Mid-Term Review

1. Find the domain of $f(x) = \frac{1}{\sqrt{3+2x}}$

$3+2x > 0$

$2x > -3$

$x > -3/2$

$(-3/2, \infty)$

2. What is the range of $f(x) = 3(x-2)^2 + 5$?

vertical shift up5

$[5, \infty)$

3. Describe the symmetry of $y = |x| - 2$

Even symmetry

Symmetric to y-axis

4. Describe the symmetry of $f(x) = \frac{x^2}{x^2+1}$

$f(x) = f(-x)$

$\frac{x^2}{x^2+1} = \frac{(-x)^2}{(-x)^2+1} = \frac{x^2}{x^2+1}$ ✓

Even symmetry

Symmetric to y-axis

5. Find the horizontal asymptote of $f(x) = \frac{3x^2+2x-16}{x^2-7}$

HA: $y=3$

$\lim_{x \to \pm\infty} f(x) = 3$

6. Find the vertical asymptote of $y = \frac{2}{x-3}$

$x-3 \neq 0$

VA: $x=3$

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = \infty$

7. If $f(x) = 2x^2 + 1$ and $g(x) = x + 2$, then $(f \circ g)(x) =$

$f(g(x)) = 2(x+2)^2 + 1$

$= 2(x^2+4x+4) + 1$

$= 2x^2 + 8x + 8 + 1$

$f(g(x)) = 2x^2 + 8x + 9$

8. If $f(x) = \frac{2x+1}{3}$, find $f^{-1}(x)$

$x = \frac{2y+1}{3}$

$3x = 2y + 1$

$\frac{3x-1}{2} = y$

$f^{-1}(x) = \frac{3x-1}{2} = \frac{3}{2}x - 1$

9. $\lim_{x \to 3} \frac{x^2-8x+15}{(x-3)^2} =$ $\frac{\#}{0}$

$\lim_{x \to 3} \frac{(x-5)\cancel{(x-3)}}{(x-3)^2}$

$\lim_{x \to 3} \frac{x-5}{x-3} = \frac{3-5}{3-3} = \frac{-2}{0}$ DNE

$\lim_{x \to 3^+} \frac{x-5}{x-3} = -\infty$ $\lim_{x \to 3^-} \frac{x-5}{x-3} = +\infty$

10. $\lim_{x \to 27} \frac{\left(x^{\frac{1}{3}} - 3\right)}{x-27} =$

$\lim_{x \to 27} \frac{\sqrt[3]{x} - 3}{(\sqrt[3]{x}-3)(\sqrt[3]{x^2} + 3\sqrt[3]{x} + 9)}$

$\lim_{x \to 27} \frac{1}{x^{2/3} + 3x^{1/3} + 9}$

$\lim_{x \to 27} \frac{1}{(27)^{2/3} + 3(27)^{1/3} + 9} = \frac{1}{9+9+9} = \frac{1}{27}$

11. $\lim_{x \to 0^-} \frac{1}{x} =$

$\frac{1}{-.0001} = -\infty$

12. $\lim_{x \to 0^+} \frac{1}{x} =$

$\frac{1}{.0001} = \infty$

13. Given a function is defined by $f(x) = \frac{2x+2}{x^2+5x+4}$, for what value(s) of x does the function have one or more vertical asymptotes?

$f(x) = \frac{2\cancel{(x+1)}}{(x+4)\cancel{(x+1)}}$ hole at $x = -1$

$f(x) = \frac{2}{x+4}$ $x+4 \neq 0$

VA: $x = -4$

14. Given a function defined by $f(x) = \frac{2x+1}{x^2+5x+4}$, for what values of x is the function discontinuous?

$f(x) = \frac{2x+1}{(x+4)(x+1)}$ $x \neq -4, -1$

The function is discontinuous at $x = -4, -1$

15. If $f(x) = -\frac{4}{\sqrt[4]{x}}$, then $f'(16) =$

$f(x) = -4x^{-1/4}$

$f'(x) = \frac{1}{x^{5/4}}$

$f'(16) = \frac{1}{\sqrt[4]{16}^5} = \frac{1}{32}$

16. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)(3) - 3x(2x)}{(x^2+1)^2}$

