# Review Sheet for Meaning of Integration Test

**Antiderivatives**

1. $\int \frac{3}{(2x)^4} dx$

$\int \frac{3}{16x^4} dx$

$\frac{3}{16}\int x^{-4} dx$

$\frac{3x^{-3}}{16 \cdot -3} + C = \frac{-1}{16x^3} + C$

2. $\int \sec^2 x \, dx$

$\tan x + C$

3. $\int (x^{-1} + \csc x \cdot \cot x) \, dx$

$\ln|x| - \csc x + C$

4. $\int \frac{e}{1+x^2} dx$

$e\int \frac{1}{1+x^2} dx$

$e\tan^{-1} x + C$

5. $\int \frac{1}{\sec^2 x} dx$

omit

6. $\int \frac{5x^3 - 3\sqrt{x}}{\sqrt[3]{x^2}} dx$

$\int \frac{5x^3}{x^{2/3}} - \frac{3x^{1/2}}{x^{2/3}} dx$

$\int 5x^{7/3} - 3x^{-1/6} dx$

$\frac{3}{10} \cdot 5x^{10/3} - 3x^{5/6} \cdot \frac{6}{5} + C$

$\frac{3}{2}x^{10/3} - \frac{18}{5}x^{5/6} + C$

7. A particle moves s that its acceleration is $a(t) = 3t^2 - t - 6$. The velocity at $t = 1$ is $2.5\, m/sec$. Its position at time $t = 0$ is 5 meters. Find both the velocity function and the position function.

$v(t) = \int a(t)\, dt$

$v(t) = \int 3t^2 - t - 6\, dt$

$v(t) = t^3 - \frac{1}{2}t^2 - 6t + C$

$2.5 = (1)^3 - \frac{1}{2}(1)^2 - 6(1) + C$

$2.5 = -5.5 + C$

$C = 8$

$v(t) = t^3 - \frac{1}{2}t^2 - 6t + 8$

$s(t) = \int v(t)\, dt$

$s(t) = \int t^3 - \frac{1}{2}t^2 - 6t + 8\, dt$

$s(t) = \frac{1}{4}t^4 - \frac{1}{6}t^3 - 3t^2 + 8t + C$

$5 = 0 - 0 - 0 + 0 + C$

$5 = C$

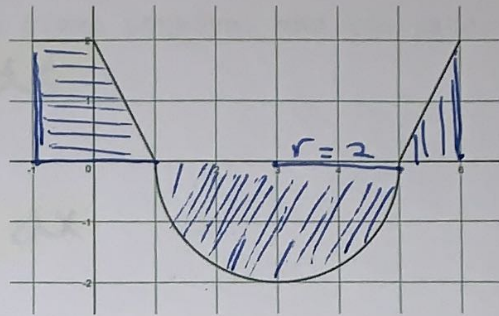
$s(t) = \frac{1}{4}t^4 - \frac{1}{6}t^3 - 3t^2 + 8t + 5$

8. Explain the meaning of $\int_{10}^{60} r(t)\, dt$ if $r(t)$ is the rate of change in the amount of oil in a tanker in gallons per minute measured from $t = 0$ is $8{:}00\ am$

The total gallons of oil in the tanker (entering or leaving) between 8:10 am and 9:00am.

9. a. Find the area under the curve of $f(x)$ from $[-1,6]$

b. Find $\int_1^6 f(x)\, dx$




a. $\int_{-1}^{6} f(x) \approx \frac{1}{2}(2)(1+2) - \frac{1}{2}\pi(2)^2 + \frac{1}{2}(1)(2)$

$\approx 3 - 2\pi + 1$

$\approx 4 - 2\pi$

b. $\int_{1}^{6} f(x)dx \approx -2\pi + 1$

10.

| $x$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | -42 | -37 | -33 | -25 | -22 | -15 | -13 | -9 | -2 |

a. Use a Right Riemann Sum with 4 subintervals to estimate $\int_0^{80} f(x)dx$

$\Delta x = \frac{80-0}{4} = 20$ $\quad A \approx 20(-33-22-13-2)$

$A \approx -1400$

b. Use a midpoint Riemann estimate with 4 subintervals.

$A \approx 20(-37-25-15-9)$

$A \approx -1720$

c. Use a trapezoid estimate with 8 subintervals.

