Unit 7

Integration Applications

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Average Function Value & Mean Value Theorem
- ❖ Area Between 2 Curves
- ❖ Volume by Known Cross Sections
- ❖ Volumes of Revolutions – Disk & Washer Method

Quiz is _____

Test is _____

Name: _____

Average Function Value/Mean Value Theorem

For each problem, find the average value of the function over the given interval.

1. $f(x) = -x^2 - 2x + 5;\ [-4, 0]$

2. $f(x) = -x^4 + 2x^2 + 4;\ [-2, 1]$

3. $f(x) = 4 - x^2;\ [-2, 2]$

4. $f(x) = \dfrac{x^2 + 5}{x}; [1, 2]$

5. $f(x) = \sin x\,;\ [0, \pi]$

6. $f(x) = \cos x\,;\ \left[0, \frac{\pi}{2}\right]$

For each problem, find the values of c that satisfy the Mean Value Theorem for Integrals.

7. $f(x) = -x + 2;\ [-2, 2]$

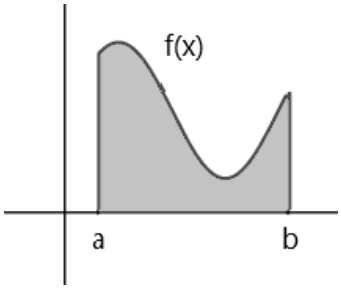
8. $f(x) = \frac{4}{x^2};\ [-4, -2]$

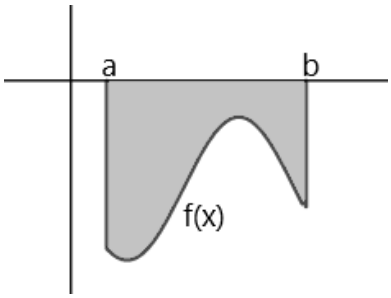
9. $f(x) = 4\sqrt{x};\ [0,4]$

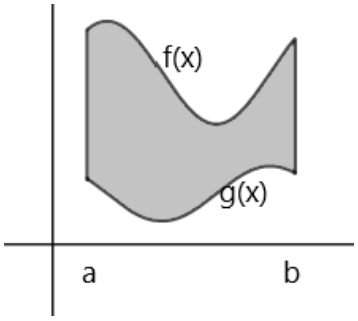
10. $f(x) = -3(2x - 6)^{\frac{1}{2}};\ [3, 5]$

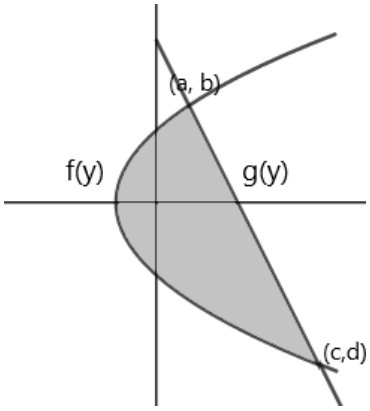
Area Between Curves Introduction Worksheet

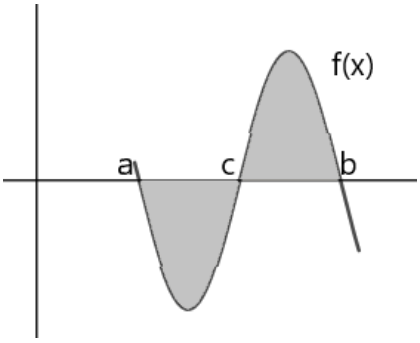
Draw the arbitrary rectangle and set up the integral to find the area for the shaded region.

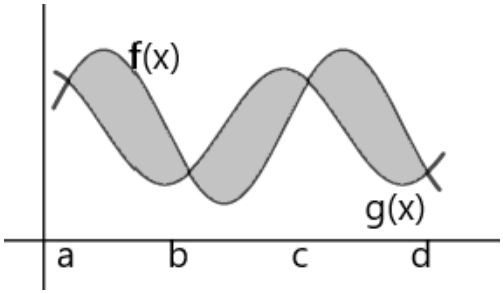
1. 

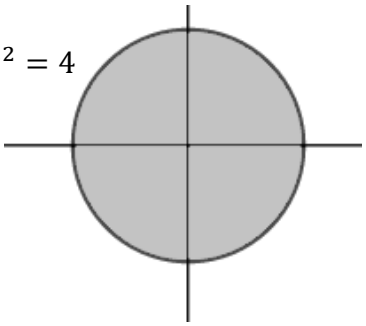
2. 

