Unit 2

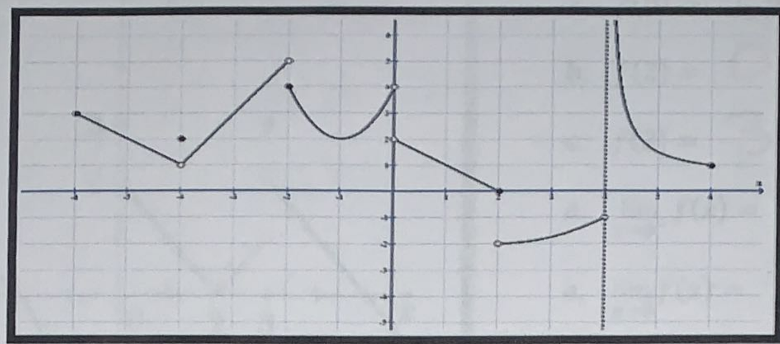
Bonanni

AP Calculus AB - Limits
Fall 2020

Day	Topic / Essential Question	Assignment
Thursday, August 20th	**2.1 Limits from Graphs and Graphs from Limits** *E.Q: How can I estimate limits from graphs and estimate graphs based on limit statements?*	Graphs from limits and limits from graphs worksheet **Ticket Out the Door**
Friday, August 21st	**Creative Factoring** **2.2 Algebraic Limits** *E.Q: How can limits of a function be found algebraically or from a table of values?*	**Skills Check** 2.1 Algebraic Limits Worksheet #1-9
Monday, August 24th	**2.2 Algebraic Limits** *E.Q: How can limits of a function be found algebraically or from a table of values?*	Algebraic Limits Worksheet #10-69
Tuesday, August 25th	**2.3 Intermediate Value Theorem and Continuity** *E.Q.: What types of functions are continuous? What are the types of discontinuities and what happens in functions to create them?*	**Skills Check** 2.2 Continuity and Intermediate Value Theorem Worksheet
Wednesday, August 26th	**2.4 One-sided Limits** *E.Q.: How do I evaluate limits in piecewise functions and absolute value functions?*	Skills Check 2.3 One-sided Limits Graphically & Algebraically Worksheet
Thursday, August 27th	**2.5 Limits Involving Infinity** *E.Q.: What happens to functions as x approaches infinity and what causes y to approach infinity?*	**Skills Check** 2.4 Vertical and Horizontal Asymptotes Worksheet Infinite Limits Worksheet
Friday, August 28th	**Review** *E.Q.: How can we put all the limit concepts together?*	**Skills Check** 2.5 Released Multiple Choice Questions - Limits Worksheet Limits Practice Test
Monday, August 31st	**Unit 2 Test**	This is your opportunity to show what you know!

Graphs from Limit and Limits from Graphs

1. Use the graph to evaluate the limits below



a. $f(-4) = 2$

b. $\lim_{x\to-4^-} f(x) = 1$

c. $\lim_{x\to-4^+} f(x) = 1$

d. $\lim_{x\to-4} f(x) = 1$

e. $f(-2) = 4$

f. $\lim_{x\to-2^-} f(x) = 5$

g. $\lim_{x\to-2^+} f(x) = 3$

h. $\lim_{x\to-2} f(x)$ DNE

i. $f(0)$ DNE

j. $\lim_{x\to0^-} f(x) = 4$

k. $\lim_{x\to0^+} f(x) = 2$

l. $\lim_{x\to0} f(x)$ DNE

m. $f(2) = 0$

n. $\lim_{x\to2^-} f(x) = 0$

o. $\lim_{x\to2^+} f(x) = -2$

p. $\lim_{x\to2} f(x)$ DNE

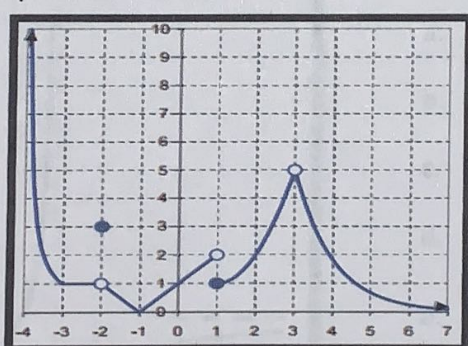
q. $f(4)$ DNE

r. $\lim_{x\to4^-} f(x) = -1$

s. $\lim_{x\to4^+} f(x) = \infty$

t. $\lim_{x\to4} f(x)$ DNE

2. Use the graph to evaluate the expressions below.



a. $f(-2) = 3$

b. $\lim_{x\to-2^+} f(x) = 1$

c. $\lim_{x\to-2} f(x) = 1$

d. $\lim_{x\to-1^+} f(x) = 0$

e. $\lim_{x\to-1^-} f(x) = 0$

f. $\lim_{x\to-1} f(x) = 0$

g. $\lim_{x\to1^+} f(x) = 1$

h. $\lim_{x\to1^-} f(x) = 2$

i. $\lim_{x\to1} f(x)$ DNE

j. $f(3)$ DNE

k. $\lim_{x\to3^+} f(x) = 5$

l. $\lim_{x\to3^-} f(x) = 5$

m. $\lim_{x\to3} f(x) = 5$

n. $\lim_{x\to-4^+} f(x) = \infty$

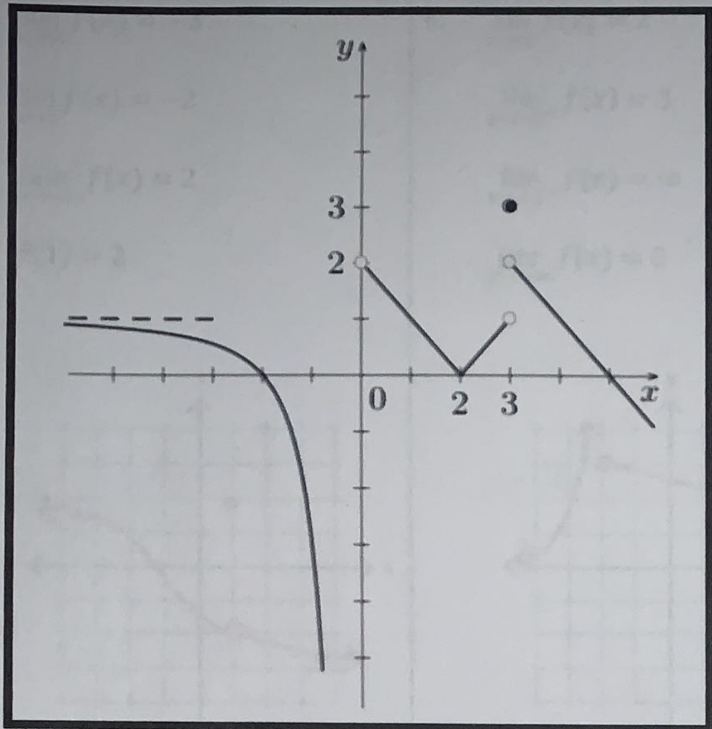
o. $\lim_{x\to\infty} f(x) = 0$

p. $f(1) = 1$

q. $\lim_{x\to-3} f(x) = 1$

r. $f(-4)$ DNE

3. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$ DNE

b. $f(2) =$ 0

c. $f(3) =$ 3

d. $\lim_{x\to 0^-} f(x) =$ $-\infty$

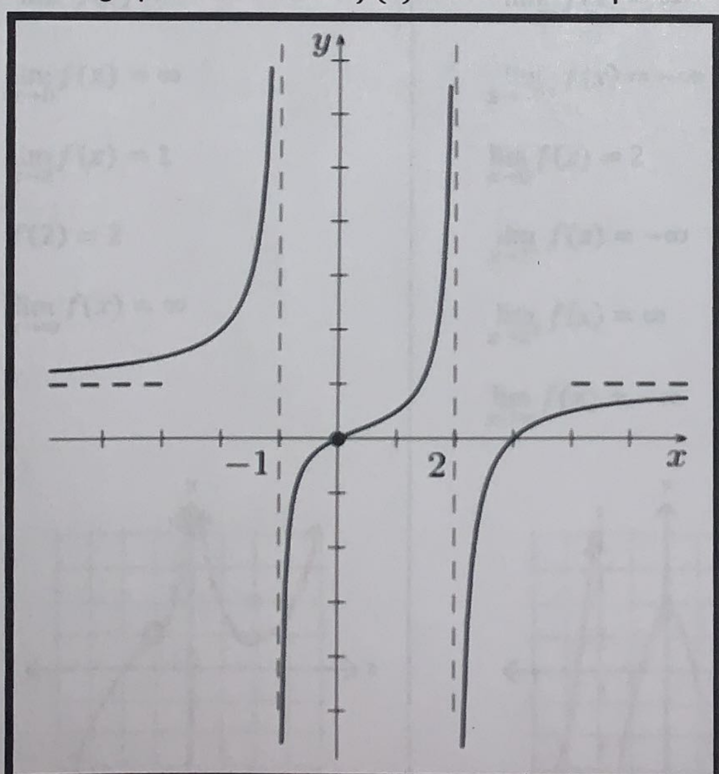
e. $\lim_{x\to 0} f(x) =$ DNE

f. $\lim_{x\to 3^+} f(x) =$ 2

g. $\lim_{x\to 3} f(x) =$ DNE

h. $\lim_{x\to -\infty} f(x) =$ 1

4. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$ 0

b. $f(2) =$ DNE

c. $f(3) =$ 0

d. $\lim_{x\to -1} f(x) =$ DNE

e. $\lim_{x\to 0} f(x) =$ 0

f. $\lim_{x\to 2^+} f(x) =$ $-\infty$

g. $\lim_{x\to \infty} f(x) =$ 1

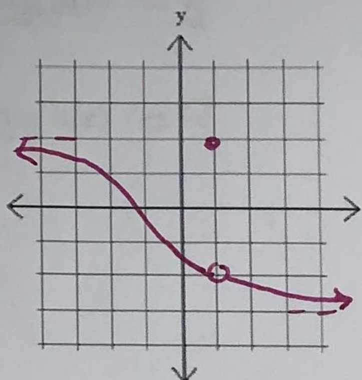
Draw a graph of a function with the give limits.

