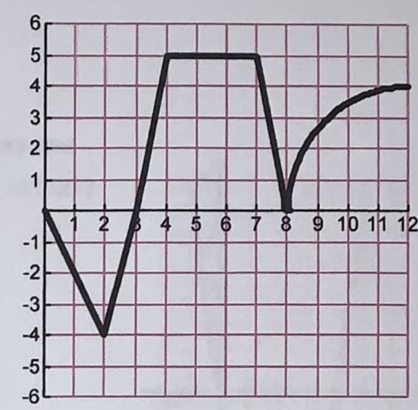
Meaning of Integration - Extra Review

1. What is the meaning of $\int_1^6 R(t)dt$, if R(t) is the rate that tide removes sand from Sandy Point Beach. R(t) has units cubic yards per hour and t is measured in hours with t=0 is number of hours after midnight?

The total cubic yards of sand removed from Sandy Point Beach between 1AM and 6AM.

2. Zach Zalar is pushing a hockey puck across an uneven surface. Below is a graph of the velocity v (in cm/sec as a function of the time t (in seconds).



a. Determine the displacement of the puck in $0 \le t \le 12$ seconds.

$\int_0^{12} v(t)\,dt = -\frac{1}{2}(3)(4) + \frac{1}{2}(5)(3+5) + \frac{1}{4}\pi(4)^2$

$= -6 + 20 + 4\pi$

$= 14 + 4\pi$ cm

b. Determine the total distance traveled by the puck in $0 \le t \le 5$ seconds.

$\int_0^{12} |v(t)|\,dt = \frac{1}{2}(3)(4) + \frac{1}{2}(5)(3+5) + \frac{1}{4}\pi(4)^2$

$= 6 + 20 + 4\pi$

$= 26 + 4\pi$ cm

3. A particle moves along a straight line with acceleration $a(t) = 6t^2 - 4t + 3$. The velocity at t = 1 second is 5 m/sec. Its position at time t = 0 is 3 meters. Find both the velocity function and the position function

$v(t) = 2t^3 - 2t^2 + 3t + C$

$5 = 2 - 2 + 3 + C$

$C = 2$

$v(t) = 2t^3 - 2t^2 + 3t + 2$

$s(t) = \frac{1}{2}t^4 - \frac{2}{3}t^3 + \frac{3}{2}t^2 + 2t + C$

$3 = 0 - 0 + 0 + 0 + C$

$C = 3$

$s(t) = \frac{t^4}{2} - \frac{2t^3}{3} + \frac{3t^2}{2} + 2t + 3$

4. If $\int_3^9 f(x)dx = 14$, $\int_3^9 g(x) = -6$, $\int_5^9 f(x)dx = 8$ and $\int_1^3 f(x)dx = 4$, find the following. a

a. $\int_3^9 [2f(x) + 3g(x)] =$

$2\int_3^9 f(x)\,dx + 3\int_3^9 g(x)\,dx$

$2(14) + 3(-6)$

10

b. $\int_9^3 3g(x)dx =$

$-1 \cdot 3\int_3^9 g(x)\,dx$

$-3(-6)$

18

c. $\int_1^3 (-4f(x) + 2)dx =$

$-4\int_1^3 f(x)\,dx + \int_1^3 2\,dx$

$-4(4) + 2x|_1^3$

$-16 + 6 - 2$

-12

d. $\int_1^5 f(x)dx =$

14, 4, 6, 8

1, 3, 5, 9

$4 + 6$

10

5. Express the limit as a definite integral on the given interval and evaluate.

$$\lim_{n\to\infty} \sum_{i=1}^{n} [3(x_i^*)^2 - 2x_i^*]\Delta x,\ [2, 3]$$

a b

$\int_2^3 3x^2 - 2x\,dx$

6. Express the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} \sec^2\left(\frac{k}{n}\right)$ as an integral and evaluate.

$\int_0^1 \sec^2 x\,dx$

7. Evaluate the definite integrals.

a. $\int_1^{27} \frac{4}{\sqrt[3]{t^2}}\,dt = \int_1^{27} 4t^{-2/3}$

$12t^{1/3}\Big|_1^{27} = 12\sqrt[3]{27}-12$

$36-12=24$

b. $\int_{\pi/4}^{3\pi/2} (\sin x+1)\,dx$

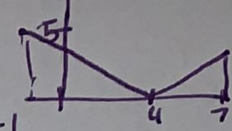
$-\cos x + x \Big|_{\pi/4}^{3\pi/2}$

$-\cos(\frac{3\pi}{2}) + \frac{3\pi}{2} + \cos(\frac{\pi}{4}) - \frac{\pi}{4}$

$0+\frac{3\pi}{2}+\frac{\sqrt{2}}{2}-\frac{\pi}{4}$

$\frac{5\pi+2\sqrt{2}}{4}$

c. $\int_{-1}^{7} |4-x|\,dx$



$\frac{1}{2}(5)(5)+\frac{1}{2}(3)(3)$

$\frac{25}{2}+\frac{9}{2}=\frac{34}{2}$

$=17$

8. Use the Fundamental Theorem of Calculus to simplify.

a. If $g(x)=\int_x^2 \cos^3 t\,dt$, find $g'(x)$.