$\frac{dy}{dx} = \frac{3x^2+3-6x^2}{(x^2+1)^2}$

$\frac{dy}{dx} = \frac{-3x^2+3}{(x^2+1)^2}$

17. If $y = -\frac{4}{\sqrt[3]{x+5}}$, then $\frac{dy}{dx} =$

$y = -4(x+5)^{-1/3}$

$\frac{dy}{dx} = \frac{4}{3}(x+5)^{-4/3}$

$\frac{dy}{dx} = \frac{4}{3(x+5)^{4/3}}$

18. Find the derivative of $y = \sqrt[3]{x^2+x}$

$y' = \frac{2x+1}{3\sqrt[3]{x^2+x}^2}$ or $\frac{2x+1}{3(x^2+x)^{2/3}}$

19. Find the derivative of $y = (x^2+2x+5)^6$

$y' = 6(x^2+2x+5)^5(2x+2)$

$y' = (12x+12)(x^2+2x+5)^5$

20. Find $f'(x)$ for $f(x) = (2x^2+5)^7$

$f'(x) = 7(2x^2+5)^6(4x)$

$f'(x) = 28x(2x^2+5)^6$

21. Given $y = \sin(\sin x)$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x$

22. If $y = \cos(e^x)$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = -\sin(e^x)\, e^x$

$\frac{dy}{dx} = -e^x \sin(e^x)$

23. Find $f'(x)$ fiven $f(x) = \sin^3(4x)$

$f'(x) = 3\sin^2(4x)\cos(4x) \cdot 4$

$f'(x) = 12\sin^2(4x)\cos(4x)$

24. Given $y = \sin^2 x^3$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = 2\sin(x^3)\cos(x^3) \cdot 3x^2$

$\frac{dy}{dx} = 6x^2 \sin(x^3)\cos(x^3)$

25. Find $\frac{dy}{dx}$ if $y = x^2 \cdot e^x$

$\frac{dy}{dx} = x^2 e^x + 2x e^x$

or $\frac{dy}{dx} = x e^x (x+2)$

26. $\frac{d}{dx} \ln \frac{5}{5-x} =$

$\frac{d}{dx}(\ln 5 - \ln(5-x))$

$0 - \frac{-1}{5-x} = \frac{1}{5-x}$

27. If $y = e^{\frac{1}{x}}$, then $y' =$

$y' = e^{1/x} \cdot -\frac{1}{x^2}$

$y' = \frac{-e^{1/x}}{x^2}$

28. $\frac{d}{dx} e^{\ln 5x} =$

$\frac{d}{dx} 5x = 5$

29. $\frac{d}{dx} \ln(e^{x^2})$

$\frac{d}{dx} x^2 = 2x$

30. Find $\frac{dy}{dx}$ given $y = \frac{x^3}{3^x}$

$\frac{dy}{dx} = \frac{3^x \cdot 3x^2 - x^3 \cdot 3^x \ln 3}{(3^x)^2}$

$\frac{dy}{dx} = \frac{x^2 3^x (3 - x\ln 3)}{3^{2x}}$

$\frac{dy}{dx} = \frac{x^2(3 - x\ln 3)}{3^x}$

31. If $y = \log_3(2x^2 - 5)$, then $\frac{dy}{dx} =$

$y = \frac{\ln(2x^2-5)}{\ln 3}$

$y = \frac{1}{\ln 3}\ln(2x^2-5)$

$y' = \frac{1}{\ln 3}\cdot\frac{4x}{2x^2-5}$

$y' = \frac{4x}{\ln 3(2x^2-5)}$

32. Find $\frac{d^2y}{dx^2}$ for $y = \frac{1-x}{x-3}$

$\frac{dy}{dx} = \frac{(x-3)(-1)-(1-x)}{(x-3)^2}$

$\frac{dy}{dx} = \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2} = 2(x-3)^{-2}$