5. $\lim_{x\to\infty} f(x) = -3$

$\lim_{x\to 1} f(x) = -2$

$\lim_{x\to -\infty} f(x) = 2$

$f(1) = 2$

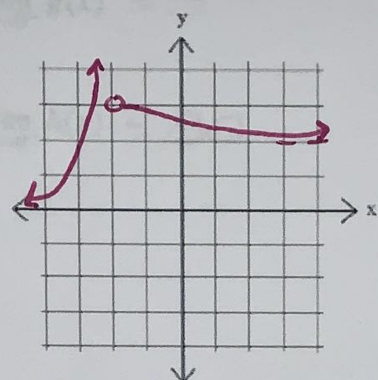


6. $\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to -2^+} f(x) = 3$

$\lim_{x\to -2^-} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 0$

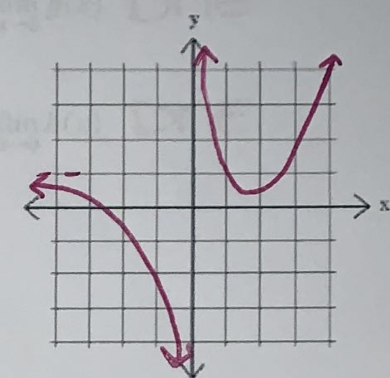


7. $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = 1$



8. $\lim_{x\to -\infty} f(x) = -\infty$

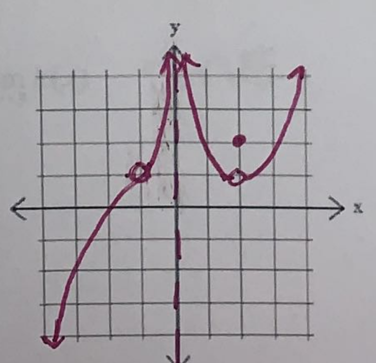
$\lim_{x\to -1} f(x) = 1$

$\lim_{x\to 0} f(x) = \infty$

$\lim_{x\to 2} f(x) = 1$

$f(2) = 2$

$\lim_{x\to\infty} f(x) = \infty$



9. $\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to -2^-} f(x) = \infty$

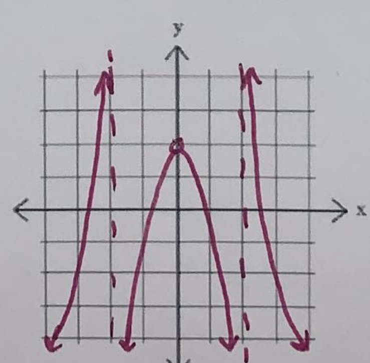
$\lim_{x\to -2^+} f(x) = -\infty$

$\lim_{x\to 0} f(x) = 2$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$



10. $\lim_{x\to -\infty} f(x) = -2$

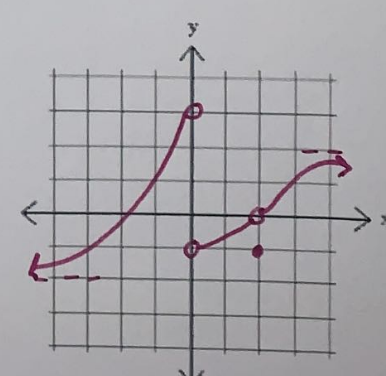
$\lim_{x\to 0^-} f(x) = 3$

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 2} f(x) = 0$

$\lim_{x\to\infty} f(x) = 2$

$f(2) = -1$



11. Use the table of values to evaluate the limit.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	7.018	7.008	7.002	20	7.002	7.008	7.018
g(x)	4.126	4.789	4.989	8	8.0015	8.1016	8.546
h(x)	4971	8987	9972	undefined	8.997	8.987	8.971

a. $\lim_{x\to 0^+} f(x) = 7$

b. $\lim_{x\to 0^-} f(x) = 7$

c. $\lim_{x\to 0} f(x) = 7$

d. $\lim_{x\to 0^+} g(x) = 8$

e. $\lim_{x\to 0^-} g(x) = 5$

f. $\lim_{x\to 0} g(x)$ DNE

g. $\lim_{x\to 0^+} h(x) = 9$

h. $\lim_{x\to 0^-} h(x) = \infty$

i. $\lim_{x\to 0} h(x)$ DNE

12. Use the table of values to evaluate the limit.

X	2.75	2.9	2.99	2.999	3	3.001	3.01	3.1	3.25
f(x)	5.313	5.710	5.970	5.997	8	6.003	6.030	6.310	6.813
g(x)	1.99499	1.99950	1.99995	1.99999	und	2.00005	2.00050	2.00499	2.01
h(x)	199	540	700	854	2	6.003	6.030	6.310	6.813

a. $\lim_{x\to 3} f(x) = 6$

b. $\lim_{x\to 3} g(x) = 2$

c. $\lim_{x\to 3} h(x)$ DNE

Creative Factoring and Other Interesting Algebra

Difference of Squares

Example: $x-16=\left(\sqrt{x}+4\right)\left(\sqrt{x}-4\right)$

1. $x-9$

$(\sqrt{x}+3)(\sqrt{x}-3)$

2. x^2-5

$(x+\sqrt{5})(x-\sqrt{5})$

3. $x^{16}-1$

$(x^8+1)(x^8-1)$

$(x^8+1)(x^4+1)(x^4-1)$

$(x^8+1)(x^4+1)(x^2+1)(x^2-1)$

$(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)$

4. $(x+5)^2-25$

$[(x+5)+5][(x+5)-5]$

$(x+10)x$

$x(x+10)$

5. $9y-a^4$

$(3\sqrt{y}+a^2)(3\sqrt{y}-a^2)$

Sums or Differences of Cubes **"SOAP"**

Example: $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

Example: $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

6. $64a^3+125b^3$

$(4a+5b)(16a^2-20ab+25b^2)$

7. $64a^3x^3-125$

$(4ax-5)(16a^2x^2+20ax+25)$

8. $(x+1)^3+64$

$[(x+1)+4][(x+1)^2-4(x+1)+16]$

$(x+5)(x^2+2x+1-4x-4+16)$

$(x+5)(x^2-2x+13)$

9. $8c^3-(a+b)^3$

$[2c-(a+b)][4c^2+2c(a+b)+(a+b)^2]$

Factor: $x^6 - y^6$:

10. as a difference of squares

$(x^3+y^3)(x^3-y^3)$

$(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

11. as a difference of cubes

$(x^2-y^2)(x^4+x^2y^2+y^4)$

$(x+y)(x-y)(x^4+x^2y^2+y^4)$

Rationalize the Numerator

12. $\dfrac{\sqrt{x+2}-\sqrt{2}}{x}$

$$\frac{(\sqrt{x+2}-\sqrt{2})(\sqrt{x+2}+\sqrt{2})}{x(\sqrt{x+2}+\sqrt{2})}$$

$$\frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$$

$$\frac{1}{\sqrt{x+2}+\sqrt{2}}$$

13. $\dfrac{\sqrt{x+3}+\sqrt{3}}{x}$

$$\frac{(\sqrt{x+3}+\sqrt{3})(\sqrt{x+3}-\sqrt{3})}{x(\sqrt{x+3}-\sqrt{3})}$$

$$\frac{x+3-3}{x(\sqrt{x+3}-\sqrt{3})}$$

$$\frac{1}{\sqrt{x+3}-\sqrt{3}}$$

Factor completely. **Use synthetic division to help find all factors.**

14. $x^3+6x^2+5x-12$

P/q: $\pm1, \pm2, \pm3, \pm6, \pm12$

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 5 & -12 \\ & \downarrow & 1 & 7 & 12 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$(x-1)(x^2+7x+12)$

$(x-1)(x+3)(x+4)$

15. $x^3+x^2-8x-12$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -12 \\ & \downarrow & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$(x+2)(x^2-x-6)$

$(x+2)(x-3)(x+2)$

or $(x+2)^2(x-3)$

16. $x^3+6x^2-9x-14$

P/q: $\pm1, \pm2, \pm7, \pm14$

$$\begin{array}{r|rrrr} 2 & 1 & 6 & -9 & -14 \\ & \downarrow & 2 & 16 & 14 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

$(x-2)(x^2+8x+7)$

$(x-2)(x+1)(x+7)$

Simplify:

17. $\dfrac{2x^3+7x^2+8x+3}{x+1}$

$$\begin{array}{r|rrrr} -1 & 2 & 7 & 8 & 3 \\ & \downarrow & -2 & -5 & -3 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$$\frac{\cancel{(x+1)}(2x^2+5x+3)}{\cancel{(x+1)}}$$

$(2x+3)(x+1)$

18. $\dfrac{2x^3+x^2-13x+6}{x+3}$

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -13 & 6 \\ & \downarrow & -6 & 15 & -6 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

$2x^2-5x+2$

$(2x-1)(x-2)$

Algebraic Limits Worksheet

Given $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$, and $\lim_{x\to a} h(x) = 8$, find each limit if it exists.