$\Delta x = 10$ $\quad A \approx \frac{1}{2}\cdot 10(-42+2(-37)+2(-33)+2(-25)+2(-22)+2(-15)+2(-13)+2(-9)-2)$

$\approx -1760$

11. Find the following given the integral below:

$$\int_0^2 (x^2 + 2x - 3)dx$$

a. A midpoint Riemann sum with 4 subintervals

$\Delta x = \frac{2}{4} = \frac{1}{2}$ $\quad [0, \frac{1}{2}]\,[\frac{1}{2}, 1]\,[1, \frac{3}{2}]\,[\frac{3}{2}, 2]$

$A \approx \frac{1}{2}\left(f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4})\right] \approx \frac{1}{2}\left(-\frac{39}{16} - \frac{15}{16} + \frac{17}{16} + \frac{57}{16}\right) \approx \frac{5}{8}$

b. A trapezoid sum with 4 subintervals

$A = \frac{1}{2}\cdot\frac{1}{2}\left(f(0) + 2f(\frac{1}{2}) + 2f(1) + 2f(\frac{3}{2}) + f(2)\right)$

$A = \frac{1}{4}\left(-3 + 2(-\frac{7}{4}) + 2(0) + 2(\frac{9}{4}) + 5\right) \approx \frac{3}{4}$

c. The actual value of the integral

$\frac{x^3}{3} + x^2 - 3x \Big|_0^2 = \left(\frac{2^3}{3} + 2^2 - 3(2)\right) - \left(\frac{0^3}{3} + 0^2 - 3(0)\right)$

$\frac{8}{3} + 4 - 6 \approx \frac{2}{3}$

12. Express the limit as a definite integral on the given interval and evaluate.

a. $\lim_{n\to\infty} \sum_{i=0}^{n}[2(x_i^*)^2 - 5(x_i^*)]\Delta x$ on $[0,1]$ $\int_0^1 2x^2 - 5x\,dx = \frac{2x^3}{3} - \frac{5x^2}{2}\Big|_0^1$

$\frac{2}{3} - \frac{5}{2} = \frac{-11}{6}$

b. $\lim_{n\to\infty} \sum_{i=0}^{n}[5(x_i^*)^2 - 2(x_i^*)]\Delta x$ on $[-1,2]$ $\int_{-1}^{2} 5x^2 - 2x\,dx = \frac{5x^3}{3} - x^2\Big|_{-1}^{2}$

$\frac{40}{3} - 4 + \frac{5}{3} + 1 = 12$

c. $\lim_{n\to\infty} \sum_{i=0}^{n}\left[\frac{1}{(x_i^*)}\right]\Delta x$ on $[1,4]$ $\int_1^4 \frac{1}{x}\,dx = \ln|x|\Big|_1^4 = \ln 4 - \ln 1 = \ln 4$

M.C.

13. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}} + \sqrt{\frac{2}{50}} + \sqrt{\frac{3}{50}} + .. + \sqrt{\frac{50}{50}}\right)$ is the Riemann sum approximation for

a. $\int_0^1 \sqrt{\frac{x}{50}}\,dx$ b. $\int_0^1 \sqrt{x}\,dx$ c. $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$ d. $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$ e. $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\sqrt{\frac{k}{n}} \to \int_0^1 \sqrt{x}\,dx$

14. A particle moves with acceleration $a(t) = 5 + 4t - 2t^2\ m/s^2$. Find the velocity function and the position function if $v(0) = 3\ m/s$ and $s(0) = 10\ m$.

$v(t) = 5t + 2t^2 - \frac{2t^3}{3} + C$ $s(t) = -\frac{t^4}{6} + \frac{2t^3}{3} + \frac{5t^2}{2} + 3t + C$

$3 = C$ $10 = C$

$v(t) = -\frac{2t^3}{3} + 2t^2 + 5t + 3$ $s(t) = -\frac{t^4}{6} + \frac{2t^3}{3} + \frac{5t^2}{2} + 3t + 10$

15. Let $f(t)$ be the rate at which oil is being pumped into a holding tank in *gal/min*. Approximate $\int_0^{70} f(t)\,dt$ using (including units)

| $t$ | $f(t)$ |
|---|---|
| 0 | 25 |
| 30 | 35 |
| 40 | 40 |
| 50 | 55 |
| 70 | 60 |

a. Right Riemann sum with 4 subintervals

$[0,30]\ [30,40]\ [40,50]\ [50,70]$

$A \approx 30(35) + 10(40) + 10(55) + 20(60) \approx 3200$ gallons

b. Trapezoids with 4 subintervals

$A = \frac{1}{2}\cdot 30(25+35) + \frac{1}{2}\cdot 10(35+40) + \frac{1}{2}\cdot 10(40+55) + \frac{1}{2}\cdot 20(55+60)$

$\approx 2900$ gallons

c. Using your approximation in part b. If there were originally 3500 gallons of oil in the holding tank, how many gallons are in the tank after 70 minutes?

