AP Calculus Analyzing Graphs in Class Practice Problems

Consider the function $h'(x) = 2x - x\sin(2x)$ on the open interval $-5 < x < 5$ to answer the following questions.

1. Based on the graph of $h'(x)$, how many relative extrema does the graph of $h(x)$ have? Give a reason for your answer.

2. Based on the graph of $h'(x)$, how many points of inflection does the graph of $h(x)$ have? Give a reason for your answer.

3. Find the equation of $h''(x)$ and then graph it on your calculator. Explain why the graph of $h''(x)$ confirms your response to question #2 above.

Based on these relationships between a function and its first and second derivative, complete the following statements.

1. $f(x)$ is increasing $\leftrightarrow$ $f'(x)$ ____________________
2. $f(x)$ is decreasing $\leftrightarrow$ $f'(x)$ ____________________
3. $f(x)$ has a relative maximum or minimum $\leftrightarrow$ $f'(x)$ ____________________
4. $f(x)$ has a point of inflection $\leftrightarrow$ $f'(x)$ ____________________
5. $f(x)$ is concave up $\leftrightarrow$ $f''(x)$ ____________________
6. $f(x)$ is concave down $\leftrightarrow$ $f''(x)$ ____________________
7. $f(x)$ has a point of inflection $\leftrightarrow$ $f''(x)$ ____________________
8. $f'(x)$ is increasing $\leftrightarrow$ $f''(x)$ ____________________
9. $f'(x)$ is decreasing $\leftrightarrow$ $f''(x)$ ____________________
10. $f'(x)$ has a relative maximum or minimum $\leftrightarrow$ $f''(x)$ ____________________
11. $f'(x)$ has a point of inflection $\leftrightarrow$ $f''(x)$ ____________________
12. $f'(x)$ changes from negative to positive $\leftrightarrow$ $f(x)$ ____________________
13. $f'(x)$ changes from positive to negative $\leftrightarrow$ $f(x)$ ____________________
14. $f'(x)$ has a relative maximum or minimum $\leftrightarrow$ $f(x)$ ____________________
15. $f''(x)$ changes from positive to negative $\leftrightarrow$ $f'(x)$ ____________________
16. $f''(x)$ changes from negative to positive $\leftrightarrow$ $f'(x)$ ____________________

1. For the function $h(x) = \dfrac{x^2 - 3x - 4}{x - 2}$, determine the open intervals on which the given function is increasing or decreasing and the *x*–values of any relative extrema. Show your analysis and explain your reasoning.

2. If $F'(x) = (x-1)^2(x-2)(x-4)$, where is the graph of *F*(*x*) increasing, decreasing, and/or reaching a relative maximum or minimum? Show your work and justify your reasoning.

3. If *h*(*x*) is a twice differentiable function such that $h(x) < 0$ for all values of *x*, then at what value(s) does the graph of *g*(*x*) have a relative maximum if $g'(x) = \left(9 - x^2\right) \cdot h(x)$?

For exercises 4 – 5, identify the intervals where the function, *g*(*x*), is concave up and concave down. Also, identify the *x* – values of any points of inflection. Show your work and justify your reasoning.

4. $g'(x) = \sqrt{8x - x^2}$
5. $g(x) = xe^{2x}$

For exercises 6 and 7, use the Second Derivative Test to find the local extrema for the given function.

Show your analysis and justify your reasoning.

6. $g(x) = 3x - x^3 + 5$	7. $h(x) = x^3 + 3x^2 - 2$

Calculator Active Questions

The function $f'(x) = \cos(\ln x)$ is the first derivative of a twice differentiable function, *f*(*x*).

a. On the interval 0 < *x* < 10, find the *x* – value(s) where *f*(*x*) has a relative maximum.
Justify your answer.

b. On the interval 0 < *x* < 10, find the *x* – value(s) where *f*(*x*) has a relative minimum.

Justify your answer.

c. On the interval 0 < *x* < 10, find the *x* – value(s) where *f*(*x*) has a point of inflection.

Justify your answer.