## Algebra 2 Unit 5: Logarithms and Exponential Expressions

## ICAN:

- Rewrite expressions from exponential to logarithmic form, and vice versa
- Evaluate logarithms
- Apply properties of logarithms to expand or condense expressions
- Solve equations involving logarithmic expressions
- Solve equations involving exponential expressions
- Apply logarithms to real-world scenarios and solve application problems

SHOWERS...


BRING (hay FLOWERS

*THIS PLAN IS SUBJECT TO CHANGE. PLEASE REFER TO CTLS DIGITAL CLASSROOM FOR UPDATES.*

Let $\boldsymbol{b}$ and $\boldsymbol{a}$ be positive numbers, $b \neq 1$.
The logarithm of $a$ with base $b$ is denoted $\log _{\boldsymbol{b}} \boldsymbol{a}$ and is defined as the exponent $x$ that makes the equation $\boldsymbol{b}^{\boldsymbol{x}}=\boldsymbol{a}$ true.

- $b$ is called the $\qquad$
- $a$ is called the $\qquad$
- $x$ is called the $\qquad$

ex: If $\log _{3} 9=2$, then $3^{2}=9$


Ex 1: Rewrite each logarithmic equation in exponential form.
a. $\log _{2} 8=3$
b. $\log _{3} 729=6$
C. $\log _{5} \frac{1}{25}=-2$

Ex 2: Rewrite each exponential equation in logarithmic form.

a. $3^{3}=27$
b. $15^{3}=3375$
C. $4^{\frac{1}{2}}=2$

Special Logarithms
A logarithm with base 10 is called a $\qquad$ logarithm and can be written without a base. When no base is shown, it is understood base $\qquad$ _.

$$
\text { ex: } \log 1000=3 \text { because } 10^{3}=1000
$$

A logarithm with base $\boldsymbol{e}$ is called a $\qquad$ logarithm and can be written using the notation $\qquad$ . You may choose to write it with base $\qquad$ .

- $e$ represents Euler's number, an irrational constant, similar to $\pi$
- the decimal approximation of $e$ is 2.718

Ex 3: Evaluate each logarithm without using a calculator.
*THINK IT THROUGH: "2 to what power gives me 4?"
*WORK IT OUT: Rewrite in exponential form with $x$ as the exponent.

| a. $\log _{2} 4$ | b. $\log _{3} 81$ | c. $\log 100$ | d. $\log _{8} 8$ |
| :--- | :--- | :--- | :--- |
| e. $\log _{7} 1$ | f. $\log _{2} \frac{1}{2}$ | g. $\log _{9} 3$ | h. $\log _{8} 2$ |
| i. $\log _{x} x$ | j. $\log _{64} 2$ | k. $\log _{5} 5^{3}$ | I. $\log .001$ |
| m. $\ln e^{4}$ | n. $\log _{\frac{1}{3}} 3$ | o. $\log 10$ | p. $\log _{5} 125$ |

You Try:

1. Rewrite in exponential form: $\log _{4} 1024=5$
2. Write a true logarithmic equation (not shown above) using a base of 2 .
3. Fill in the blank to make the equation true.
a. $\log$ $\qquad$ $625=4$
b. $\log _{3} 9=$ $\qquad$
C. $\log _{6}$ $\qquad$ $=2$
d. $\log _{\ldots}=-1 \quad{ }^{*}$ this is a common log
$\qquad$ Period $\qquad$

## Rewrite each equation in exponential form.

1) $\log _{3} 81=4$
2) $\log _{13} 169=2$
3) $\log _{8} 8=1$
4) $\log _{49} \frac{1}{7}=-\frac{1}{2}$
5) $\log _{18} 324=2$
6) $\log _{4} 64=3$
7) $\log _{17} 289=2$
8) $\log _{16} 256=2$
9) $\log _{7} \frac{1}{49}=-2$
10) $\log _{169} \frac{1}{13}=-\frac{1}{2}$

Rewrite each equation in logarithmic form.
11) $18^{2}=324$
12) $18^{-2}=\frac{1}{324}$
13) $20^{-2}=\frac{1}{400}$
14) $5^{3}=125$
15) $19^{-1}=\frac{1}{19}$
16) $5^{4}=625$
17) $14^{2}=196$
18) $4^{-3}=\frac{1}{64}$
19) $15^{2}=225$
20) $3^{3}=27$