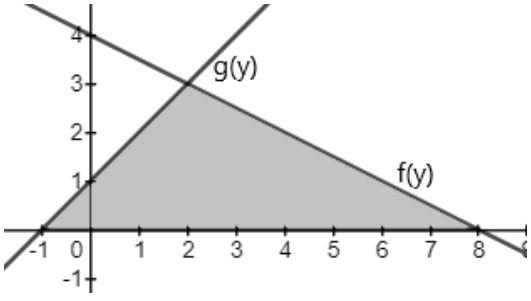
3. 

4. 

5. 

6. 

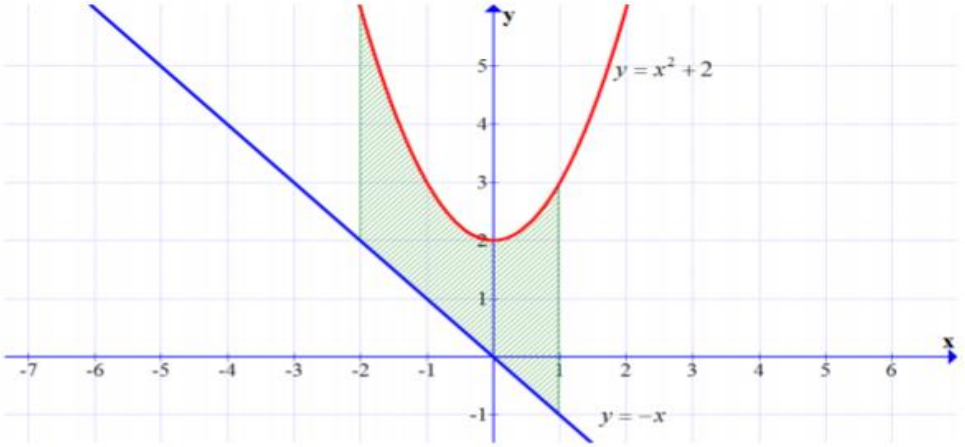
7. $x^2 + y^2 = 4$ 

8. 

Area between Two Curves

Compute the area of the shaded region.

1.



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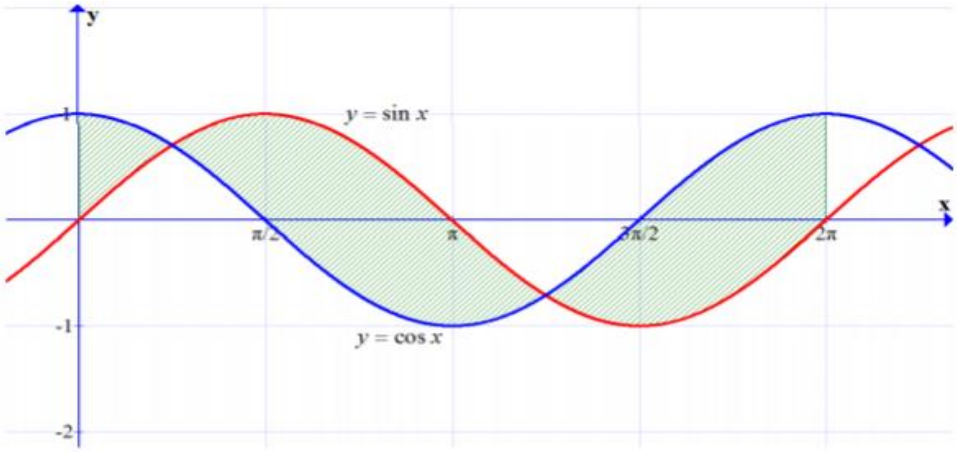

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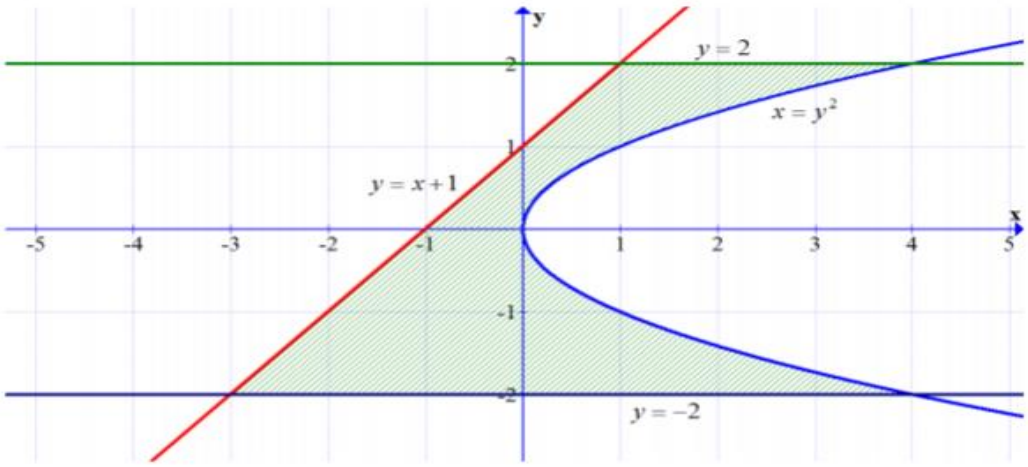


