = x(x+4)$

$0 = x^2 + 4x$ $\quad x = 0, -4$

6. Let $f(x) = (2x + 1)^3$ and let g be the inverse of f. Given that $f(0) = 1$, what is the value of $g'(1)$?
   a. $-\frac{2}{27}$
   b. $\frac{1}{54}$
   c. $\frac{1}{27}$
   d. $\frac{1}{6}$
   e. 6

$g(x) = f^{-1}(x)$

$g(x) = \frac{1}{f'(f^{-1}(x))}$

$g'(1) = \frac{1}{f'(f^{-1}(1))}$

$1 = (2x+1)^3$ $\quad g'(1) = \frac{1}{f'(0)}$

$1 = 2x + 1$

$0 = 2x$ $\quad g'(1) = \frac{1}{6}$

$x = 0$

$f^{-1}(1) = 0$

$f'(x) = 3(2x+1)^2 \cdot 2$

$f'(0) = 3(1)^2 \cdot 2$

$= 6$

7. The $\lim_{h\to 0} \frac{\ln[\sin(x+h)]-\ln(\sin x)}{h}$ is …

a. $\sin x$
b. x
c. $\frac{1}{x}$
d. $\cot x$
e. $\tan x$

$f(x) = \ln(\sin x)$

$f'(x) = \frac{1}{\sin x} \cdot \cos x$

8. The $\lim_{x\to\frac{\pi}{2}} \frac{\sin(x)-\sin\left(\frac{\pi}{2}\right)}{x-\frac{\pi}{2}}$ has a value of …

a. 0
b. 1
c. $\frac{\sqrt{2}}{2}$
d. -1
e. 2

$\lim_{x\to\frac{\pi}{2}} \sin(\frac{\pi}{2}) - \sin(\frac{\pi}{2}) = 0$

$\lim_{x\to\frac{\pi}{2}} \frac{\pi}{2} - \frac{\pi}{2} = 0$

LH: $\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{1} = \cos\frac{\pi}{2} = 0$

9. The equation of the normal line to the graph of $y = e^{2x}$ when $\frac{dy}{dx} = 2$ is …

a. $y = -\frac{1}{2}x + 1$
b. $y = 2\left(x - \frac{\ln 2}{2}\right) + 2$
c. $y = 2x + 1$
d. $y = -\frac{1}{2}\left(x - \frac{\ln 2}{2}\right) + 2$

$y = e^{2(0)}$
$y = 1$
$(0, 1)$

$\frac{dy}{dx} = 2e^{2x}$
$2 = 2e^{2x}$
$1 = e^{2x}$
$x = 0$

$y'(0) = 2e^{2(0)}$
$m_{tan} = 2$ ✓
$m_{norm} = -\frac{1}{2}$

10. If $f(x) = 5\cos^2(\pi - x)$, then $f'\left(\frac{\pi}{2}\right)$ is …

a. 0
b. $-\frac{2}{3}$
c. $\frac{2}{3}$
d. $-\frac{5}{6}$
e. 1

$f'(x) = 5[\cos(\pi - x)]^2$
$= 2\cos(\pi - x) \cdot -\sin(\pi - x) \cdot -1$
$= 2\cos(\pi - x)\sin(\pi - x)$
$f'(\pi/2) = 2\cos(\pi/2)\sin(\pi/2)$
$= 2(0)(1)$
$= 0$

11. For what value(s) of k does the graph of $g(x) = ke^{2x} + 3x$ have a normal line whose slope is $-\frac{1}{5}$ when $x = 1$?

a. e
b. $\frac{1}{e^2}$
c. $-\frac{8}{5e^2}$
d. $\frac{2}{e^2}$
e. 0

$g'(x) = 2ke^{2x} + 3$
$5 = 2ke^{2(1)} + 3$
$1 = ke^2$
$k = \frac{1}{e^2}$

12. If $f'(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

a. $2\sqrt{3}$
b. 4
c. $\sqrt{3}$
d. 8
e. 6

$f'\left(\frac{\pi}{6}\right) = \tan\left(\frac{2\pi}{6}\right)$
$= \tan\left(\frac{\pi}{3}\right)$
$= \sqrt{3}$