Evaluate each expression.
21) $\log _{2} 64$
22) $\log _{4} 64$
23) $\log _{4} 16$
24) $\log _{5} \frac{1}{25}$
25) $\log _{2} 32$
26) $\log _{7} \frac{1}{343}$
27) $\log _{3} 243$
28) $\log _{7} 1$
29) $\log _{5} 25$
30) $\log _{2} 16$


Name:
Date:

## Topic:

Class:

| Main Ideas/Questions | Notes/Examples |  |  |
| :---: | :---: | :---: | :---: |
| Product <br> Property | Condense into a single logarithm. Simplify if possible. |  |  |
|  | 1. $\log _{2} 7+\log _{2} 4$ | 2. $\log 25+\log 4$ | 3. $\log _{4} 2 x+\log _{4} 4 x^{2}$ |
| $\log _{b}(m \cdot n)=$ | Expand using the product property. |  |  |
|  | 4. $\log 6$ | 5. $\log _{7} 45$ | 6. $\log _{2}(5 x)$ |
| Quotient <br> Property | Condense into a single logarithm. Simplify if possible. |  |  |
|  | 7. $\log _{3} 24-\log _{3} 8$ | 8. $\log _{2} 15-\log _{2} 15$ | 9. $\log _{4} x^{9}-\log _{4} x^{2}$ |
| $\log _{b}\left(\frac{m}{n}\right)=$ | Expand using the quotient property. |  |  |
|  | 10. $\log _{8} 4$ | 11. $\log _{5} \frac{1}{3}$ | 12. $\log \left(\frac{m}{7}\right)$ |
| Power Property | Condense into a single logarithm. Simplify if possible. |  |  |
|  | 13. $5 \cdot \log _{4} 2$ | 14. $7 \cdot \log _{2} x$ | 15. $\frac{1}{3} \cdot \log 8$ |
| $\log _{b} m^{n}=$ | Expand using the power property. Simplify if possible. |  |  |
|  | 16. $\log _{2} 8^{7}$ | 17. $3 \cdot \log 4^{x-1}$ | 18. $\log _{7} \sqrt{w}$ |

## Putting it All Together!

|  | Directions: Rewrite as a single logarithm. Simplify if possible. |  |
| :--- | :--- | :--- |
| 19. $2 \cdot \log 6-\log 9$ | 20. $4 \cdot \log _{4} a+2 \cdot \log _{4} b$ |  |
|  | 21. $7 \cdot \log _{4} u-3 \cdot \log _{4} v^{2}$ |  |
| 23. $\log _{3} 4+\log _{3} y+\frac{1}{2} \cdot \log _{3} 49$ | 22. $\log _{2} 15+\log _{2} 4-\log _{2} 6$ |  |

$\qquad$ Period $\qquad$

## Expand each logarithm.

1) $\log _{9}\left(x^{6} \cdot y\right)^{5}$
2) $\log _{9}\left(c^{6} \sqrt[3]{a}\right)$
3) $\log _{4}\left(x \cdot y \cdot z^{3}\right)$
4) $\log _{5}\left(3^{2} \sqrt[3]{10}\right)$
5) $\log _{9}\left(11^{6} \sqrt{2}\right)$
6) $\log _{5}\left(u v^{2}\right)^{4}$
7) $\log _{6}\left(x^{3} y^{6}\right)$
8) $\log _{8} \frac{5^{3}}{7^{6}}$
9) $\log _{9}\left(x \cdot y \cdot z^{3}\right)$
10) $\log _{4}\left(\frac{7}{10^{3}}\right)^{6}$
11) $\log _{4}\left(11^{4} \sqrt[3]{3}\right)$
12) $\log _{9}\left(\frac{3}{2^{5}}\right)^{5}$
13) $\log _{5}\left(u^{6} \cdot v\right)^{5}$
14) $\log _{3}\left(\frac{8}{7^{5}}\right)^{3}$
15) $\log _{6} \sqrt[3]{u \cdot v \cdot w}$
16) $\log _{3}\left(5 \cdot 6^{2}\right)^{3}$