1. $\lim_{x\to a}[f(x) + h(x)]$

$\lim_{x\to a} f(x) + \lim_{x\to a} h(x)$

$-3 + 8 = 5$

2. $\lim_{x\to a}[f(x)]^2$

$[\lim_{x\to a} f(x)]^2$

$(-3)^2 = 9$

3. $\lim_{x\to a} \sqrt[3]{h(x)}$

$\sqrt[3]{\lim_{x\to a} h(x)}$

$\sqrt[3]{8} = 2$

4. $\lim_{x\to a} \frac{1}{f(x)}$

$\lim_{x\to a} [f(x)]^{-1}$

$[\lim_{x\to a} f(x)]^{-1}$

$-3^{-1} = -\frac{1}{3}$

5. $\lim_{x\to a} \frac{g(x)}{h(x)}$

$\frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{0}{8} = 0$

6. $\lim_{x\to a} \frac{h(x)}{g(x)}$

$\frac{\lim_{x\to a} h(x)}{\lim_{x\to a} g(x)} = \frac{8}{0}$ DNE

7. $\lim_{x\to a} \frac{2f(x)}{h(x)-f(x)}$

$\frac{2\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)}$

$\frac{2(-3)}{8--3} = \frac{-6}{11}$

8. $\lim_{x\to a}[f(x)h(x)]$

$\lim_{x\to a} f(x) \cdot \lim_{x\to a} h(x)$

$-3 \cdot 8 = -24$

9. $\lim_{x\to a}\left[\frac{g(x)+h(x)}{f(x)}\right]$

$\frac{\lim_{x\to a} g(x) + \lim_{x\to a} h(x)}{\lim_{x\to a} f(x)}$

$\frac{0+8}{-3} = -\frac{8}{3}$

Evaluate the limits:

10. $\lim_{x\to 0} \frac{x^2+7x+6}{x+3}$

$\frac{(0)^2+7(0)+6}{0+3}$

$=2$

11. $\lim_{x\to 2} \frac{\frac{2}{x^2}-\frac{1}{2}}{x-2}$

$\lim_{x\to 2} \frac{\frac{4}{2x^2} - \frac{x^2}{2x^2}}{x-2}$

$\lim_{x\to 2} \frac{\frac{4-x^2}{2x^2}}{x-2}$

$\lim_{x\to 2} \frac{-1(x+2)(x-2)}{2x^2} \cdot \frac{1}{x-2}$

$\lim_{x\to 2} \frac{-1(x+2)}{2x^2} = \frac{-(2+2)}{2(2)^2}$

$= -\frac{1}{2}$

12. $\lim_{x\to 2} \frac{(2x+1)^2-25}{x-2}$

$\lim_{x\to 2} \frac{(2x+1+5)(2x+1-5)}{x-2}$

$\lim_{x\to 2} \frac{(2x+6)(2x-4)}{x-2}$

$\lim_{x\to 2} \frac{2(x+3)2(x-2)}{x-2}$

$\lim_{x\to 2} 4(x+3)$

$4(2+3) = 20$

13. $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$

$\lim_{h\to 0} \frac{(2+h-2)((2+h)^2+2(2+h)+4)}{h}$

$\lim_{h\to 0} \frac{\cancel{h}((2+h)^2+2(2+h)+4)}{\cancel{h}}$

$(2+0)^2+2(2+0)+4$

$4 + 4 + 4$

$=12$

14. $\lim_{x\to 3} \frac{x^2-9}{x+3}$

$\lim_{x\to 3} \frac{(x+3)(x-3)}{x+3}$

$\lim_{x\to 3} x-3$

$=3-3$

$=0$

15. $\lim_{h\to 0} \frac{(1+h)^2-1^2}{h}$

$\lim_{h\to 0} \frac{(1+h-1)(1+h+1)}{h}$

$\lim_{h\to 0} \frac{h(h+2)}{h}$

$\lim_{h\to 0} h+2$

$=0+2$

$=2$

16. $\lim_{h\to 0}\frac{(-5+h)^2-25}{h}$

$\lim_{h\to 0}\frac{(-5+h+5)(-5+h-5)}{h}$

$\lim_{h\to 0}\frac{h(h-10)}{h}$

$\lim_{h\to 0} h-10$

$=0-10$

$=-10$

17. $\lim_{t\to 2}\frac{t^2-4}{t^3-8}$

$\lim_{t\to 2}\frac{(t+2)(t-2)}{(t-2)(t^2+2t+4)}$

$\lim_{t\to 2}\frac{t+2}{t^2+2t+4}$

$\frac{4}{4+4+4}$

$=\frac{1}{3}$

18. $\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2}$

$\lim_{u\to 2}\frac{(\sqrt{4u+1}-3)(\sqrt{4u+1}+3)}{(u-2)(\sqrt{4u+1}+3)}$

$\lim_{u\to 2}\frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)}$

$\lim_{u\to 2}\frac{4u-8}{(u-2)(\sqrt{4u+1}+3)}$

$\lim_{u\to 2}\frac{4(u-2)}{(u-2)(\sqrt{4u+1}+3)}$

$\lim_{u\to 2}\frac{4}{\sqrt{4u+1}+3}$

$\frac{4}{\sqrt{4(2)+1}+3}=\frac{4}{6}=\frac{2}{3}$

19. $\lim_{x\to 3}\frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

$\lim_{x\to 3}\frac{\frac{3}{3x}-\frac{x}{3x}}{x-3}$

$\lim_{x\to 3}\frac{\frac{-1(x-3)}{3x}}{x-3}$

$\lim_{x\to 3}\frac{-1(x-3)}{3x}\cdot\frac{1}{x-3}$

$\lim_{x\to 3}\frac{-1}{3x}$

$=\frac{-1}{3(3)}=\frac{-1}{9}$

20. $\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$

$\lim_{h\to 0}\frac{\frac{2}{2(2+h)}-\frac{2+h}{2(2+h)}}{h}$

$\lim_{h\to 0}\frac{\frac{2-2-h}{2(2+h)}}{h}$

$\lim_{h\to 0}\frac{\frac{-h}{2(2+h)}}{h}$

$\lim_{h\to 0}\frac{-1}{2(2+h)}$

$=\frac{-1}{2(2+0)}=\frac{-1}{4}$

21. $\lim_{x\to 2}\frac{x^4-2x^2-8}{x^2-x-6}$

$\frac{(2)^4-2(2)^2-8}{(2)^2-2-6}$

$\frac{16-8-8}{4-4-6}=\frac{0}{-6}$

$=0$

22. $\lim_{x\to -2}\frac{\frac{x}{x+4}+1}{x+2}$

$\lim_{x\to -2}\frac{\frac{x}{x+4}+\frac{x+4}{x+4}}{x+2}$

$\lim_{x\to -2}\frac{\frac{2x+4}{x+4}}{x+2}$

$\lim_{x\to -2}\frac{2(x+2)}{x+4}\cdot\frac{1}{x+2}$

$\lim_{x\to -2}\frac{2}{x+4}$

$=\frac{2}{-2+4}=1$

23. $\lim_{x\to 3}\frac{x^2-9}{2x^2+7x+3}$

$\lim_{x\to 3}\frac{(x+3)(x-3)}{(2x+1)(x+3)}$

$\lim_{x\to 3}\frac{x-3}{2x+1}$

$\frac{3-3}{2(3)+1}$

0

24. $\lim_{x\to 1}\frac{x^2-x-2}{x-2}$

$\frac{(1)^2-1-2}{1-2}$

$\frac{-2}{-1}=2$

25. $\lim_{x\to 1} \frac{4x^4-5x^2+1}{x^2+2x-3}$

$\lim_{x\to 1} \frac{(4x^2-1)(x^2-1)}{(x+3)(x-1)}$

$\lim_{x\to 1} \frac{(2x+1)(2x-1)(x+1)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$

$\lim_{x\to 1} \frac{(2x+1)(2x-1)(x+1)}{x+3}$

$\frac{(2\cdot 1+1)(2\cdot 1-1)(1+1)}{1+3}$

$\frac{(3)(1)(2)}{4} = \frac{3}{2}$

26. $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4}$

$\lim_{x\to 4} \frac{x(x-4)}{(x-4)(x+1)}$

$\lim_{x\to 4} \frac{x}{x+1}$

$\frac{4}{4+1}$

$\frac{4}{5}$

27. $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$

$\lim_{h\to 0} \frac{(3+h+3)(3+h-3)}{h}$

$\lim_{h\to 0} \frac{h(h+6)}{h}$

$\lim_{h\to 0} h+6$

$0+6 = 6$

28. $\lim_{h\to 0} \frac{\frac{1}{(h+2)^2}-\frac{1}{4}}{h}$

$\lim_{h\to 0} \frac{\frac{4-(h+2)^2}{4(h+2)^2}}{h}$

$\lim_{h\to 0} \frac{\frac{4-h^2-4h-4}{4(h+2)^2}}{h}$

$\lim_{h\to 0} \frac{-h^2-4h}{4(h+2)^2}\cdot\frac{1}{h}$

$\lim_{h\to 0} \frac{-h(h+4)}{4(h+2)^2}\cdot\frac{1}{h}$

$\lim_{h\to 0} \frac{-(h+4)}{4(h+2)^2} = \frac{-(0+4)}{4(0+2)^2} = \frac{-1}{4}$

29. $\lim_{x\to 1} \frac{x^2-3x+2}{x^3-1}$

$\lim_{x\to 1} \frac{(x-2)(x-1)}{(x-1)(x^2+x+1)}$

$\lim_{x\to 1} \frac{x-2}{x^2+x+1}$

$\frac{1-2}{1+1+1}$

$\frac{-1}{3}$

30. $\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}$

$\lim_{x\to 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{(16x-x^2)(4+\sqrt{x})}$