$3500 + 2900 = 6400$ gallons

16. $\int_6^{10} dx$

$x \Big|_6^{10}$

$10 - 6 = 4$

17. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$

$\frac{x^2}{4} + 3x \Big|_{-2}^{4}$

$\frac{4^2}{4} + 3(4) - \frac{(-2)^2}{4} - 3(-2)$

$4 + 12 - 1 + 6$

$21$

18. $\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos^2 x} dx$

$\int_0^{\pi/4} \sec^2 x \, dx$

$\tan x \Big|_0^{\pi/4}$

$\tan \pi/4 - \tan 0$

$1 - 0$

$1$

19. $\int_{\frac{\sqrt{3}}{3}}^{1} \frac{2}{1+x^2} dx$

$2\tan^{-1} x \Big|_{\frac{\sqrt{3}}{3}}^{1}$

$2\tan^{-1} 1 - 2\tan^{-1} \frac{\sqrt{3}}{3}$

$2\left(\frac{\pi}{4}\right) - 2\left(\frac{\pi}{6}\right)$

$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

20. $\int_8^{27} \frac{dx}{\sqrt[3]{x}} = \int_8^{27} x^{-1/3} dx$

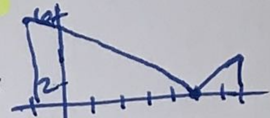
$\frac{3x^{2/3}}{2} \Big|_8^{27}$

$\frac{3}{2}\sqrt[3]{27}^2 - \frac{3}{2}\sqrt[3]{8}^2$

$\frac{27}{2} - \frac{12}{2}$

$\frac{15}{2}$

21. $\int_{-1}^{7} |5 - x| dx$

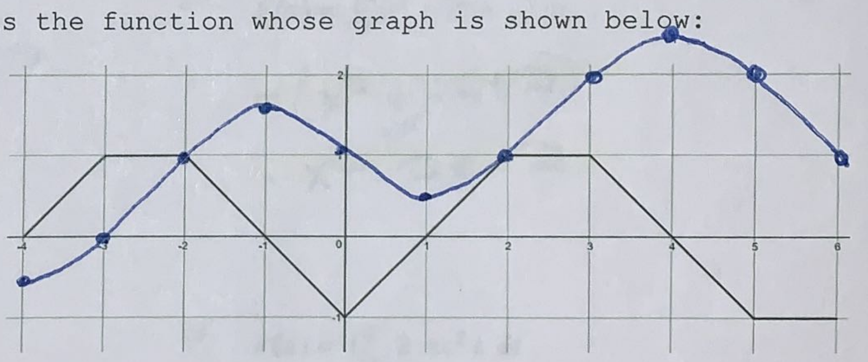


$\frac{1}{2}(6)(6) + \frac{1}{2}(2)(2)$

$18 + 2$

$20$

22. Let $g(x) = \int_{-3}^{x} f(t)dt$, where $f$ is the function whose graph is shown below:



a. Find $g(0)$ $\int_{-3}^{0} f(t)\,dt$

$1 + \frac{1}{2} - \frac{1}{2} = 1$

b. Find $g'(0) = f(0)$

$= -1$

c. At what value(s) of $x$ foes $g$ attain a local maximum and/or local minimum?

$g'(x) = f(x)$

f(x) ↗ ↘ ↗ ↘

g'(x) + | − | + | −

-1, 1, 4

$x = -1, 4$ local max bc $g'(x)$ changes from + to −

$x = 1$ local min bc $g'(x)$ changes from − to +

d. Sketch a graph of $g(x)$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -1/2$