$\frac{d^2y}{dx^2} = -4(x-3)^{-3}$

$\frac{d^2y}{dx^2} = \frac{-4}{(x-3)^3}$

33. Find the slope of the tangent line to the graph f $f(x) = 2x(2x^2-1)$ at the point where $x = 1$

$f(x) = 4x^3 - 2x$

$f'(x) = 12x^2 - 2$

$f'(1) = 10$

34. Find an equation of the tangent line to the curve $f(x) = -x^2 + 12$ passing through the point $(4,0)$

$f'(x) = -2x$

$f'(4) = -2(4) = -8$

$y - 0 = -8(x-4)$

$y = -8x + 32$

35. Find the critical numbers of $f(x) = x^3 - 12x^2$

$f'(x) = 3x^2 - 24x$

$0 = 3x(x-8)$

$x = 0, 8$

36. Let $f(x) = x^2(x-3)$. Over what interval is the function decreasing?

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

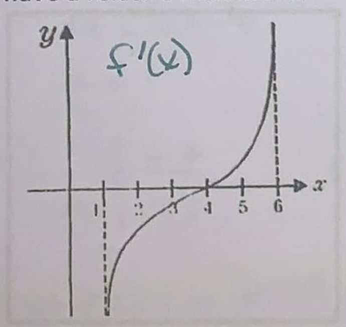
$0 = 3x(x-2)$

$x = 0, 2$

f' + − +
0 2

Decreasing on (0,2) since $f'(x)$ is negative

37. The figure shows the graph of ff', the derivative of the function f. The domain of the function f is $-10 \le x \le 10$. For what value(s) does the function have a relative maximum?



f(x) does not have a relative maximum

f' − − + +
4

f ↘ ↗

38. Refer to the previous figure. For what value(s) does the function have a relative minimum?

f(x) has a relative minimum at x=4 since f'(x) changes from negative to positive

39. A particle's motion is described by $x(t) = 4t^3 - 5t^2$, $t \geq 0$, where t is in seconds and distance in meters. Find the velocity in the third second.

$v(t) = 12t^2 - 10t$

$v(3) = 12(3)^2 - 10(3)$

$v(3) = 78$ m/s

40. The position of a particle moving in a straight line at any time t is $x(t) = 2t^2 + 6t + 5$. What is the acceleration of the particle at $t = 3$?

$v(t) = 4t + 6$

$a(t) = 4$ m²/s

41. Find all points of inflection for $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

$x = 0, 2$

f'' + | - | +
0 2
f ∪ ∩ ∪

$f(0) = 2$

$f(2) = 16 - 32 + 2 = -14$

POI @ (0,2) & (2,-14)

42. Find the interval(s) on which the curve $y = x^3 - 3x^2 - 9x + 6$ is concave upward or concave downward.

$y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

$0 = 6x - 6$

$x = 1$

y'' - | +
1

Concave up (1,∞) since y'' is positive

Concave down (-∞,1) bc y'' is negative

43. Given that $f(x) = \frac{4}{x}$, determine where the function is concave up and concave down.

$f'(x) = \frac{-4}{x^2}$

$f''(x) = \frac{8}{x^3}$

f'' - | +
0

conc. up (0,∞) bc f'' is positive

conc. down (-∞,0) bc f'' is negative

44. Given that $f(x) = -x^2 + 12x - 34$ has a relative maximum at $x = 6$, determine where $f'(x)$ is positive and negative.

