Calculator

AP Calculus Analyzing Graphs in Class Practice Problems

Consider the function $h'(x) = 2x - x\sin(2x)$ on the open interval $-5 < x < 5$ to answer the following questions.

1. Based on the graph of $h'(x)$, how many relative extrema does the graph of $h(x)$ have? Give a reason for your answer. $h'(x)=0$

$2x - x\sin(2x) = 0$

$x(2-\sin(2x))=0$

$x=0 \quad 2-\sin 2x = 0$

$\sin 2x = 2$

DNE

x-int.

$\therefore$ $h'(x)$ has one zero so $h(x)$ has one relative extrema

2. Based on the graph of $h'(x)$, how many points of inflection does the graph of $h(x)$ have? Give a reason for your answer.

The graph of $h'(x)$ has 5 extrema b/n $(-5,5)$

$\therefore$ $h(x)$ has 5 POI

3. Find the equation of $h''(x)$ and then graph it on your calculator. Explain why the graph of $h''(x)$ confirms your response to question #2 above.

$h''(x) = 2 - (x\cos(2x)2 + \sin(2x))$

$h''(x) = 2 - 2x\cos(2x) + \sin(2x)$

$h''(x)$ has 5 zeros b/n $(-5,5)$ $\therefore$ $h(x)$ has 5 POIs sign or type of point

Based on these relationships between a function and its first and second derivative, complete the following statements.

1. $f(x)$ is increasing $\leftrightarrow$ $f'(x)$ positive
2. $f(x)$ is decreasing $\leftrightarrow$ $f'(x)$ negative
3. $f(x)$ has a relative maximum or minimum $\leftrightarrow$ $f'(x)$ zero
4. $f(x)$ has a point of inflection changes concavity so f' changes directions $\leftrightarrow$ $f'(x)$ relative extrema
5. $f(x)$ is concave up $\leftrightarrow$ $f''(x)$ positive
6. $f(x)$ is concave down $\leftrightarrow$ $f''(x)$ negative
7. $f(x)$ has a point of inflection $\leftrightarrow$ $f''(x)$ zero
8. $f'(x)$ is increasing $\leftrightarrow$ $f''(x)$ positive
9. $f'(x)$ is decreasing $\leftrightarrow$ $f''(x)$ negative
10. $f'(x)$ has a relative maximum or minimum $\leftrightarrow$ $f''(x)$ zero
11. $f'(x)$ has a point of inflection (like 4) $\leftrightarrow$ $f''(x)$ relative extrema
12. $f'(x)$ changes from negative to positive $\leftrightarrow$ $f(x)$ local min

    $-|+$
13. $f'(x)$ changes from positive to negative $\leftrightarrow$ $f(x)$ local max
14. $f'(x)$ has a relative maximum or minimum $\leftrightarrow$ $f(x)$ POI

    $f''(x)=0$ so it's a POI at $f(x)$
15. $f''(x)$ changes from positive to negative $\leftrightarrow$ $f'(x)$ local max

    ↗↘
16. $f''(x)$ changes from negative to positive $\leftrightarrow$ $f'(x)$ local min

    ↘↗

no calc.

1. For the function $h(x) = \dfrac{x^2 - 3x - 4}{x - 2}$, determine the open intervals on which the given function is increasing or decreasing and the x–values of any relative extrema. Show your analysis and explain your reasoning. $x \neq 2$ VA at $x=2$

$h'(x) = \dfrac{(x-2)(2x-3) - (x^2-3x-4)}{(x-2)^2}$

$h'(x) = \dfrac{2x^2-7x+6-x^2+3x+4}{(x-2)^2}$

$h'(x) = \dfrac{x^2-4x+10}{(x-2)^2}$

$0 = x^2-4x+10$

$\dfrac{4 \pm \sqrt{16-4(1)(10)}}{2}$ imaginary

f': $+$ | $+$ at 2

Increasing $(-\infty, 2) \cup (2, \infty)$ since $f'(x)$ is positive.

no Decrease or extrema.

2. If $F'(x) = (x-1)^2(x-2)(x-4)$, where is the graph of F(x) increasing, decreasing, and/or reaching a relative maximum or minimum? Show your work and justify your reasoning.