$g(-3) = \int_{-3}^{-3} f(t)\,dt = 0$

$g(-2) = \int_{-3}^{-2} f(t)\,dt = 1$

$g(-1) = \int_{-3}^{-1} f(t)\,dt = 1\frac{1}{2}$

$g(0) = \int_{-3}^{0} f(t)\,dt = 1\frac{1}{2} - \frac{1}{2} = 1$

$g(1) = \int_{-3}^{1} f(t)\,dt = 1 - \frac{1}{2} = \frac{1}{2}$

$g(2) = \int_{-3}^{2} f(t)\,dt = \frac{1}{2} + \frac{1}{2} = 1$

$g(3) = \int_{-3}^{3} f(t)\,dt = 1 + 1 = 2$

$g(4) = \int_{-3}^{4} f(t)\,dt = 2 + \frac{1}{2} = 2.5$

$g(5) = \int_{-3}^{5} f(t)\,dt = 2.5 + -.5 = 2$

$g(6) = \int_{-3}^{6} f(t)\,dt = 2 - 1 = 1$

23. Suppose $\int_1^9 f(x)\,dx = -1$, $\int_7^9 f(x)dx = 5$, $\int_7^9 h(x)dx = 4$ Find the following:

a. $\int_1^9 -2f(x)dx$

$-2\int_1^9 f(x)\,dx$

$-2(-1) = 2$

b. $\int_7^9 [2f(x) - 3h(x)]dx$

$2\int_7^9 f(x)\,dx - 3\int_7^9 h(x)dx$

$2(5) - 3(4)$

$-2$

c. $\int_9^7 h(x)dx$

$-1\int_7^9 h(x)dx$

$-1 \cdot 4$

$-4$

d. $\int_9^9 h(x)dx$

$0$

e. $\int_1^7 f(x)dx$

$\int_1^9 f(x)dx - \int_7^9 f(x)\,dx$

$-1 - 5$

$-6$

24. On a particular day, suppose that $t$ hours after midnight, the outside temperature is changing at a rate $r(t) = 5\cos(0.05t^2 + 2)$ degrees Fahrenheit per hour. If the temperature is 72°F at 10 a.m., what is the temperature at 1 p.m.? Set up and use your calculator to evaluate.

$72 + \int_{10}^{13} r(t)\,dt$

$72 - 6.01223 \approx 65.988°F$

25. The temperature of a tank of water is changing at a rate of $r(t) = e^{\sqrt{t}} - 6$ degrees $F/hour$, where the time $t$ is in hours. Suppose the temperature of the water at $t = 0$ is 56°F. Find the temperature at $t = 4$

$56 + \int_0^4 e^{\sqrt{t}} - 6\,dt \approx 48.778\,°F$

For each problem, find $F'(x)$.

26. $F(x) = \int_x^{x^2} 4(t-2)^{\frac{1}{2}}\,dt$

$4(x^2-2)^{1/2} \cdot 2x - 4(x-2)^{1/2}$

$8x\sqrt{x^2-2} - 4\sqrt{x-2}$

27. $F(x) = \int_x^3 (t^2 + 2t + 2)\,dt$

$-(x^2+2x+2)$

or

$-x^2 - 2x - 2$

28. $F(x) = \int_x^{2x} \frac{5}{t^3}\,dt$

$\frac{2 \cdot 5}{(2x)^3} - \frac{5}{x^3}$

$\frac{10}{8x^3} - \frac{5}{x^3} = \frac{5}{4x^3} - \frac{5}{x^3}$ or $\frac{-15}{4x^3}$

29. $F(x) = \int_{3x^4}^{x^2} 2\csc^2 t\,dt$

$2x \cdot 2\csc^2(x^2) - 12x^3 \cdot 2\csc^2(3x^4)$

$4x\csc^2(x^2) - 24x^3\csc^2(3x^4)$

30. $F(x) = \int_x^{2\cos x} 2(t-2)^{\frac{1}{2}}\,dt$

$-2\sin x\,(2(2\cos x - 2)^{1/2}) - 2(x-2)^{1/2}$

$-4\sin x\sqrt{2\cos x - 2} - 2\sqrt{x-2}$

31. $F(x) = \int_{3x}^{4x^2} 2\cos t\,dt$

$8x \cdot 2\cos(4x^2) - 3 \cdot 2\cos(3x)$

$16x\cos(4x^2) - 6\cos(3x)$

32. $F(x) = \int_x^{x^2} \frac{3}{(t+2)^2}\, dt$

$2x \cdot \frac{3}{(x^2+2)^2} - \frac{3}{(x+2)^2}$

$F'(x) = \frac{6x}{(x^2+2)^2} - \frac{3}{(x+2)^2}$

33. $F(x) = \int_{\sin x}^{\cos x} e^t\, dt$

$F'(x) = -\sin x \cdot e^{\cos x} - \cos x \cdot e^{\sin x}$

34. $F(x) = \int_2^x \sec^2 t\, dt$

$F'(x) = \sec^2 x$

35. $F(x) = \int_2^{x^2} \sqrt{-3t \cdot e^{t-2}}\, dt$

$F'(x) = 2x\sqrt{-3x^2 \cdot e^{x^2-2}}$