Advanced Algebra/Trig

6.1 - 6.2 Review

Name _____ Date _____

1. If $\cos\theta = -\frac{2}{5}$ and θ lies in quadrant II, find $\csc\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + (-\frac{2}{5})^2 = 1$

$\sin^2\theta = \frac{21}{25}$

$\sin\theta = \frac{\sqrt{21}}{5} \rightarrow \csc\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

2. If $\tan\beta = 3$ and β lies in quadrant I, find $\sec\beta$.

$1 + \tan^2\beta = \sec^2\beta$

$1 + (3)^2 = \sec^2\beta$

$1 + 9 = \sec^2\beta$

$10 = \sec^2\beta$

$\sqrt{10} = \sec\beta$

3. If $\sin\theta = \frac{2}{3}$ and θ lies in quadrant II, find $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$(\frac{2}{3})^2 + \cos^2\theta = 1$

$\frac{4}{9} + \cos^2\theta = 1$

$\cos^2\theta = \frac{5}{9}$

$\cos\theta = \frac{\sqrt{5}}{3}$

4. If $\cot\theta = -2$ and θ lies in quadrant IV, find $\cos\theta$.

① $\cot\theta = -2$

$\tan\theta = -\frac{1}{2}$

② $1 + \tan^2\theta = \sec^2\theta$

$1 + (-\frac{1}{2})^2 = \sec^2\theta$

$1 + \frac{1}{4} = \sec^2\theta$

$\sqrt{\frac{5}{4}} = \sqrt{\sec^2\theta}$

$\frac{\sqrt{5}}{2} = \sec\theta$

③ $\sec\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\frac{2\sqrt{5}}{5}$

Simplify.

5. $(\sin^2\theta + \cos^2\theta) + \tan^2\theta$

$(1 + \tan^2\theta)$

$\sec^2\theta$

6. $\sin\theta\csc\theta + \tan\theta\cot\theta$

$\sin\theta \cdot \frac{1}{\sin\theta} + \tan\theta \cdot \frac{1}{\tan\theta}$

$1 + 1$

2

7. $\tan^2\theta \cdot \cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$

$\sin^2\theta$

8. $\dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$

$\frac{1}{\sin^2\theta} = \csc^2\theta$

9. $\csc\theta - \cos\theta\cot\theta$

$\frac{1}{\sin\theta} - \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta} = \frac{1 - \cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta}$

$= \sin\theta$

10. $\cot\theta\tan\theta - \sec\theta\cos\theta$

$\frac{1}{\tan\theta} \cdot \tan\theta - \frac{1}{\cos\theta} \cdot \cos\theta$

$1 - 1$

0

11. $(1 + \cos\theta)(\csc\theta - \cot\theta)$

$\csc\theta - \cot\theta + \cos\theta\csc\theta - \cos\theta\cot\theta$

$\csc\theta - \cot\theta + \cos\theta \cdot \frac{1}{\sin\theta} - \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta} = \frac{1 - \cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \sin\theta$

12. $(1 - \sin\theta)(\sec\theta + \tan\theta)$

$\sec\theta + \tan\theta - \sin\theta\sec\theta - \sin\theta\tan\theta$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$

$\frac{1 + \sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta}$

$= \cos\theta$

SHOW ALL Steps!

Verify each identity.

13. $\sin\theta\cot\theta\sec\theta = 1$

$\sin\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} = 1$

$\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = 1$

$1 = 1$ ✓

14. $\frac{\sin\theta}{1-\cos^2\theta} = \csc\theta$

$\frac{\sin\theta}{\sin^2\theta} = \csc\theta$

$\frac{1}{\sin\theta} = \csc\theta$

$\csc\theta = \csc\theta$ ✓

15. $\frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} = 1$

$\frac{1}{\sin^2\theta} - \cot^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