$0 = (x-1)^2(x-2)(x-4)$

$x = 1, 2, 4$

$+$ | $+$ | $-$ | $+$ at 1, 2, 4

$f(x)$ is increasing on $(-\infty, 1) \cup (1, 2) \cup (4, \infty)$ since $f'(x)$ is positive

$f(x)$ is decreasing on $(2, 4)$ since $f'(x)$ is negative

$f(x)$ has a local max at $x=2$ since $f'(x)=0$ & it changes from $-$ to $+$

$f(x)$ has a local min at $x=4$ since $f'(x)=0$ & it changes from $+$ to $-$

3. If $h(x)$ is a twice differentiable function such that $h(x) < 0$ for all values of x, then at what value(s) does the graph of g(x) have a relative maximum if $g'(x) = (9 - x^2) \cdot \overline{h(x)}$?

$0 = (9-x^2) \cdot h(x)$

$x^2 = 9$ $\quad h(x) = 0$

$x = \pm 3$ $\quad$ DNE

g': $+$ | $-$ | $+$ at -3, 3

$g(x)$ has a relative maximum at $x = -3$ since $g'(x) = 0$ and changes from $+$ to $-$ at -3

For exercises 4 – 5, identify the intervals where the function, g(x), is concave up and concave down. Also, identify the x – values of any points of inflection. Show your work and justify your reasoning.

4. $g'(x) = \sqrt{8x - x^2}$

$g''(x) = \dfrac{8-2x}{2\sqrt{8x-x^2}} = 0$

$8 - 2x = 0$

$x = 4$

$8x - x^2 = 0$

$x(8-x) = 0$

$x = 0, 8$

DNE | $+$ | $-$ | DNE at 0, 4, 8

$g(x)$ is concave up on $(0, 4)$ since $g''(x)$ is positive

$g(x)$ is concave down on $(4, 8)$ since $g''(x)$ is negative

POI at $x = 4$ since g'' changes signs

5. $g(x) = xe^{2x}$

$g'(x) = x \cdot 2e^{2x} + e^{2x}$

$g'(x) = 2xe^{2x} + e^{2x}$

$g''(x) = 2x \cdot 2e^{2x} + 2e^{2x} + 2e^{2x}$

$g''(x) = 4xe^{2x} + 4e^{2x}$

$0 = 4e^{2x}(x+1)$

$x = -1$

g'': $-$ | $+$ at -1

$g(x)$ is conc up on $(-1, \infty)$ since $g''(x)$ is pos.

$g(x)$ is conc down on $(-\infty, -1)$ since $g''(x)$ is neg.

POI at $x = -1$ on $g(x)$ since g'' changes signs

No Calc

For exercises 6 and 7, use the Second Derivative Test to find the local extrema for the given function.

Show your analysis and justify your reasoning.

6. $g(x) = 3x - x^3 + 5$	7. $h(x) = x^3 + 3x^2 - 2$
$g'(x) = 3 - 3x^2$ $x = \pm 1$ $g''(x) = -6x$ $g''(-1) = -6(-1) = 6$ conc up $g''(1) = -6(1) = -6$ conc down $\therefore x = -1$ rel. min $x = 1$ rel. max	$h'(x) = 3x^2 + 6x$ $0 = 3x(x+2)$ $x = 0, -2$ $h''(x) = 6x + 6$ $h''(0) = 6$ conc up $h''(-2) = -6$ conc down $\therefore x = 0$ rel. min $x = -2$ rel. max

Calculator Active Questions

The function $f'(x) = \cos(\ln x)$ is the first derivative of a twice differentiable function, *f*(*x*).

a. On the interval ~~$0 < x < 10$~~ $0.1 < x < 5$, find the x – value(s) where *f*(*x*) has a relative maximum. Justify your answer.

calc (bounds 0, 5)

$\cos(\ln x) = 0$ look at graph's x-int

$x = .208, 4.81$

$- \quad + \quad -$

$.208 \quad 4.81$

f(x) has a rel. max at x = 4.81 since f'(x) goes from – to +

b. On the interval ~~$0 < x < 10$~~ $0.1 < x < 5$, find the x – value(s) where *f*(*x*) has a relative minimum.

Justify your answer.

f(x) has a rel. min at x = 0.208 since f'(x) goes from pos. to neg.

c. On the interval ~~$0 < x < 10$~~ $0.1 < x < 5$, find the x – value(s) where *f*(*x*) has a point of inflection.

Justify your answer.

$f''(x) = -\dfrac{\sin(\ln x)}{x}$ or $\frac{d}{dx} f(x)$ in graph

look at zeros

$x = .0432, 1$ bc $f''(x) = 0$