## Condense each expression to a single logarithm.

17) $6 \log _{6} a+2 \log _{6} b$
18) $\frac{\log _{5} 7}{2}+\frac{\log _{5} 10}{2}+\frac{\log _{5} 3}{2}$
19) $12 \log _{3} x+4 \log _{3} y$
20) $4 \log _{7} a+24 \log _{7} b$
21) $5 \log _{5} 7+\frac{\log _{5} 12}{3}$
22) $4 \log _{3} z+\frac{\log _{3} x}{2}$
23) $15 \log _{5} 8+5 \log _{5} 7$
24) $\log _{8} 11+\frac{\log _{8} 7}{2}+\frac{\log _{8} 12}{2}$
25) $\log _{5} x+\log _{5} y+5 \log _{5} z$
26) $\log _{6} x+\log _{6} y+4 \log _{6} z$
27) $\frac{\log _{6} 7}{2}+\frac{\log _{6} 8}{2}+\frac{\log _{6} 3}{2}$
28) $\log _{7} 5+\log _{7} 8+6 \log _{7} 3$
29) $\ln a+\ln b+4 \ln c$
30) $3 \log _{4} 12+15 \log _{4} 5$
31) $4 \log _{4} w+\frac{\log _{4} u}{2}$
*Recall that in $\log _{b} a=x$, both $\boldsymbol{a}$ and $\boldsymbol{b}$ must be positive numbers, and $b \neq 1$.
If a solution to a log equation causes a negative base or argument, it is extraneous!

$$
\angle O G=\angle O G
$$

Each term in the equation is a logarithmic expression.

One-to-One Property:
If $\log _{b} m=\log _{b} n$, then $\qquad$ $=$ $\qquad$ B $\qquad$ , O $\qquad$ B $\qquad$
Step 1: Condense and isolate the log Step 2: BOB it!

Step 3: Solve
Step 4: Check for extraneous solutions.
b. $\log _{2}(x-4)=6$
C. $\log _{9}(6-3 w)=\log _{9}(-2 w)$
d. $\log _{4}(4 x+8)-7=-5$

| e. $\log _{3}(2 p-5)=\log _{3} 6-\log _{3} 2$ | f. $2=\log _{6}(x+9)+\log _{6} x$ |
| :--- | :--- |
|  |  |

## Solving Log Equations Practice

Date $\qquad$ Block $\qquad$

## Solve each equation.

1) $\log _{4}(5 a+4)=\log _{4}(3 a+8)$
2) $\log _{15}(5 a+8)=\log _{15}-3 a$
3) $\log _{6}(4-k)=\log _{6}(k+3)$
4) $\log _{19} 5 k=\log _{19}(3 k+10)$
5) $\log _{6} k=4$
6) $\log _{2} n=0$
7) $\log _{8}(x+2)=-2$
8) $\log _{8}-6 x=1$
9) $\log _{5}-9 r+5=9$
10) $5 \log _{8}(m-6)=-5$
11) $4 \log _{9}(-9 v-5)=12$
12) $\log _{12}(8 n+10)-9=-7$
13) $3+\log _{3}(-10 n+8)=1$
14) $-9 \log _{12}(9 k-2)=-36$
15) $-2 \log _{2}(-2 n-4)-6=-8$
16) $1+5 \log _{12}(9-3 p)=16$
17) $\log _{8} 4+\log _{8}(x-9)=2$
18) $\log _{9}(x-5)-\log _{9} x=2$
19) $\log _{6} 2 x^{2}+\log _{6} 8=4$
20) $\ln (x+5)-\ln x=1$
21) $\log _{4}(-5 x-8)-\log _{4} 3=3$
22) $\log _{8}\left(4 x^{2}-4\right)-\log _{8} 5=2$
23) $\log _{9} 6-\log _{9}(4 x-2)=1$
24) $\ln (5 x+5)-\ln 8=3$
25) $\ln 3+\ln \left(3 x^{2}+5\right)=\ln 15$
26) $\log (x-3)-\log (x-5)=1$

Any logarithm can be rewritten or evaluated using the Change of Base Formula: $\log _{b} a=\frac{\log a}{\log }$ ex: $\log _{3} 7$

| Strategy 1: <br> Make the bases match... | Strategy 2: <br> The bases cannot match... |
| :---: | :---: |
| Property of Equality of Exponential Functions <br> If $b^{x}=b^{y}$, then $\qquad$ = $\qquad$ | BOB it! Rewrite in logarithmic form: <br> B $\qquad$ , O $\qquad$ , B $\qquad$ |
| Step 1: Rewrite each exponential expression as a power with a common base. <br> Step 2: Set the exponents equal <br> Step 3: Solve <br> Step 4: Check | Step 1: Isolate the exponential expression <br> Step 2: BOB it! <br> Step 3: Solve <br> Step 4 Check |
| a. $2^{x+1}=2^{9}$ | b. $7^{x}=20$ |


| e. $7^{4 x+11}=\frac{1}{7}$ | f. $4^{3 w}-5=3$ |
| :--- | :--- |
|  |  |

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## Solving Exponential Equations Practice