$\lim_{x\to 16} \frac{16-x}{x(16-x)(4+\sqrt{x})}$

$\lim_{x\to 16} \frac{1}{x(4+\sqrt{x})}$

$\frac{1}{16(4+\sqrt{16})} = \frac{1}{16(8)}$

$= \frac{1}{128}$

31. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

$\lim_{x\to 4} \frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)}$

$\lim_{x\to 4} \frac{1}{\sqrt{x}+2}$

$\frac{1}{\sqrt{4}+2}$

$\frac{1}{4}$

32. $\lim_{x\to 3} \frac{3(x+1)^{-1}-3(4)^{-1}}{x-3}$

$\lim_{x\to 3} \frac{\frac{3}{x+1}-\frac{3}{4}}{x-3}$

$\lim_{x\to 3} \frac{\frac{12-3(x+1)}{4(x+1)}}{x-3}$

$\lim_{x\to 3} \frac{\frac{12-3x-3}{4(x+1)}}{x-3}$

$\lim_{x\to 3} \frac{-3(x-3)}{4(x+1)}\cdot\frac{1}{x-3}$

$\lim_{x\to 3} \frac{-3}{4(x+1)}$

$= \frac{-3}{4(3+1)}$

$= -3/16$

33. $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

$\lim_{t\to 0} \frac{(\sqrt{t^2+9}-3)(\sqrt{t^2+9}+3)}{t^2(\sqrt{t^2+9}+3)}$

$\lim_{t\to 0} \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)}$

$\lim_{t\to 0} \frac{t^2}{t^2(\sqrt{t^2+9}+3)}$

$\lim_{t\to 0} \frac{1}{\sqrt{t^2+9}+3}$

$\frac{1}{\sqrt{0+9}+3}$

$\frac{1}{6}$

34. $\lim_{x\to 1} \dfrac{\frac{2x}{x+1}-1}{x-1}$

$\lim_{x\to 1} \dfrac{\frac{2x-(x+1)}{x+1}}{x-1}$

$\lim_{x\to 1} \dfrac{x-1}{x+1}\cdot\dfrac{1}{x-1}$

$\lim_{x\to 1} \dfrac{1}{x+1}$

$\dfrac{1}{1+1}$

$\dfrac{1}{2}$

35. $\lim_{x\to 2} \dfrac{x^3+x^2-4x-4}{x^2+x-6}$

$\lim_{x\to 2} \dfrac{x^2(x+1)-4(x+1)}{(x+3)(x-2)}$

$\lim_{x\to 2} \dfrac{(x+2)(x-2)(x+1)}{(x+3)(x-2)}$

$\lim_{x\to 2} \dfrac{(x+2)(x+1)}{(x+3)}$

$\dfrac{(2+2)(2+1)}{(2+3)}$

$\dfrac{12}{5}$

36. $\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$

$\lim_{h\to 0} \dfrac{(x+h+x)(x+h-x)}{h}$

$\lim_{h\to 0} \dfrac{(2x+h)h}{h}$

$\lim_{h\to 0} 2x+h$

$2x+0$

$2x$

37. $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

$\lim_{t\to -3} \dfrac{(t+3)(t-3)}{(2t+1)(t+3)}$

$\lim_{t\to -3} \dfrac{t-3}{2t+1}$

$\dfrac{-3-3}{2(-3)+1}$

$\dfrac{6}{5}$

38. $\lim_{x\to 0} \dfrac{\frac{3}{x+5}-\frac{3}{5}}{x}$

$\lim_{x\to 0} \dfrac{\frac{15-3(x+5)}{5(x+5)}}{x}$

$\lim_{x\to 0} \dfrac{\frac{15-3x-15}{5(x+5)}}{x}$

$\lim_{x\to 0} \dfrac{-3x}{5(x+5)}\cdot\dfrac{1}{x}$

$\lim_{x\to 0} \dfrac{-3}{5(x+5)}$

$\dfrac{-3}{5(0+5)}=\dfrac{-3}{25}$

39. $\lim_{h\to 0} \dfrac{(3+h)^3-27}{h}$

$\lim_{h\to 0} \dfrac{(3+h-3)[(3+h)^2+3(3+h)+9]}{h}$

$\lim_{h\to 0} \dfrac{h[(3+h)^2+3(3+h)+9]}{h}$

$\lim_{h\to 0} (3+h)^2+9+3h+9$

$(3+0)^2+9+3(0)+9$

$=27$

40. $\lim_{p\to -2} \dfrac{(p+4)^{-1}-2^{-1}}{p+2}$

$\lim_{p\to -2} \dfrac{\frac{1}{p+4}-\frac{1}{2}}{p+2}$

$\lim_{p\to -2} \dfrac{\frac{2-p-4}{2(p+4)}}{p+2}$

$\lim_{p\to -2} \dfrac{-p-2}{2(p+4)}\cdot\dfrac{1}{p+2}$

$\lim_{p\to -2} \dfrac{-1}{2(p+4)}$

$\dfrac{-1}{2(-2+4)}$

$\dfrac{-1}{4}$

41. $\lim_{t\to 0} \dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}$

$\lim_{t\to 0} \dfrac{(\sqrt{1+t}-\sqrt{1-t})(\sqrt{1+t}+\sqrt{1-t})}{t(\sqrt{1+t}+\sqrt{1-t})}$

$\lim_{t\to 0} \dfrac{1+t-1+t}{t(\sqrt{1+t}+\sqrt{1-t})}$

$\lim_{t\to 0} \dfrac{2t}{t(\sqrt{1+t}+\sqrt{1-t})}$

$\lim_{t\to 0} \dfrac{2}{\sqrt{1+t}+\sqrt{1-t}}$

$\dfrac{2}{\sqrt{1+0}+\sqrt{1-0}}$

$=1$

42. $\lim_{x\to 3} \dfrac{\sqrt{x+6}-x}{x^3-3x^2}$

$\lim_{x\to 3} \dfrac{(\sqrt{x+6}-x)(\sqrt{x+6}+x)}{(x^3-3x^2)(\sqrt{x+6}+x)}$

$\lim_{x\to 3} \dfrac{x+6-x^2}{(x^3-3x^2)(\sqrt{x+6}+x)}$

$\lim_{x\to 3} \dfrac{-1(x-3)(x+2)}{x^2(x-3)(\sqrt{x+6}+x)}$

$\lim_{x\to 3} \dfrac{-1(x+2)}{x^2(\sqrt{x+6}+x)}$

$\dfrac{-1(3+2)}{(3)^2(\sqrt{3+6}+3)}=\dfrac{-5}{9(6)}$

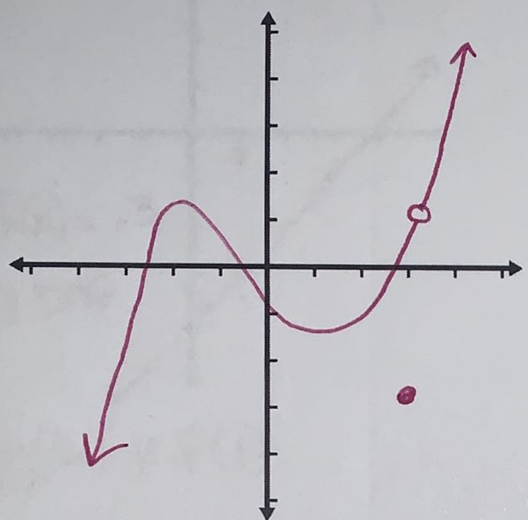
$\dfrac{-5}{54}$

Continuity and Intermediate Value Theorem

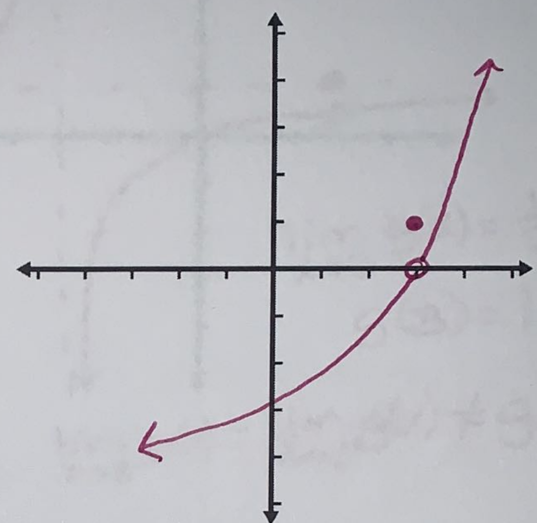
Answers will vary on this page.

Sketch the graph of a function f that satisfies the stated conditions.

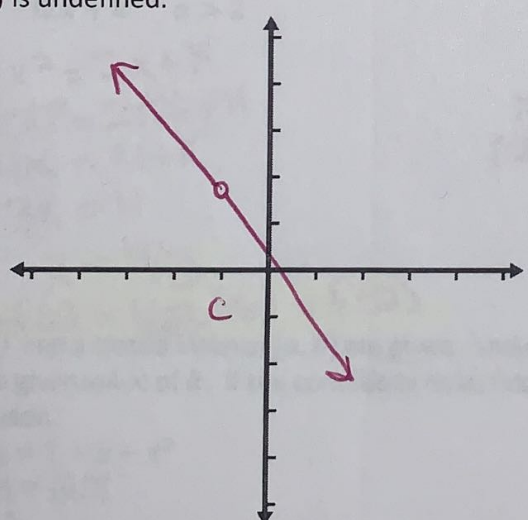
1. f has a limit at $x = 3$, but is not continuous at $x = 3$



2. f is not continuous at $x = 3$, but if its value at $x = 3$ is changed from $f(3) = 1$ to $f(3) = 0$, f becomes continuous at $x = 3$.



3. f has a removable discontinuity at $x = c$ for which $f(c)$ is undefined.



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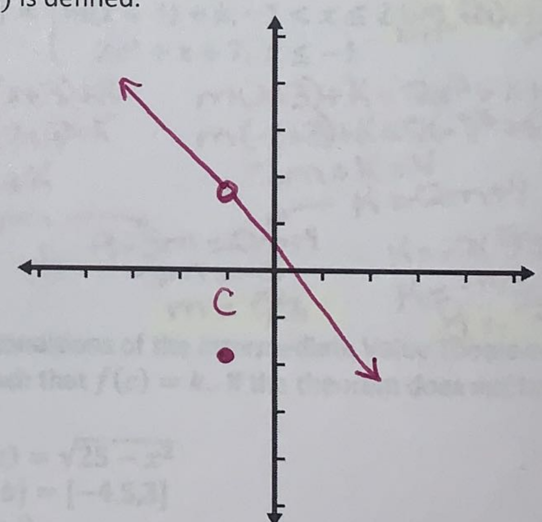


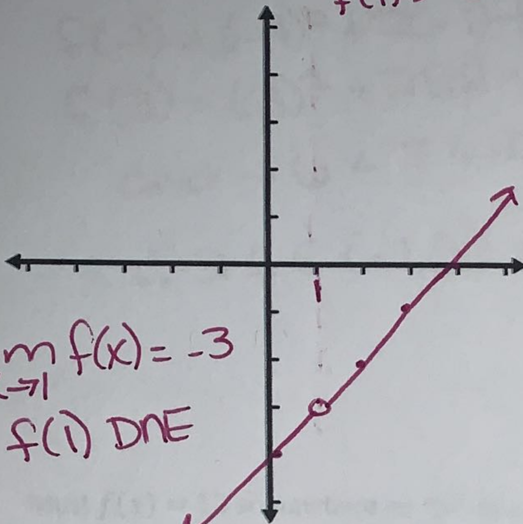
Use the definition of continuity to prove that the function is discontinuous at the given value of a. Sketch the graph of the function.