$1 = 1$ ✓

16. $\cos^2\theta\tan^2\theta + \sin^2\theta\cot^2\theta = 1$

$\cos^2\theta \cdot \frac{\sin^2\theta}{\cos^2\theta} + \sin^2\theta \cdot \frac{\cos^2\theta}{\sin^2\theta} = 1$

$\frac{\cos^2\theta\sin^2\theta}{\cos^2\theta} + \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta} = 1$

$\sin^2\theta + \cos^2\theta = 1$

$1 = 1$ ✓

17. $\sin\theta(1+\cot^2\theta) = \csc\theta$

$\sin\theta \cdot \csc^2\theta = \csc\theta$

$\sin\theta \cdot \frac{1}{\sin^2\theta} = \csc\theta$

$\frac{\sin\theta}{\sin^2\theta} = \csc\theta$

$\frac{1}{\sin\theta} = \csc\theta$

$\csc\theta = \csc\theta$ ✓

18. $\frac{\cot\theta}{1+\cot^2\theta} = \cos\theta\sin\theta$

$\frac{\cot\theta}{\csc^2\theta} = \cos\theta\sin\theta$

$\cot\theta \div \csc^2\theta = \cos\theta\sin\theta$

$\cot\theta \cdot \frac{1}{\csc^2\theta} = \cos\theta\sin\theta$

$\frac{\cos\theta}{\sin\theta} \cdot \sin^2\theta = \cos\theta\sin\theta$

$\frac{\cos\theta\sin^2\theta}{\sin\theta} = \cos\theta\sin\theta$

$\cos\theta\sin\theta = \cos\theta\sin\theta$ ✓

19. $\tan\theta\sin\theta + \cos\theta = \sec\theta$

$\frac{\sin\theta}{\cos\theta} \cdot \sin\theta + \cos\theta = \sec\theta$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1} = \sec\theta$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} = \sec\theta$

$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$\sec\theta = \sec\theta$ ✓

20. $\frac{\tan^2\theta}{\sec\theta+1} = \frac{1-\cos\theta}{\cos\theta}$

$\frac{\sec^2\theta - 1}{\sec\theta+1} = \frac{1-\cos\theta}{\cos\theta}$

$\frac{(\sec\theta+1)(\sec\theta-1)}{(\sec\theta+1)} = \frac{1-\cos\theta}{\cos\theta}$

$\sec\theta - 1 = \frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}$

$\sec\theta - 1 = \sec\theta - 1$ ✓

Solve each equation for $0 \le \theta < 2\pi$.

21) $-4\sin\theta = 2$

$\frac{-4\sin\theta}{-4} = \frac{2}{-4}$

$\sin\theta = -1/2$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

22) $-3 + \tan\theta = -4$

$+3 \quad +3$

$\tan\theta = -1$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

23) $3 = 4 + 2\cos\theta$

$-4 \quad -4$

$-1 = 2\cos\theta$

$-\frac{1}{2} = \cos\theta$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

24) $\frac{1}{2} = \frac{2\cos^2\theta}{2}$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

25) $5 = -4\sin^2\theta + 4\sin\theta + 4$

$0 = -4\sin^2\theta + 4\sin\theta - 1$

$\frac{0}{-1} = \frac{-1}{-1}(4\sin^2\theta - 4\sin\theta + 1)$

$0 = (2\sin\theta - 1)(2\sin\theta - 1)$

$2\sin\theta - 1 = 0$

$\sin\theta = 1/2$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

26) $0 = \cot^2\theta - 1$

$\pm\sqrt{1} = \sqrt{\cot^2\theta}$

$\cot\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

27) $2\sin^2\theta - 1 = \sin\theta$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0 \quad \sin\theta - 1 = 0$

$\sin\theta = -\frac{1}{2} \quad \sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

28) $3\csc^2\theta = 4$

$\sqrt{\csc^2\theta} = \pm\sqrt{\frac{4}{3}}$

$\csc\theta = \pm\frac{2}{\sqrt{3}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$