Compute the area of the region enclosed by the given curves.

4. $y = 4x,\ y = 6x^2$

5. $y = 2x^2, y = x^2 + 2$

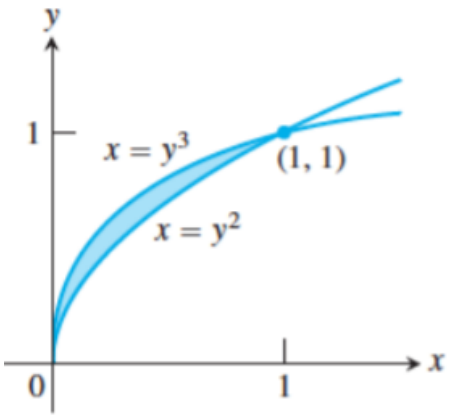
6. $x = 4 - y^2, x = y^2 - 4$

7. $y = x^4, y = |x|$

Area between Curves 2

Find the area of the shaded region analytically.

1.



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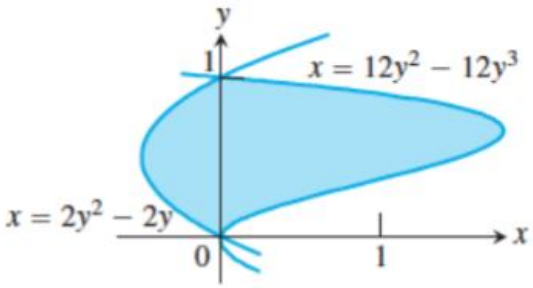

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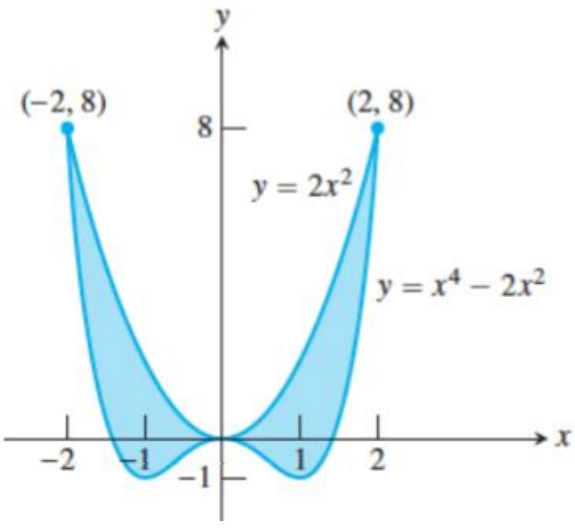

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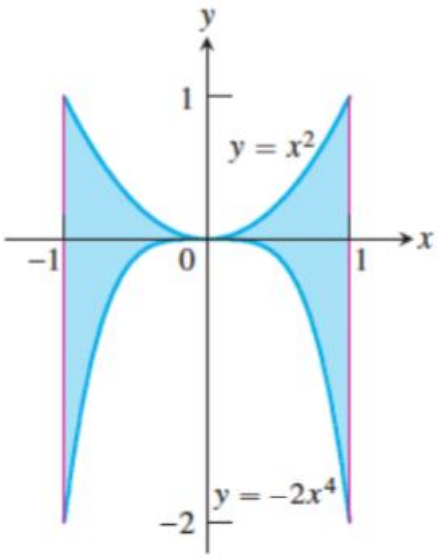

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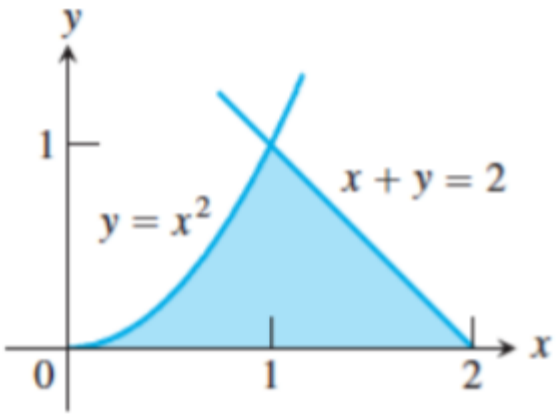