5. $f(x) = \frac{x^2-5x+4}{x-1}, a = 1$

$\frac{(x-4)(x-1)}{x-1}$

$f(x) = x-4$

$f(1) = -3$ hole



$\lim_{x\to 1} f(x) = -3$

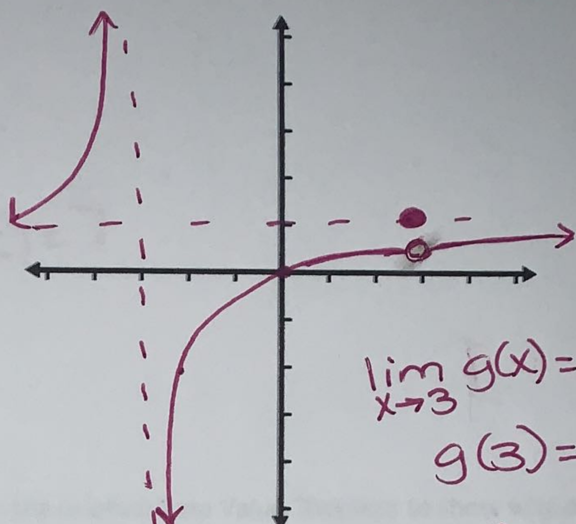
$f(1)$ DNE

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \neq f(1)$

6. $g(x) = \begin{cases} \frac{x^2-3x}{x^2-9} & if\ x \neq 3 \\ 1 & if\ x = 3 \end{cases}, a = 3$

$\frac{x(x-3)}{(x+3)(x-3)}$ hole at $(3, \frac{1}{2})$

$\frac{x}{x+3}$ if $x \neq 3$ VA at $x = -3$



$\lim_{x\to 3} g(x) = \frac{1}{2}$

$g(3) = 1$

$\lim_{x\to 3^-} g(x) = \lim_{x\to 3^+} g(x) \neq g(3)$

Use the definition of continuity to find the values of k and/or m that will make the function continuous everywhere.

7. $f(x) = \begin{cases} kx^2 & x \leq 2 \\ 2x + k & x > 2 \end{cases}$

$Kx^2 = 2x + K$

$K(2)^2 = 2(2) + K$

$4K = 4 + K$

$3K = 4$

$K = 4/3$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

8. $g(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x + 3) + k, & -1 < x \leq 2 \\ 2x^3 + x + 7, & x \leq -1 \end{cases}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = f(2)$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x) = f(-1)$

$x^2 + 5 = m(x+3) + K$

$(2)^2 + 5 = m(2+3) + K$

$9 = 5m + K$

$K = 9 - 5m$

$m(x+3) + K = 2x^3 + x + 7$

$m(-1+3) + K = 2(-1)^3 + (-1) + 7$

$2m + K = 4$

$K = -2m + 4$

$9 - 5m = -2m + 4$

$-3m = -5$

$m = 5/3$

$K = -2(5/3) + 4$

$K = -10/3 + 12/3$

$K = 2/3$

A function f and a closed interval $[a, b]$ are given. Show whether the conditions of the Intermediate Value Theorem hold for the given value of k. If the conditions hold, find a number c such that $f(c) = k$. If the theorem does not hold, give the reason.

9. $f(x) = 2 + x - x^2$
$[a, b] = [0,3]$
$k = 1$

Since $f(x)$ is continuous from $(0,3)$

$f(0) = 2 + 0 - 0^2 = 2$

$f(3) = 2 + 3 - 3^2 = -4$

and $-4 < 1 < 2$

$\therefore \exists c \in (0,3)$ s.t. $f(c) = 1$

$f(c) = 2 + c - c^2$

$1 = 2 + c - c^2$

$c^2 - c - 1 = 0$

$c = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

$c = \frac{1 \pm \sqrt{5}}{2}$

$c = \frac{1+\sqrt{5}}{2}$

10. $f(x) = \sqrt{25 - x^2}$
$[a, b] = [-4.5,3]$
$k = 3$

Since $f(x)$ is continuous from $(-4.5, 3)$

$f(-4.5) = \sqrt{25-(-4.5)^2} = \sqrt{20.25} \approx 2.18$

$f(3) = \sqrt{25-(3)^2} = 4$

and $2.18 < 3 < 4$

$\therefore \exists c \in (-4.5, 3)$ s.t. $f(c) = 3$

$f(c) = \sqrt{25-c^2}$

$3 = \sqrt{25-c^2}$

$9 = 25 - c^2$

$-16 = -c^2$

$16 = c^2 \quad c = \pm 4$

$c = -4$

For Questions 11 and 12: Given the function $f(x) = x^2 + 2x - 5$.

11. Does $f(x) = 7$ somewhere on the interval $[-1,3]$? Use the Intermediate Value Theorem to show why or why not.

Since $f(x)$ is continuous on $(-1,3)$

$f(-1) = (-1)^2 + 2(-1) - 5 = -6$

$f(3) = (3)^2 + 2(3) - 5 = 10$

and $-6 < 7 < 10$

$\therefore \exists\, c \in (-1,3)$ s.t. $f(c) = 7$

12. Must $f(x) = 12$ somewhere on the interval $[-1,3]$? Use the Intermediate Value Theorem to show why or why not.

Since $f(x)$ is continuous on $(-1,3)$

$f(-1) = -6$

$f(3) = 10$

and 12 is not between -6 and 10

$\therefore \nexists\, c \in (-1,3)$ s.t. $f(c) = 12$

13. The amount of money raised during a fund-raising campaign is modeled by the function M defined by $M(t) = \frac{240(2^t - 1)}{2^t + 36}$, where $M(t)$ is measured in United States dollars and t is the time in days since that campaign began. According to this model, is there a time t, for $0 \le t \le 2$ at which the amount of money raised is 10 dollars? Justify your answer.

Since $m(t)$ is continuous from $(0,2)$

$m(0) = \frac{240(2^0 - 1)}{2^0 + 36} = 0$

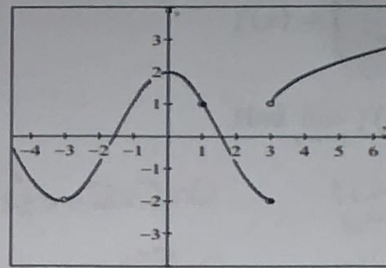
$m(2) = \frac{240(2^2 - 1)}{2^2 + 36} = 18$

and $0 < 10 < 18$

$\therefore \exists\, t \in (0,2)$ s.t. $m(t) = 10$

One-Sided Limits Graphically and Algebraically

1. Given the graph of $f(x)$, determine the following.



a. $\lim_{x\to -3^-} f(x) = -2$ b. $\lim_{x\to -3^+} f(x) = -2$ c. $\lim_{x\to -3} f(x) = -2$

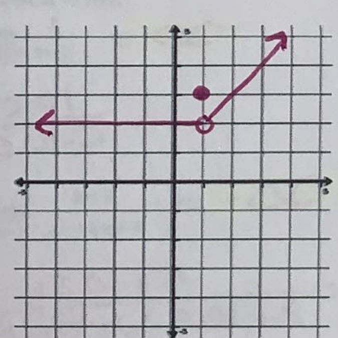
d. $\lim_{x\to 1^-} f(x) = 1$ e. $\lim_{x\to 1^+} f(x) = 1$ f. $\lim_{x\to 1} f(x) = 1$

g. $\lim_{x\to 3^-} f(x) = -2$ h. $\lim_{x\to 3^+} f(x) = 1$ i. $\lim_{x\to 3} f(x)$ DNE

j. $f(-3)$ DNE k. $f(1) = 1$ l. $f(3) = -2$

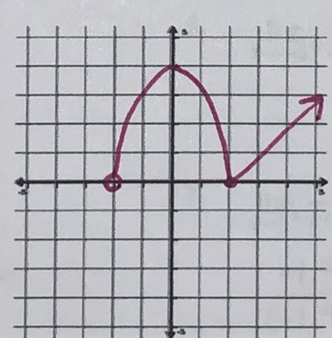
2. Sketch each piecewise function below and determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a. $f(x) = \begin{cases} 2, & x < 1 \\ 3, & x = 1 \\ x+1, & x > 1 \end{cases}$



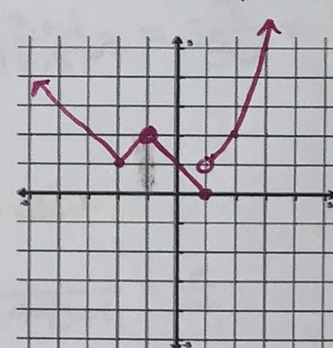
$\lim_{x\to 1} f(x) = 2$

b. $f(x) = \begin{cases} 4 - x^2, -2 < x \le 2 \\ x - 2, \quad x > 2 \end{cases}$



$\lim_{x\to 2} f(x) = 0$

c. $f(x) = \begin{cases} |x+2| + 1, & x < -1 \\ -x + 1, & -1 \le x \le 1 \\ x^2 - 2x + 2, & x > 1 \end{cases}$



$\lim_{x\to 1} f(x)$ DNE

3. For each function below, determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a. $f(x) = \begin{cases} 2x - 1, \ x \le -2 \\ -x + 2, \ x > -2 \end{cases}$

Find $\lim_{x\to -2^+} f(x) = -(-2)+2$

$= 4$

b. $f(x) = \begin{cases} -x^2 + 4x - 3, \ x < 1 \\ x - 7, \ x \ge 1 \end{cases}$

Find $\lim_{x\to 1^-} f(x) = -(1)^2 + 4(1) - 3$

$= 0$

c. $f(x) = \begin{cases} x+3, & x \in (-\infty, 0] \\ -x+2, & x \in (0,2) \\ (x-2)^2, & x \in [2,\infty) \end{cases}$

Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 2} f(x)$

$\lim_{x\to 0^-} f(x) = 0+3 = 3$

$\lim_{x\to 0^+} f(x) = -0+2 = 2$

$\lim_{x\to 0} f(x)$ DNE

$\lim_{x\to 2^-} f(x) = -2+2 = 0$

$\lim_{x\to 2^+} f(x) = (2-2)^2 = 0$

$\lim_{x\to 2} f(x) = 0$

d. $f(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ -\frac{x}{2} + \frac{7}{2}, & x \geq -1 \end{cases}$

Find $\lim_{x\to -1} f(x)$

$\lim_{x\to -1^-} f(x) = (-1)^2 - 2(-1) + 1 = 4$

$\lim_{x\to 1^+} f(x) = \frac{1}{2} + \frac{7}{2} = 4$

$\lim_{x\to 1} f(x) = 4$

e. $f(x) = \begin{cases} (x+1)^2 - 1, & -2 \leq x < 0 \\ \frac{5}{4}\sin\left(\frac{\pi x}{2}\right), & 0 \leq x < 2 \\ (x-3)^2 - 1, & 2 \leq x \leq 4 \end{cases}$

Find $\lim_{x\to 2} f(x)$

$\lim_{x\to 2^-} f(x) = \frac{5}{4}\sin\left(\frac{2\pi}{2}\right) = 0$

$\lim_{x\to 2^+} f(x) = (2-3)^2 - 1 = 0$

$\lim_{x\to 2} f(x) = 0$

Evaluate each limit.