$f'(x) = -2x + 12$

f(x) ∩

f' + + 0 - -
6

f'(x) is positive (-∞,6) since f(x) is increasing

f'(x) is neg. (6,∞) since f(x) is positive

45. Find the point of inflection of $f(x) = x^3 - 3x^2 - x + 7$

$f'(x) = 3x^2 - 6x + 1$

$f''(x) = 6x - 6$

$0 = 6x - 6$

$x = 1$

f'' - - 0 + +
1
f ∩ | ∪

$f(1) = (1)^3 - 3(1)^2 - 1 + 7$

$f(1) = 4$

f(x) has a POI at (1,4) since f''(x) changes signs

46. Given a function defined by $f(x) = 3x^5 - 5x^3 - 8$, for what value(s) of x is there a point of relative minimum?

$f'(x) = 15x^4 - 15x^2$

$0 = 15x^2(x^2 - 1)$

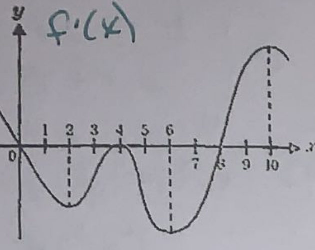
$x = 0, -1, 1$

f'(x) + | - | - | +
-1 0 1
↗ ↘ ↘ ↗

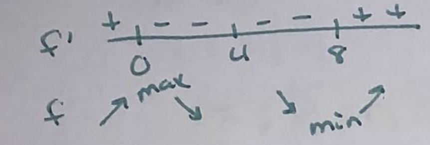
$f(1) = 3(1)^5 - 5(1)^3 - 8$
$= 3 - 5 - 8$
$= -10$

f(x) has a relative minimum at x=1 since f'(x) changes from neg. to pos.

47. The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \le x \le 10$. For what value(s) does the function have a relative minimum?



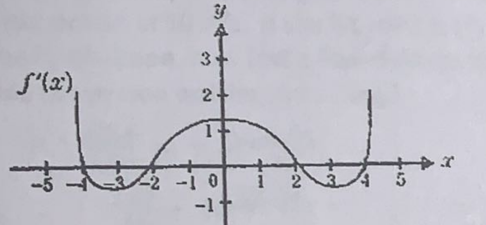
f(x) has a relative min. at $x=8$ bc $f'(x)$ changes from negative to positive



48. Refer to the previous figure. For what value(s) does the function have a relative maximum?

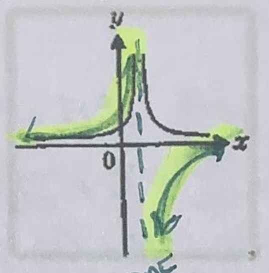
f(x) has a relative maximum at $x=0$ since $f'(x)$ changes from positive to negative

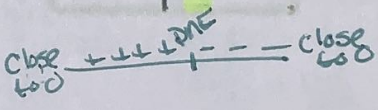
49. The graph $f(x)$ has horizontal tangents when $x =$



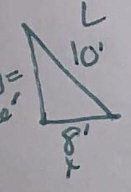
$x=-4,-2,2,4$ because $f'(x)=0$

50. The graph of the derivative is shown. Draw the graph of f.





51. A ladder 10 feet long is leaning against a wall, with the foot of the ladder 8 feet away from the wall. If the foot of the ladder is being pulled away from the wall at 3 ft/sec how fast is the top of the ladder sliding down the wall?



K: $dx/dt = 3$ ft/sec

F: dy/dt

W: $x=8'$, $y=6'$, $L=10'$

$x^2+y^2=L^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$8(3)+6\,dy/dt=0$

$\frac{dy}{dt}=\frac{-24}{6}$

$\frac{dy}{dt}=-4$ ft/sec

52. Find all value(s) of x (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{1+x}$ on the interval [0,1].

f(x) is continuous on [0,1]

f(x) is differentiable on (0,1)

$\therefore \exists x \in [0,1]$ s.t. $f'(x)=\frac{f(1)-f(0)}{1-0}$

$m_{sec}=\frac{f(1)-f(0)}{1-0}=\frac{\frac{1}{2}-1}{1}=-\frac{1}{2}$

$f'(x)=\frac{-1}{(1+x)^2}$

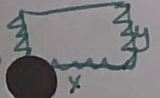
$-\frac{1}{2}=\frac{-1}{(1+x)^2}$

$2=(1+x)^2$

$x=\pm\sqrt{2}-1$

$x=\sqrt{2}-1$

53. A farmer has 20 feet of fence, and he wishes to make from it a rectangular pen for his pig Wilbur, using a barn as one of the sides. In square feet, What is the maximum area possible for his pet?