13. If $y = 3x(3^{-2x})$, then $\frac{dy}{dx} =$

a. $-\frac{6\ln 3}{3^{2x}}$
b. $\frac{3\ln 3}{3^{2x}}$
c. $\frac{3(1-2x\cdot\ln 3)}{3^{2x}}$
d. $\frac{1+x\cdot\ln 3}{9^{2x}}$
e. $\frac{1+x\cdot\ln 3}{3^{2x}}$

$y' = 3x \cdot 3^{-2x}\ln 3 \cdot -2 + 3 \cdot 3^{-2x}$

$y' = \frac{-6x\ln 3}{3^{2x}} + \frac{3}{3^{2x}}$

$= \frac{3(1-2x\ln 3)}{3^{2x}}$

14. If $f(x) = \log_5(5x+1)^4$, then what is the value of $f'(1)$?

a. $\frac{10}{3\ln 5}$
b. $\frac{4}{\ln 6}$
c. $\frac{2}{3\ln 5}$
d. $\frac{4}{\ln 5}$
e. $\frac{5}{\ln 4}$

$f'(x) = \frac{4\ln(5x+1)}{\ln 5}$

$= \frac{4}{\ln 5} \cdot \frac{5}{5x+1}$

$f'(1) = \frac{20}{\ln(5)(5\cdot 1+1)}$

$= \frac{20}{6\ln 5}$

$= \frac{10}{3\ln 5}$

Calculator Section

15. The graph of $y = e^{\tan x} - 2$ crosses the x –axis at one point in the interval $[0,1]$. What is the slope of the graph at this point?
   a. 0.606
   b. 2
   c. 2.242
   d. 2.961
   e. 3.747

$\text{Solve}\left(e^{\tan(x)} - 2 = 0, x\right) \Big| 0 < x < 1$

$x = 0.606$

$\frac{d}{dx}\left(e^{\tan x} - 2\right) \Big| x = 0.606$ Enter

16. Given that $f(x) = x^2 e^x$, what is an approximate value of $f(1.1)$ if you use the equation of the tangent line to the graph of f at $x = 1$?
   a. 3.534
   b. 3.635
   c. 7.055
   d. 8.155
   e. 5.263

next unit

17. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangents?
   a. -0.701
   b. -0.567
   c. -0.391
   d. -0.302
   e. -0.327

$f'(x) = 6e^{2x}$

$g'(x) = 18x^2$

$f'(x) = g'(x)$

$\text{Solve}\left(6e^{2x} = 18x^2, x\right)$

≈ 0.391

18. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$.
   a. $y = 8x - 5$
   b. $y = x + 7$
   c. $y = x + 0.763$
   d. $y = x - 0.122$
   e. $y = 2x - 3.407$

$\leftarrow m_{tan}$

$f'(x) = 4x^3 + 4x$

$1 = 4x^3 + 4x$

$\text{Solve}(1 = 4x^3 + 4x, x)$

$x \approx .237$

$f(.237) = (.237)^4 + 2(.237)^2$

$f(.237) \approx 0.115$

$y - 0.115 = 1(x - .237)$

$y = x - 0.122$

19. On the interval $-4 < x < 4$, for what value(s) of x will the graphs of $y = \log_4\left(\frac{2x}{2x+3}\right)$ and $y = x^4 + 3xe^x$ have parallel tangent lines?
   a. -0.395 only
   b. -1.568 and -0.395
   c. -0.480 only
   d. -0.817 and 0.159
   e. 0.159 only

$\text{Solve}\left(\left(\frac{d}{dx}\left(\log_4\left(\frac{2x}{2x+3}\right)\right) = \frac{d}{dx}\left(x^4 + 3xe^x\right)\right), x\right) \Big| -4 < x < 4$

FREE RESPONSE #1

Non-Calculator

Consider the piece-wise defined function below to answer the questions that follow.

$$f(x)=\begin{cases} ax^2+bx+2, & x\le 2 \\ ax+b, & x>2 \end{cases}$$

a. If $a=-3$ and $b=4$, will $f(x)$ be continuous at $x=2$? Justify your answer.

$-3(2)^2+4(2)+2=-2$

$-3(2)+4=-2$

$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=\lim_{x\to 2} f(x)=f(2)=-2$ $\therefore$ f(x) is continuous at $x=2$

b. If $a=-3$ and $b=4$, will $f(x)$ be differentiable at $x=2$? Justify your answer.