4. $\lim_{x\to 2^+} \frac{x}{x-2} = \infty$

$\frac{2.01}{2.01-2}$

5. $\lim_{x\to -3^+} \frac{x+1}{x^2-6x+9} = \frac{-3+1}{9+18+9} = \frac{-2}{36} = \frac{-1}{18}$

6. $\lim_{x\to -3^-} \frac{x+2}{x^2+6x+9} = -\infty$

$\frac{-3.01+2}{(-3.01)^2+6(-3.01)+9} = \frac{-\#}{+\#}$

7. $\lim_{x\to -2^+} \frac{x-2}{x^2+4x+4} = -\infty$

$\frac{-1.99-2}{(-1.99)^2+4(-1.99)+4} = \frac{-}{+}$

8. $\lim_{x\to -3^-} \frac{x^2}{3x+9} = -\infty$

$\frac{(-3.01)^2}{3(-3.01)+9} = \frac{+}{-}$

9. $\lim_{x\to 2^+} \frac{x^2}{2x-4} = \infty$

$\frac{(2.01)^2}{2(2.01)-4} = \frac{+}{+}$

10. $\lim_{x\to -2^+} \frac{1}{x^2-4} = -\infty$

$\frac{1}{(-1.99)^2-4} = \frac{1}{-\#}$

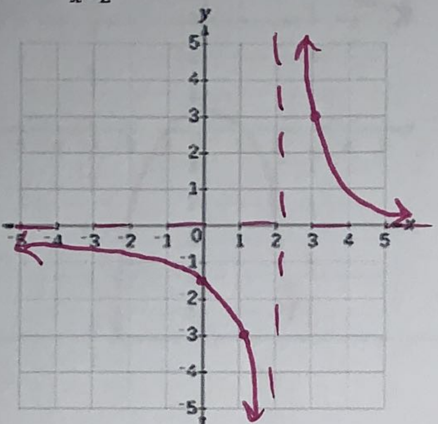
11. $\lim_{x\to 1^-} -\frac{2}{x^2-1} = \infty$

$\frac{-2}{(.99)^2-1} = \frac{-2}{-\#}$

Vertical and Horizontal Asymptotes Worksheet

State the vertical, horizontal, or slant asymptotes for the following (justify using limits). Sketch the graph and find the end behavior.

1. $f(x) = \frac{3}{x-2}$



Vertical Asymptote: $x=2$

Horizontal Asymptote: $y=0$

Slant Asymptote: none

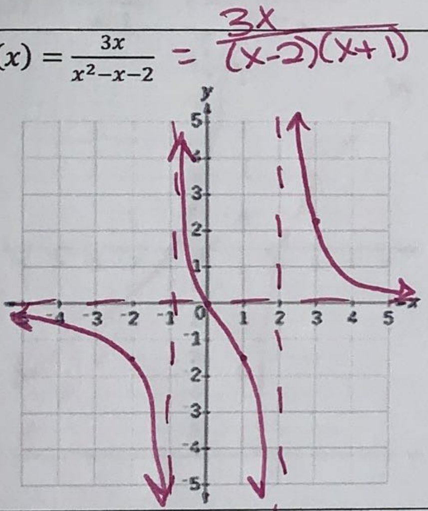
End Behavior: $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0$

$\lim_{x\to2^+} f(x) = \infty$

$\lim_{x\to2^-} f(x) = -\infty$

$\lim_{x\to\pm\infty} f(x) = 0$

2. $f(x) = \frac{3x}{x^2-x-2} = \frac{3x}{(x-2)(x+1)}$



Vertical Asymptote: $x=2,\ x=-1$

Horizontal Asymptote: $y=0$

Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0$

$\lim_{x\to-1^-} f(x) = -\infty$

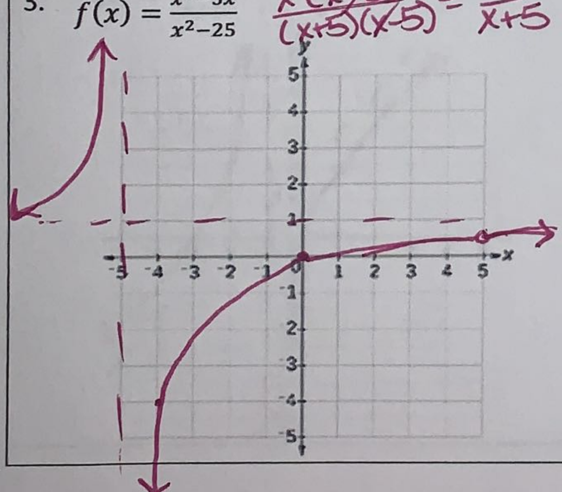
$\lim_{x\to-1^+} f(x) = \infty$

$\lim_{x\to2^-} f(x) = -\infty$

$\lim_{x\to2^+} f(x) = \infty$

$\lim_{x\to\pm\infty} f(x) = 0$

3. $f(x) = \frac{x^2-5x}{x^2-25}$ $\frac{x(x-5)}{(x+5)(x-5)} = \frac{x}{x+5}$



Vertical Asymptote: $x=-5$

Horizontal Asymptote: $y=1$

Slant Asymptote: none

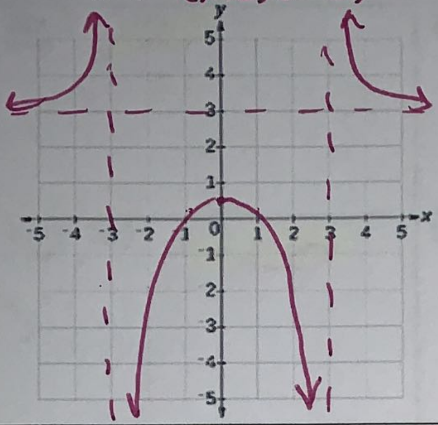
End Behavior: $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to-\infty} f(x) = 1$

$\lim_{x\to-5^-} f(x) = \infty$

$\lim_{x\to-5^+} f(x) = -\infty$

$\lim_{x\to\pm\infty} f(x) = 1$

4. $f(x) = \frac{3x^2-4}{x^2-9}$ $\frac{3x^2-4}{(x+3)(x-3)}$



Vertical Asymptote: $x=-3, x=3$

Horizontal Asymptote: $y=3$

Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) =$ 3

$\lim_{x\to-\infty} f(x) =$ 3

$\lim_{x\to-3^-} f(x) = \infty$

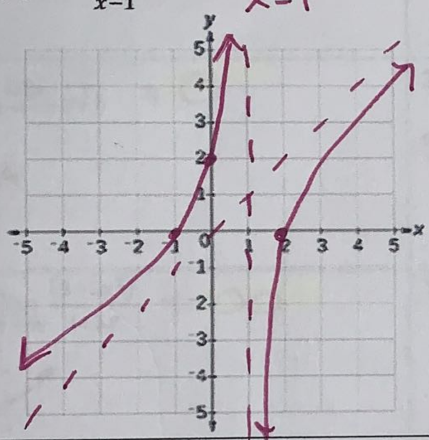
$\lim_{x\to-3^+} f(x) = -\infty$

$\lim_{x\to3^-} f(x) = -\infty$

$\lim_{x\to3^+} f(x) = \infty$

$\lim_{x\to\pm\infty} f(x) = 3$

5. $f(x) = \frac{x^2-x-2}{x-1}$ $\frac{(x-2)(x+1)}{x-1}$



Vertical Asymptote: $x=1$

Horizontal Asymptote: none

Slant Asymptote: $y=x$

End Behavior: $\lim_{x\to\infty} f(x) =$ ∞

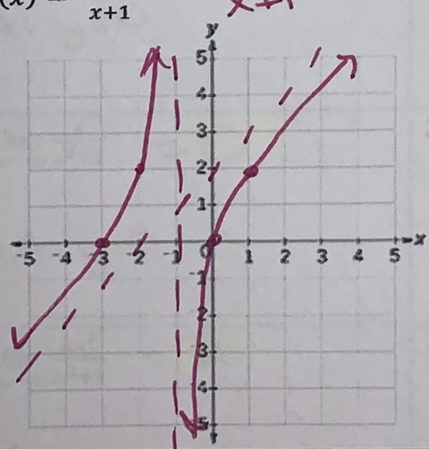
$\lim_{x\to-\infty} f(x) =$ $-\infty$

$\lim_{x\to1^-} f(x) = \infty$

$\lim_{x\to1^+} f(x) = -\infty$

1 | 1 -1 -2
  | ↓ 1 0
  1 0 ; -2

6. $f(x) = \frac{x^2+3x}{x+1}$ $\frac{x(x+3)}{x+1}$



Vertical Asymptote: $x=-1$

Horizontal Asymptote: none

Slant Asymptote: $y=x+2$

End Behavior: $\lim_{x\to\infty} f(x) =$ ∞

$\lim_{x\to-\infty} f(x) =$ $-\infty$

$\lim_{x\to-1^-} f(x) =$

$\lim_{x\to-1^+} f(x) =$

-1 | 1 3 0
   | ↓ -1 -2
   1 2 :

Infinite Limits Worksheet

Find the Limit.