$-\cos^3 x$

b. Find $\frac{d}{dx}\int_3^{x^3}\sqrt{t^2-4t}\,dt$ $\quad 3x^2\sqrt{x^6-4x^3}$

c. Find $f'(x)$ when $f(x) = \int_{x^2}^{e^{3x}} \frac{1}{\sqrt{1-t^2}}\,dt$

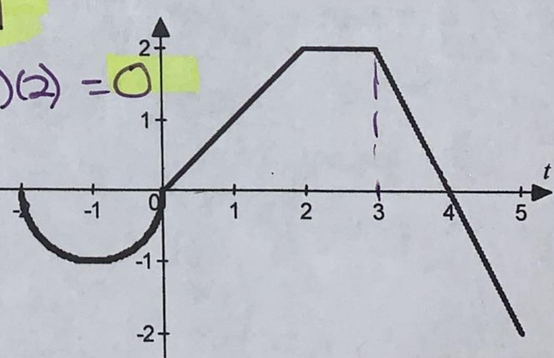
$\frac{3e^{3x}}{\sqrt{1-e^{6x}}} - \frac{2x}{\sqrt{1-x^4}}$

9. Suppose $A(x)=\int_3^x f(t)\,dt$. Use the graph of f shown to answer the following.

a. $A(-1)= \int_3^{-1} f(t)\,dt = -1 \cdot -\frac{1}{4}\pi(1)^2 + \frac{1}{2}\cdot 2(1+3) = \frac{1}{4}\pi+4$ (negative)

b. $A(4)=\int_3^4 f(t)\,dt = \frac{1}{2}(1)(2) = 1$

c. $A(5)=\int_3^5 f(t)\,dt = \frac{1}{2}(1)(2)-\frac{1}{2}(1)(2) = 0$



d. $A'(-1) = -1$

e. $A''(1) = 1$

$\vec{m}$ at $x=1$

f. Identify the value(s) of x for which A has local maxima and/or local minima.

Identify which type of extrema occurs at these x value(s).

local max at $x=0$ A' changes from + to −

local min at $x=4$ bc A' changes from − to +

Calculator Section

10. Find the value of the integral: F(x)= $\int_0^{12} t^2 \cos t\,dt$

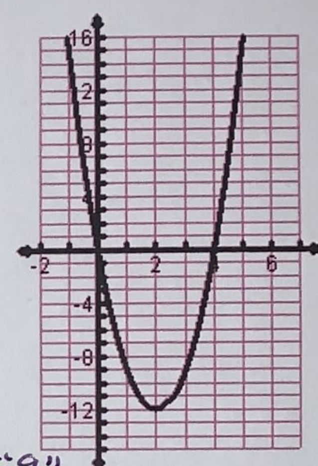
$\frac{t^3}{3}\Big|_0^{12} = 576$

11. Estimate $\int_0^{20} 0.4xe^{0.1x}dx$ using a trapezoidal approximation with 5 subintervals. $\Delta x = \frac{20-0}{5} = 4$

$A \approx \frac{1}{2}\cdot 4\left[f(0)+2f(4)+2f(8)+2f(12)+2f(16)+f(20)\right]$

$A \approx 346.803$

12. Set up and evaluate the integrals needed to determine the following. (You may use your calculator to evaluate the integral—Just remember you must show me your set up.)



A particle travels along a line. Its velocity in meters per second is given by $v(t) = 3t^2 - 12t$. Find:

a) the displacement ,s(t), from t = 0 to t = 5.

$\int_0^5 3t^2 - 12t\,dt = -25$ meters

b) the distance traveled by the particle from t = 0 to t = 5.

$\int_0^5 |3t^2 - 12t|\,dt = 39$ meter

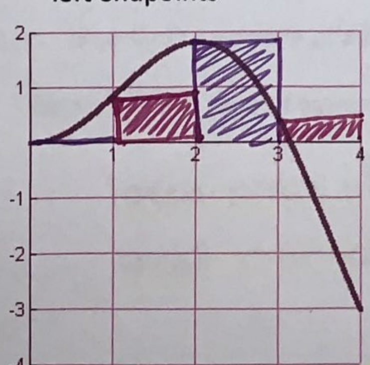
absolute value is in the boxes button next to "9"

13. Oil is leaking out of a ruptured tanker at a rate of $r(t) = -t^{3/2} - 100$ gallons per hour (for $0 \le t \le 8$). If there is initially (at t = 0) 800 gallons of oil in the tanker, write a definite integral expressing the amount of oil that is in the tanker after 5 hours. Then evaluate the integral.

$800 + \int_0^5 (-t^{3/2} - 100)\,dt = 277.693$ gallons of oil

14. Given the graphs below, **shade the rectangles/trapezoids** using 4 subdivisions and estimate the value of the integral $\int_0^4 f(x)dx$ using the method indicated.

left endpoints



$1(0+.8+1.8+.4) \approx 3$

midpoints

$1(.3+1.5+1.5-1.3 \approx 2$