$20=x+2y$

$x=20-2y$

$x=20-2(5)$

$x=10$

$A=xy$

$A=(20-2y)y$

$A=20y-2y^2$

$A'=20-4y=0$

$y=5$

$A=10(5)$

$A=50\text{ ft}^2$

54. Find the shortest distance from the point (4,0) to a point on the parabola $y^2 = 2x$.

$x=\frac{y^2}{2}$

$(4,0)$ $(x,\pm\sqrt{2x})$

$x_1\ y_1\quad x_2\ y_2$

$d=\sqrt{(x-4)^2+(\pm\sqrt{2x}-0)^2}$

$d=\sqrt{x^2-8x+16+2x}$

$d=\sqrt{x^2-6x+16}$

$d'=\frac{2x-6}{2\sqrt{x^2-6x+16}}=0$

$2x-6=0$

$x=3$

$y^2=2(3)$

$y^2=6$

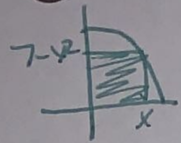
$y=\pm\sqrt{6}$

$d=\sqrt{(3-4)^2+(\sqrt{6}-0)^2}$

$d=\sqrt{1+6}$

$d=\sqrt{7}$

55. A rectangle is inscribed between the parabola $y = 7 - x^2$ and the x-axis, with its base on the x-axis. Find the value of x that maximizes the area of the rectangle.



$A = x \cdot y$

$A = x(7-x^2)$

$A = 7x - x^3$

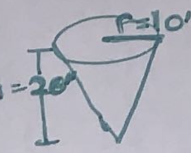
$A' = 7 - 3x^2 = 0$

$3x^2 = 7$

$x = \pm\sqrt{\frac{7}{3}}$

$x = \sqrt{\frac{7}{3}}$ or $\frac{\sqrt{21}}{3}$

56. A circular conical reservoir, vertex down, has a depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of ½ ft/hr. Find the rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 feet deep.



K: $\frac{dh}{dt} = -\frac{1}{2}$ ft/hr

F: $\frac{dV}{dt}$

W: h=8'

$\frac{r}{h} = \frac{10}{20}$

$10h = 20r$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

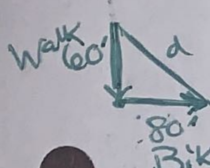
$V = \frac{1}{3}\pi\frac{1}{4}h^3$

$V' = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$V' = \frac{1}{4}\pi(8)^2\left(\frac{-1}{2}\right)$

$V' = -8\pi$ ft³/hr

57. One person is walking south toward an intersection that is 60 ft away at a rate of 2 ft/s while a second person on a bicycle is riding east away from the same intersection at 10 ft/s. If the bicyclist is 80 ft from the intersection, how fast is the distance between he and the person walking increasing?



K: $\frac{dw}{dt} = -2$ ft/s

$\frac{dB}{dt} = 10$ ft/s

F: $\frac{dd}{dt}$

W: w=60', B=80', d=100'

$w^2 + B^2 = d^2$

$\cancel{2}w\frac{dw}{dt} + \cancel{2}B\frac{dB}{dt} = \cancel{2}d\frac{dd}{dt}$

$60(-2) + 80(10) = 100\frac{dd}{dt}$

$-120 + 800 = 100\frac{dd}{dt}$

$\frac{680}{100} = \frac{dd}{dt}$

$\frac{dd}{dt} = 34/5$ ft/s or 6.8 ft/s

For Questions 58-61. Suppose that the functions f and g have values according to the following table.

	f	f'	g	g'
-1	4	7	2	3
2	3	5	4	1

58. What is the value of the derivative of $f(g(x))$ and $x = -1$

$f'(g(x)) \cdot g'(x)$

$f'(g(-1)) \cdot g'(-1)$

$f'(2) \cdot 3$

$5 \cdot 3$

15

59. Evaluate $\frac{d}{dx}[f(x)g(x)]_{x=2}$

$f(x)g'(x) + g(x)f'(x)$

$f(2)g'(2) + g(2)f'(2)$

$3 \cdot 1 + 4 \cdot 5$

$3 + 20$

23

60. Evaluate $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]_{x=-1}$

$\frac{g(-1) \cdot f'(-1) - f(-1) \cdot g'(-1)}{[g(-1)]^2}$

$\frac{2 \cdot 7 - 4 \cdot 3}{(2)^2} = \frac{14-12}{4} = \frac{1}{2}$

61. Evaluate $\frac{d}{dx}[g^{-1}(x)]_{x=2}$

$\frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(-1)} = \frac{1}{3}$