1. $\lim\limits_{x\to\infty} 3$ $= 3$	2. $\lim\limits_{x\to-\infty} 3$ $= 3$	3. $\lim\limits_{x\to-\infty} (-3)$ $= -3$
4. $\lim\limits_{x\to\infty} (-2x)$ $= -\infty$	5. $\lim\limits_{x\to\infty} (3-x)$ $= -\infty$	6. $\lim\limits_{x\to\infty} \sqrt{x}$ $= \infty$
7. $\lim\limits_{x\to-\infty} (4-x)$ $= \infty$	8. $\lim\limits_{x\to\infty} \frac{8}{5-3x}$ $= 0$ H.A.: $y=0$	9. $\lim\limits_{x\to\infty} \frac{1}{x-12}$ $= 0$ HA: $y=0$
10. $\lim\limits_{x\to-\infty} \frac{3}{x+4}$ $= 0$ HA	11. $\lim\limits_{x\to\infty} (1+2x-3x^5) =$ $-\infty$ End Beh.	12. $\lim\limits_{x\to\infty} (2x^3-110x+5) =$ ∞
13. $\lim\limits_{x\to\infty} \frac{(3+2x^2)}{4+5x}$ $= \infty$ S.A.	14. $\lim\limits_{x\to\infty} \frac{x^2+x}{3-x}$ $= -\infty$ $\frac{+}{-}$	15. $\lim\limits_{x\to\infty} \frac{x+4}{x^2-2x+5}$ $= 0$ H.A $y=0$
16. $\lim\limits_{x\to-\infty} -\frac{x-2}{x^2+2x+1}$ $= 0$ HA: $y=0$	17. $\lim\limits_{x\to\infty} \frac{7-6x^5}{x+3}$ $= -\infty$ $\frac{-6(\infty)^5}{\infty} = \frac{-}{+}$	18. $\lim\limits_{x\to\infty} \frac{6-x^3}{7x^3+3}$ $= -\frac{1}{7}$ HA: $y=-\frac{1}{7}$
19. $\lim\limits_{x\to\infty} \frac{1}{x^2+1}$ $= 0$ HA: $y=0$	20. $\lim\limits_{x\to\infty} \frac{x^4+x^2}{x^4+1}$ $= 1$ HA	21. $\lim\limits_{x\to\infty} \frac{1+x^2}{2-x^2}$ $= -1$ HA
22. $\lim\limits_{x\to\infty} \frac{2x^2}{x^2+1}$ $= 2$ HA	23. $\lim\limits_{x\to-\infty} \frac{x+4}{3x^2-5}$ $= 0$ HA	24. $\lim\limits_{x\to\infty} \frac{3x^3+25x^2-x+1}{4x^3-7x^2+2x+2}$ $= \frac{3}{4}$ HA

Released Multiple Choice Questions - Limits

1. $\lim\limits_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

a. -3 b. -2 c. 2 d. 3 e. DNE

$\lim\limits_{x\to\infty} \frac{-2x^2+7x-3}{x^2+2x-3}$

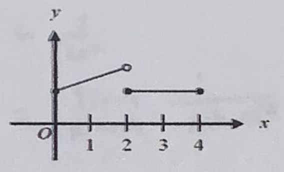
2. $\lim\limits_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$ is

a. $-\frac{1}{2}$ b. 0 c. $\frac{5}{3}$ d. $\frac{7}{6}$ e. None of These

$\lim\limits_{x\to 0} \frac{x^2(5x^2+8)}{x^2(3x^2-16)}$

$\frac{5(0)^2+8}{3(0)^2-16} = -\frac{8}{16}$

3. The figure below shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?



Graph of f

I. $\lim\limits_{x\to 2^-} f(x)$ exists. ✓ II. $\lim\limits_{x\to 2^+} f(x)$ exists. ✓ III. $\lim\limits_{x\to 2} f(x)$ exists.

a. I only b. II only c. I and II only d. I and III only e. I, II, and III

End Behavior

4. For $x \ge 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

a. $f(0) = 2$ b. $f(x) \ne 2$ for all $x \ge 0$ c. $f(2)$ is undefined d. $\lim\limits_{x\to 2} f(x) = \infty$ e. $\lim\limits_{x\to\infty} f(x) = 2$

H.A. 5. $\lim\limits_{x\to\infty} \frac{1x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$

a. 4 b. 1 c. $\frac{1}{4}$ d. 0 e. -1

6. If $f(x) = \begin{cases} \ln x & for\ 0 < x \le 2 \\ x^2 \ln 2 & for\ 2 < x \le 4 \end{cases}$, then $\lim_{x \to 2} f(x)$ is

a. $\ln 2$ b. $\ln 8$ c. $\ln 16$ d. 4 e. DNE

$\lim_{x \to 2^-} f(x) = \ln 2$ $\lim_{x \to 2^+} f(x) = 4 \ln 2$

$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) \therefore \lim_{x \to 2} f(x)$ DNE

7. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table. The equation $f(x) = \frac{1}{2}$ must have a least two solutions in the interval $[0,2]$ if $k =$

x	0	1	2
$f(x)$	1	k	2

a. 0 b. $\frac{1}{2}$ c. 1 d. 2 e. 3

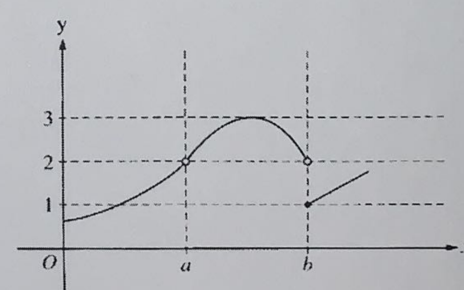
bounces off x-axis
(2 roots)

8. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

a. $\frac{1}{a^2}$ b. $\frac{1}{2a^2}$ c. $\frac{1}{6a^2}$ d. 0 e. DNE

$\lim_{x \to a} \frac{\cancel{(x+a)}\cancel{(x-a)}}{(x^2+a^2)\cancel{(x+a)}\cancel{(x-a)}} = \lim_{x \to a} \frac{1}{x^2+a^2} = \frac{1}{(a)^2+a^2} = \frac{1}{2a^2}$

9. The graph of the function f is shown in the figure to the right. Which of the following statements about f is true?



a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$ no

b. $\lim_{x \to a} f(x) = 2$

c. $\lim_{x \to b} f(x) = 2$ no

d. $\lim_{x \to b} f(x) = 1$

e. $\lim_{x \to a} f(x)$ DNE no

10. $\lim_{n \to \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

a. -5 b. -2 c. 1 d. 3 e. DNE

H.A.: $y=3$

11. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) =$

a. -4 b. -2 c. -1 d. 0 e. 2

$f(x) = \frac{(x+2)(x-2)}{(x+2)}$

$f(x) = x-2$

$f(-2) = -2-2 = -4$

12. $\lim_{n\to\infty} \frac{4n^2}{n^2+10{,}000n}$ is

a. 0 b. $\frac{1}{2500}$ c. 1 d. 4 e. DNE

13. If $\lim_{x\to a} f(x) = L$, where L is a real number which of the following must be true?

a. $f'(a)$ exists
b. $f(x)$ is continuous at $x = a$
c. $f(x)$ is defined at $x = a$
d. $f(a) = L$
e. None of these

14. If $\begin{cases} f(x) = \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, for\ x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

a. 0 b. $\frac{1}{6}$ c. $\frac{1}{3}$ d. 1 e. $\frac{7}{5}$

$$f(x) = \frac{(\sqrt{2x+5} - \sqrt{x+7})(\sqrt{2x+5} + \sqrt{x+7})}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})}$$

$$f(x) = \frac{2x+5-x-7}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})}$$

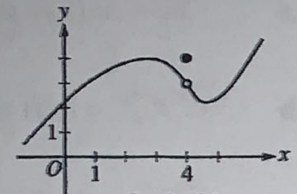
$$f(x) = \frac{1}{(\sqrt{2x+5} + \sqrt{x+7})}$$

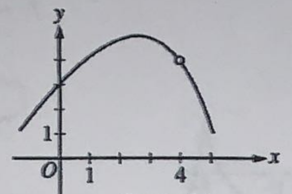
$$f(2) = \frac{1}{\sqrt{2(2)+5} + \sqrt{2+7}}$$

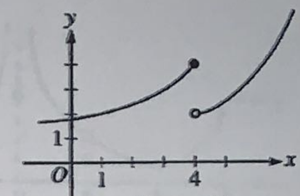
$$f(2) = \frac{1}{3+3}$$

Limits Practice Test

1. For which of the following does $\lim_{x\to 4} f(x)$ exist?

I.

Graph of f

II.

Graph of f

iii.

Graph of f

a. I only b. II only c. III only **d. I and II only** e. I, II, III

2. Use the table of values to evaluate the limit.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	7.018	7.008	7.002	20	7.002	7.008	7.018
g(x)	4.126	4.789	4.989	8	8.0015	8.1016	8.546
h(x)	4971	8987	9972	undefined	8.997	8.987	8.971

$\lim_{x\to 0^+} f(x) = 7$ $\lim_{x\to 0^-} f(x) = 7$ $\lim_{x\to 0} f(x) = 7$

$\lim_{x\to 0^+} g(x) = 8$ $\lim_{x\to 0^-} g(x) = 5$ $\lim_{x\to 0} g(x)$ DNE

$\lim_{x\to 0^+} h(x) = 9$ $\lim_{x\to 0^-} h(x) = \infty$ $\lim_{x\to 0} h(x)$ DNE

3. Find $\lim_{x\to\infty} \dfrac{-4x+2x^3}{8x^3+4x^2-3}$.

a. $\frac{1}{4}$ b. $-\frac{1}{4}$ c. $-\frac{1}{2}$ d. 0 e. ∞

4. Find $\lim_{x\to\infty} \dfrac{e^x+5}{3-2e^x}$.

a. $\frac{5}{3}$ b. $\frac{1}{3}$ c. $\frac{1}{2}$ **d.** $-\frac{1}{2}$ e. ∞

H.A.