Date $\qquad$ Block $\qquad$

## Solve each equation.

1) $6^{-3 p-1}=6^{-p}$
2) $4^{-x}=4^{x+2}$
3) $8^{-r-3}=1$
4) $\left(\frac{1}{6}\right)^{v+2}=36$
5) $36^{2-2 n}=216^{2 n+1}$
6) $6^{-2 p}=6^{-2 p}$
7) $5^{2-2 n}=125$
8) $6^{-3 m+1} \cdot 6^{3 m}=6^{2}$
9) $16 \cdot 4^{-3 a}=4$
10) $\frac{4^{3}}{64^{2 b}}=4^{2}$
11) $\left(\frac{1}{6}\right)^{2 k+3} \cdot 36^{3 k}=36$
12) $\frac{1}{81} \cdot 81^{-k}=\frac{1}{3}$
13) $15^{a}=8$
14) $6^{n}=79$
15) $3^{n-3}=56$
16) $8^{x+6}=32$
17) $13^{2 x}=56$
18) $-4 \cdot 10^{x+2}=-3$
19) $5 \cdot 10^{a+3}=65$
20) $20^{r+3}-3=46$
21) $e^{-2 b-5}-9=74$
22) $-6^{10 b-3}=0$
23) $10^{5 p+7}-2=34$
24) $2 \cdot 4^{2 n-2}=8$
25) $-7 e^{6-7 a}+7=-68$
26) $7 \cdot 12^{6 x-9}-2=56$
27) $-7 \cdot 10^{4 n+8}-8=-8$


Name: $\qquad$
Date: $\qquad$ Bell: $\qquad$ Unit 7: Exponential \& Logarithmic Functions $\qquad$

## ** This is a 2-page document! **

Directions: Solve each equation. Check all answers for extraneous solutions.

1. $\log _{4}(25-2 x)=\log _{4}(6 x+1) \quad$ 2. $\log _{9}(8 y-9)=\log _{9} 108-\log _{9} 4$
2. $6 \cdot \log _{2} 2=\log _{2} 8+\log _{2}(a-2)$
3. $\log _{6}(5 w+14)=2 \cdot \log _{6} w$
4. $\log _{27}(11-2 k)=\frac{1}{3}$
5. $\log (24 x+64)=3$
6. $5=\log _{3} 8+\log _{3}(r+6)$

| 9. $5^{x-4}=25^{x-6}$ | 10. $36^{5 v+2}=\left(\frac{1}{6}\right)^{11-v}$ |
| :--- | :--- |


| Name: |  | Date: |
| :---: | :---: | :---: |
| Topic: |  | Class: |
| Main Ideas/Questions | Notes/Examples |  |
| Exponential Growth | Occurs when a quantity exponentially increases over time. |  |
|  | Formula: | $a=$ $r=$ $t=$ |
| Examples | 1. The original value of an investment is $\$ 1,800$. If the value has increased by $7 \%$ each year, write an exponential function to model the situation. Then, find the value of the investment after 15 years. |  |
|  | 2. In 2002, there were 9 then, the number of s exponential function to enrolled in 2014. | olled at Oakview High School. Since reased by $1.5 \%$ each year. Write an uation, then find the number of students |
| Exponential Decay | Occurs when a quantity exponentially decreases over time. |  |
|  | Formula: | $a=$ $r=$ $t=$ |
| Examples | 3. An investment of $\$ 12,000$ is losing value at a rate of $4 \%$ each year. Write an exponential function to model the situation, then find the value of the investment after 9 years. |  |


| Compound Interest | Occurs when interest is calculated on both the principal amount AND the accrued interest thus far. |
| :---: | :---: |
|  | Formula: $\begin{aligned} & P= \\ & r= \\ & n= \\ & t= \end{aligned}$ |
| Examples | 5. Laura deposited $\$ 12,000$ into an account that earns $8 \%$ interest. How much money will she have in 5 years if the interest is compounded quarterly? |
|  | 6. Jack took out a 6 -year loan for $\$ 25,000$ to purchase a boat at a $4.5 \%$ interest rate. If the interest is compounded monthly, what wil he have paid total over the course of the loan? |
|  | 7. An investment account pays $3.9 \%$ interest compounded semi-annually. If $\$ 4,000$ is invested in this account, what will be the balance after 12 years? |
|  | 8. A savings account offers $0.8 \%$ interest compounded bimonthly. If Bob deposited $\$ 300$ into this account, how much interest will he earn after 10 years? |
|  | 9. Suppose you invest $\$ 750$ into an account that pays $3 \%$ interest compounded weekly. How much interest will you have earned after 20 years? |
|  |  |