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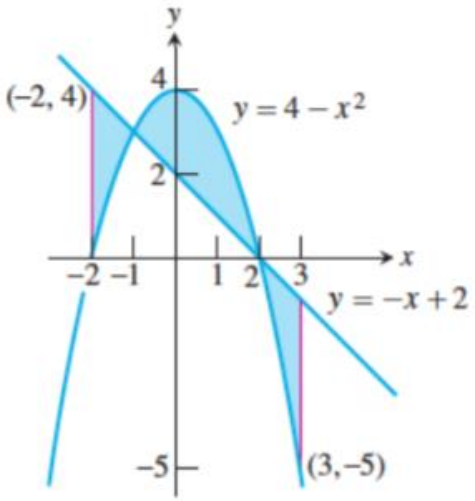


7. Find the area of the region(s) enclosed by the graphs of $x - y^2 = 0$ and $x + 2y^2 = 3$

8. Find the area of the region(s) enclosed by the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 3$

Volumes with Cross Sections

1. The base of a solid is bounded by $y = 2 - x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the x-axis are isosceles right triangles with the right angle on the x-axis. (Legs perpendicular to the x-axis). Find the volume.

2. The base of a solid is bounded by the semi-circle $y = \sqrt{4 - x^2}$ & the x-axis. Cross sections that are perpendicular to the x-axis are squares. Find the volume.

3. The base of a solid is bounded by $y = \sqrt{16 - x^2}$, the positive x-axis & the positive y-axis. Cross sections that are perpendicular to the y-axis are isosceles right triangles. Find the volume.

4. The base of a solid is a circular region in the xy-plane bounded by the graph $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane normal to the x-axis is a semi-circle.

5. The base of a solid is circular region in the xy-plane bounded by the graph of $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane normal to the x-axis is a square with one side as the base.

6. The base of a solid is bounded by $y = 2 - \frac{1}{2}x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the y-axis are semi-circles. Find the volume.

Find the Volumes of Revolution: Disk Method

1. $y = \sqrt{x}, x = 1, x = 4, y = 0$ about the x-axis

2. $y = -x + 1, y = 0, x = 0$ about the x-axis

3. $y = 4 - x^2, y = 0, x = 0$, (in the 1st quadrant) about the x-axis

4. $y = x^2, x = 0, y = 4$, (in the 1st quadrant) about the y-axis

5. $y = \sqrt{4 - x^2}, y = 0, x = 0$, (in the 1st quadrant) about the x-axis

6. $x = 4y - y^2, y = 1, x = 0$, about the y-axis

7. $y = x^{\frac{2}{3}}, y = 1, x = 0$, about the y-axis

8. $y = 5x - x^2, y = 0$, about the x-axis

Find the Volumes of Revolution: Washer Method

1. $f(x) = 2\sqrt{x}\ and\ g(x) = x^2$ about the x-axis

2. $f(x) = 2\sqrt{x}\ and\ g(x) = x^2$ about the y-axis

3. $y = x^2 + 1, y = 1, x = 1,\ x = 0$ about the y-axis

4 $y = \frac{1}{x}, y = 2, and\ x = 2$ about the y-axis

5. $y = x, y = 2 - x^2, and\ x = 0$ about the x-axis

6. $y = x^2\ and\ \ y = 2x$, about the y-axis

7. $y = x^2, and\ y = x + 2$, about the $x - axis$

8. $y = 2x + 2\ and\ y = x^2 + 2$ about the x-axis

Unit 7 Integration Applications Review

Find the average value of the function over the interval:

1. $f(x) = \frac{1}{\sqrt{x-1}}$ $[5,10]$

2. $f(x) = x^3$ $[0,2]$

Find the value of c guaranteed by the Mean Value Theorem:

3. $f(x) = -2x + 1$ $[0,4]$

4. $f(x) = \frac{2}{x^2}$ $[2,4]$

Determine the area of the bounded region:

5. $y = \frac{1}{x^2},\ y = 0,\ x = 1,\ x = 5$

6. $x = y^2 - 2y,\ x = 3$

7. $y = x,\ y = x^3$

8. $x = y^2 + 1,\ x = y + 3$

9. $y = \sin x, y = \cos x, \frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$

10. $y = x^2 + 1,\ y = -x + 7, x = 0$

11. $y = 5x - x^2,\ y = x$

12. $x = y^2 - 4y,\ x = 2y - y^2$

Find the volume by cross sections:

13. The base of a solid is the region enclosed by the circle $x^2 + y^2 = 16$. If cross sections are built up perpendicular to the x-axis, find the volume of the solid created if the cross sections are:

a) squares

b) isosceles right triangles set on the hypotenuse

c) What if cross sections are perpendicular to the y-axis and are semi-circles?