5. The graph of which function has y = 2 as an asymptote?

a. $y = e^{-x} + 2$ b. $y = \ln(x-2)$ c. $y = -\frac{2x^2}{4+x^2}$ d. $y = -\frac{2}{1-x}$ e. $y = \frac{4x}{2+x}$

V.A: X=2 HA: y=-2 H.A.: y=0 H.A: y=4

6. State all the vertical and horizontal asymptotes and justify your answer.

$$f(x) = \frac{2x^2-50}{x^2+7x+10}$$

V.A.: X=-2

$\lim_{x\to -2^-} f(x) = \infty$

$\lim_{x\to -2^+} f(x) = -\infty$

$f(x) = \dfrac{2(x+5)(x-5)}{(x+5)(x+2)}$

$f(x) = \dfrac{2(x-5)}{x+2}$

H.A.: y=2

$\lim_{x\to\pm\infty} f(x) = 2$

7. Find the indicated limits from the graph below.

a. $\lim_{x \to -1^-} f(x) =$ -2

b. $\lim_{x \to -1^+} f(x) =$ -1

c. $\lim_{x \to -1} f(x) =$ DNE

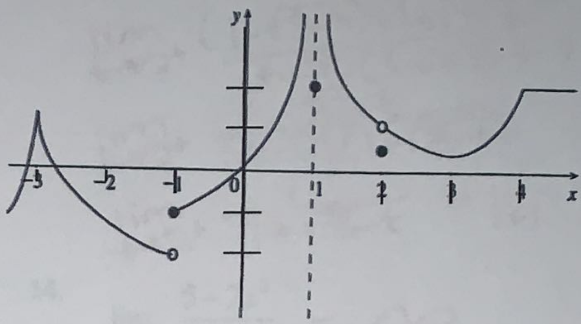
d. $\lim_{x \to 0} f(x) =$ 0

e. $\lim_{x \to 1} f(x) =$ ∞

f. $\lim_{x \to 2} f(x) =$ 1

g. $\lim_{x \to 4} f(x) =$ 2

h. $\lim_{x \to -3^-} f(x) =$ 1.3



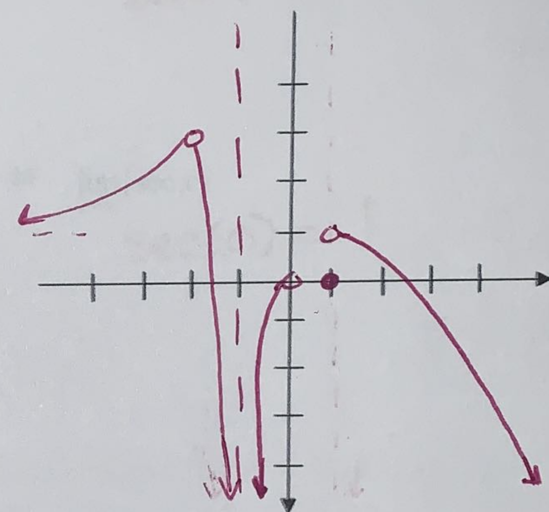
8. Draw a graph of g(x) that has the following conditions.

$\lim_{x \to \infty} g(x) = -\infty$ $\lim_{x \to -1} g(x) = -\infty$

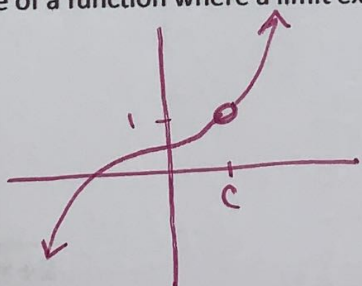
$\lim_{x \to 1^+} g(x) = 1$ $\lim_{x \to -2} g(x) = 3$

$\lim_{x \to 1^-} g(x) = 0$ $\lim_{x \to -\infty} g(x) = 1$

$g(1) = 0$



9. Draw an example of a function where a limit exists at a point but the function is still discontinuous at that point. Explain.



$\lim_{x \to c^-} f(x) = 1$

$\lim_{x \to c^+} f(x) = 1$

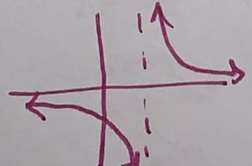
$\lim_{x \to c} f(x) = 1$

f(c) DNE because there is a removable discontinuity

∴ discontinuous at x=c because $\lim_{x \to c} f(x) \neq f(c)$

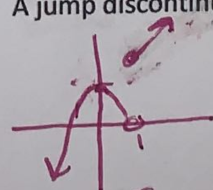
10. Draw the graph of an example of each of the following discontinuities. (Bonus for giving the equation of such a graph. The equation does not have to match the graph you draw.)

a. An infinite discontinuity



$y = \frac{1}{x-1}$

b. A jump discontinuity



$y = \begin{cases} x+1, & x \geq 1 \\ -x^2+1, & x < 1 \end{cases}$

Find the limit if it exists.

11. $\lim_{x\to 5^-} \frac{4x-20}{|x-5|}$

$\lim_{x\to 5^-} \frac{4(x-5)}{|x-5|}$

$\lim_{x\to 5^-} \frac{4(x-5)}{-1(x-5)} = -4$

12. $\lim_{t\to 2^+} \frac{1-\sqrt{3-t}}{t-2}$

$\lim_{t\to 2^+} \frac{(1-\sqrt{3-t})(1+\sqrt{3-t})}{(t-2)(1+\sqrt{3-t})}$

$\lim_{t\to 2^+} \frac{1-3+t}{(t-2)(1+\sqrt{3-t})}$

$\lim_{t\to 2^+} \frac{1}{1+\sqrt{3-t}} = \frac{1}{1+1} = \frac{1}{2}$

13. $\lim_{x\to -\infty} e^{-x} = \infty$

14. $\lim_{x\to -\infty} \frac{5-2x^2}{x+2} = \infty$

$\frac{5-2(\infty)^2}{\infty+2} = \frac{-\#}{+\#}$

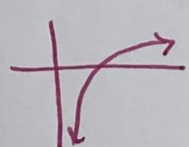
15. $\lim_{x\to -2^-} \frac{-3}{2+x} = -\infty$

$\frac{-3}{2-1.99} = \frac{-\#}{+\#}$

16. $\lim_{x\to 0} (\sec x)$

$\sec(0) = 1$

17. $\lim_{x\to 0^+} \ln x = -\infty$



18. $\lim_{x\to 0} \frac{\frac{2}{x+3} - \frac{2}{3}}{x}$ $(x+3)$

$\lim_{x\to 0} \frac{\frac{6-2x-6}{3(x+3)}}{x}$

$\lim_{x\to 0} \frac{-2x}{3(x+3)} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{-2}{3(x+3)} = \frac{-2}{9}$

19. $\lim_{x\to -\infty} \frac{\sqrt{4x^2-4x+2}}{5x-3}$

behaves like: $\lim_{x\to -\infty} \frac{\sqrt{4x^2}}{5x}$

$\lim_{x\to -\infty} \frac{2|x|}{5x}$

$\lim_{x\to -\infty} \frac{-2x}{5x} = \frac{-2}{5}$

20. $\lim_{x\to \infty} \frac{8x^3-2x}{3x^2-5x^3}$

$\lim_{x\to \infty} \frac{8x^3-2x}{-5x^3+3x^2} = -\frac{8}{5}$

21. Is the function continuous? **Justify your answer.**

$$f(x)=\begin{cases} x^2-1 & x>2 \\ 3 & x=2 \\ 4x-3 & x<2 \end{cases}$$

$\lim_{x\to 2^+} f(x) = (2)^2-1 = 3$

$\lim_{x\to 2^-} f(x) = 4(2)-3 = 5$

$f(2)=3$

f(x) is not continuous bc

$\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

22. Find the value of a and b that make the function continuous

$$f(x)=\begin{cases} ax^2-b & \text{if } x\le -1 \\ 2bx+5 & \text{if } -1<x<2 \\ bx^2+ax+1 & \text{if } x\ge 2 \end{cases}$$

$ax^2-b=2bx+5$

$a(-1)^2-b=2b(-1)+5$

$a-b=-2b+5$

$a=-b+5$

$a=-3+5$

$a=2$

$2bx+5=bx^2+ax+1$

$2b(2)+5=b(2)^2+(-b+5)(2)+1$

$4b+5=4b-2b+10+1$

$2b=6$

$b=3$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(-1)$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

23. Given $f(x)=\dfrac{2x^2+5x-3}{x^2-x-12}$, complete the chart below.

$f(x)=\dfrac{(2x-1)\cancel{(x+3)}}{(x-4)\cancel{(x+3)}}$

$f(x)=\dfrac{2x-1}{x-4}$

f(x) is discontinuous at x =	Type of discontinuity
$x=-3$	Removable
$x=4$	Infinite

Can f(x) be made continuous at any of the x values above? If so, at which x value and what point would you use to "repair" the discontinuity?

$f(-3)=1$ will repair the removable discontinuity

$f(-3)=\dfrac{2(-3)-1}{-3-4}$

24. Verify the conditions of the Intermediate Value Theorem and find the guaranteed value of c in $(-3,3)$ when $f(x)=x^2-3x-4$, and $f(c)=6$.

Since f(x) is continuous on the interval $(-3,3)$

$f(-3)=(-3)^2-3(-3)-4=14$

$f(3)=(3)^2-3(3)-4=-4$

and $-4<6<14$ $\therefore \exists\, c\in(-3,3)$ s.t. $f(c)=6$

$f(c)=x^2-3x-4$

$6=x^2-3x-4$

$0=x^2-3x-10$

$0=(x-5)(x+2)$

$x=5$ $x=-2$

$c=-2$