$f(x)=-3x^2+4x+2$ $\quad f(x)=-3x+4$ $\quad f'(x)=\begin{cases} -6x+4, & x\le 2 \\ -3, & x>2 \end{cases}$

$f'(x)=-6x+4$ $\quad f'(x)=-3$

$f'(x)=-8$ $\quad -8\ne -3$

linear slope + horizontal line so there is a corner

no f(x) is not differentiable at $x=2$

c. For what value(s) of a and b will $f(x)$ be both continuous and differentiable at $x=2$? Show your work.

$ax^2+bx+2=ax+b$

$a(2)^2+b(2)+2=a(2)+b$

$4a+2b+2=2a+b$

$2a+2=-b$

$b=-2a-2$

$2ax+b=a$

$2a(2)+b=a$

$4a+b=a$

$b=-3a$

$-2a-2=-3a$

$a=2$

$b=-3a$

$b=-3(2)$

$b=-6$

$a=2$

$b=-6$

Calculator

FREE RESPONSE #2

A rodeo performer spins a lasso in a circle perpendicular to the ground. The height from the ground of the knot, measured in units of feet, in the lasso is modeled by the function

$$H(t) = -3\cos\left(\frac{5\pi}{3}t\right) + 5,$$

where t is the time measured in seconds after the lasso begins to spin.

a. Find the value of $H(0.75)$. Using correct units, explain what this value represents in the context of this problem.

$H(0.75) \approx 7.121$ ft

After 0.75 sec, the lasso is approximately 7.121 ft above the ground.

b. Find the value of $H'(0.75)$. Using correct units, explain what this value represents in the context of this problem.

$H'(0.75) \approx -11.107$

The IROC in the height of the lasso at 0.75 sec. is decreasing by 11.107 ft/sec.

c. Find $H'(t)$ and sketch its graph on the axes to the right for the interval $0 < t < 5$ seconds.

$H'(t) = 5\pi \sin\left(\frac{5\pi x}{3}\right)$

amp $= 5\pi$

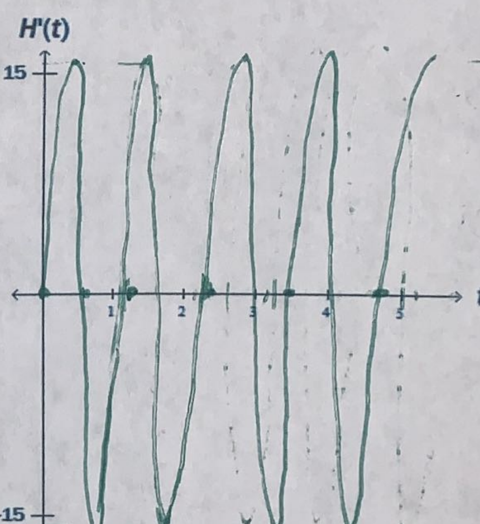
per $= \frac{6}{5}$

per $= 2\pi \div b$

$2\pi \div \frac{5\pi}{3}$

$2\pi \cdot \frac{3}{5\pi}$

no vertical shift or phase shift



d. During the first five seconds of the performer spinning the lasso, how many times is the lasso at its maximum height? Give a reason for your answer based on the graph of $H'(t)$.

The lasso reaches its maximum height 4 times between 0 & 5 seconds because $H'(t)$ changes from positive to negative 4 times.

e. What is the height of the lasso the first time it is at its minimum height on the interval $0 < t < 5$ seconds? Justify your answer and show your work.

Graph: enter function $-3\cos(\frac{5\pi}{3}x)+5 \mid 0<x<5$

Then Menu, 6 Analyze Graph, 2 Minimum

Enter

move vertical line on left of min. ENTER

move vertical line on right of min. ENTER

Enter → (1.2, 2)

$H(1.2) = -3\cos\left(\frac{5\pi}{3} \cdot 1.2\right) + 5$

$H(1.2) = -3\cos(2\pi) + 5$

$H(1.2) = -3(1) + 5$

$H(1.2) = 2$ ft

★ you can also Solve$\left(\frac{d}{dx}(f(x)) = 0, x\right) \mid 0<x<5$ to get x-value.