14. The base of a solid is the region between $y = 4 - x^2$, $x=0$, $y=0$. If cross sections are perpendicular to the y-axis and are semicircles, find the volume.

15. Find the volume of the region generated by $y = \sqrt{25 - x^2}$ and the x-axis. The cross sections are perpendicular to the x-axis:

a. Squares b. Isosceles triangles c. Semi Circles

16. Find the volume of the region generated by $y = \frac{1}{\sqrt{x}}$, $x = 0, x = 4, y = 1 \ \& \ y = 3$. The cross sections are perpendicular to the y-axis:

a. Squares b. Isosceles triangles c. Semi Circles

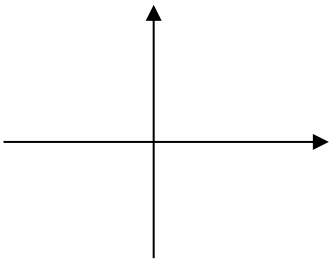
17. Find the volume of the region generated by $y = -\frac{x^2}{9} + 4 \ and \ y = 0$. The cross sections are perpendicular to the x-axis. The cross sections are rectangles with a height twice the base.

Find the volume of the revolution.

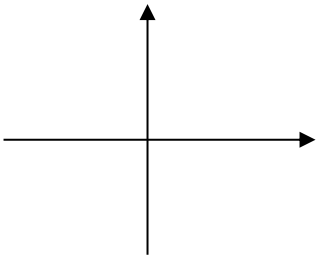
Draw the graph, draw the arbitrary cross section, set up the integral, & find the volume.

18. $y = -\sqrt{x} + 3$, y = 0, x = 0 and x = 2

a. about the x axis.

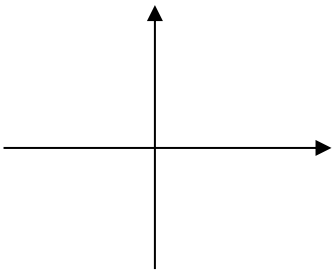


b. about the y-axis:

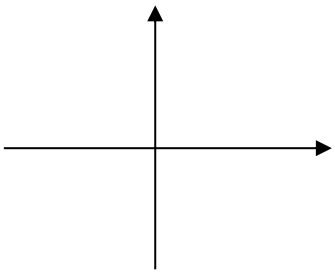


19. $y = x^2,\ x = 0,\ y = 4$

a. about the x-axis.



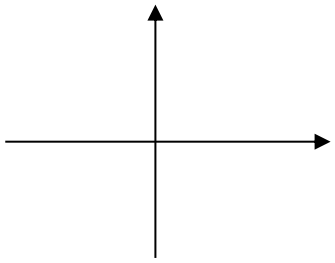
b. about the y-axis.



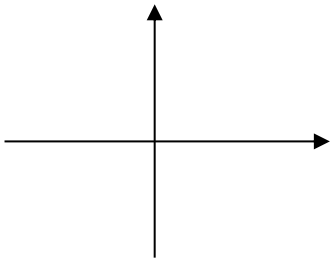
Find the volume of the solid generated by revolving the plane region bounded by the indicated equations:

20. $y = x, y = 0, x = 4$

a. x-axis

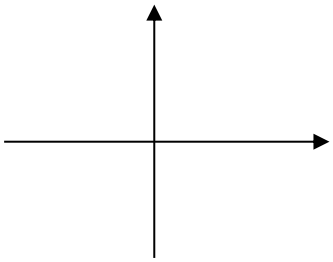


b. y-axis

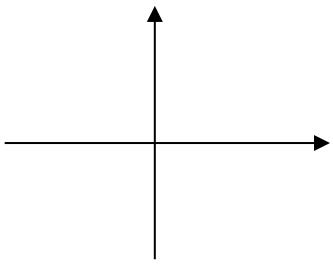


21. $y = \sqrt{x}, y = 2, x = 0$

a. x-axis



b. y-axis



22. $y = \frac{1}{x^4+1},\ y = 0,\ x = 0, x = 1$ about the x-axis