Name: $\qquad$ Unit 7: Exponential \& Logarithmic Functions $\square$
Date: $\qquad$ Bell: $\qquad$ Homework 10: Applications of Exponential Functions
** This is a 2-page document! **

## Exponential Growth \& Decay

1. Vanessa invested $\$ 2,500$ into an account that will increase in value by $3.5 \%$ each year. Write an exponential function to model this situation, then find the value of the investment after 20 years.
2. The average price of a movie ticket in 1990 was $\$ 4.22$. Since then, the price has increased by approximately $3.1 \%$ each year. Write an exponential function to model this situation, then find the price of a ticket in 2016.
3. A virus has infected 400 people in the town and is spreading to $25 \%$ more people each day. Write an exponential function to model this situation, then find the number of people that will be infected in 10 days.
4. The population of a small town was 10,800 in 2002. Since then, the population has decreased at a rate of $2.5 \%$ each year. Write an exponential function to model the situation, then find the population of the town in 2020.
5. Manny bought a brand new car in 2012 for $\$ 28,750$. If the car depreciates by $12 \%$ each year, write an exponential function to model the situation, then find the value of the car in 2018.

In 1956, scientists B. Gutenberg and C. F. Richter developed a formula to estimate the amount of energy released in an earthquake. $E$ represents the amount of energy, in ergs, released from an earthquake. The magnitude of an earthquake is given by the formula:

$$
M=\frac{\log E-11.4}{1.5}
$$

where $E$ is ergs, the energy released from an earthquake.
Richter Scale
1 only detectable by seismograph
2 hanging lamps sway
3 can be felt
4 glass breaks, buildings shiver
5 furniture falls
6 wooden houses damaged
7 buildings fall
8 catastrophic damage
Analyze:
a. The base of the logarithm in the formula is $\qquad$ .
b. Because it is a $\qquad$ logarithm, an earthquake with a rating of 7 is how much stronger than one with a rating of 4 ?
c. How much stronger is an earthquake with a rating of 8 than one with a rating of 2?

Ex 1: On April 13, 1985, the energy released by the earthquake in Indonesia was $3.981 \times 1921$ ergs. What did it measure on the Richter Scale?

Ex 2: On September 19, 1985, the energy released by the earthquake in Mexico was 1.259 x $10^{23}$ ergs. What did it measure on the Richter Scale?

Ex 3: How much more powerful was the earthquake in Mexico than the one in Indonesia?

Ex 4: The relationship between a telescope's limiting magnitude (the apparent magnitude of the dimmest star that can be seen with the telescope) and the diameter of the telescope's objective lens or mirror can be modeled by

$$
M=5 \log D+2
$$

where $M$ is the limiting magnitude and $D$ is the diameter (in millimeters) of the lens mirror. If a telescope can reveal stars with a magnitude of 12, what is the diameter of its objective lens?

Ex 5: An altimeter is an instrument that finds the height above sea level by measuring air pressure. The height and the air pressure are related by the model

$$
h=-8005 \ln \frac{P}{101,300}
$$

where $h$ is the height (in meters) above sea level and $P$ is the air pressure (in pascals). What is the air pressure when the height is 4000 meters above sea level?

Ex 6: The loudness $L$ of a sound (in decibels) is related to the intensity $I$ of the sound (in watts per square meter) by the equation

$$
L=10 \log \frac{I}{I_{0}}
$$

where $I_{0}$ is and intensity of $10^{-1}$ watt per square meter, corresponding roughly to the faintest sound that can be heard by humans.
a. What is the intensity of the sound of fans cheering in Arrowhead Stadium if the noise level measures 142.2 decibels?
b. If the decibel level of the sound of a soft whisper is 30 , what is the intensity of the sound?

## PROPERTIES OF LOGS

| Clame | Rule(s) | Example 1 | Example 2 |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { BASIC } \\ \text { LOGARITHMS } \end{gathered}$ | $\log _{b} b=\ldots \ldots \log _{b} 1=$ | Simplify $\log _{5} 5$ | Simplify $\log _{8} 1$ |
| $\begin{aligned} & \text { PRODUCT } \\ & \text { RULE } \end{aligned}$ | $\log _{b} m \cdot n=\square$ | Expand $\log _{2} 10$ | $\begin{gathered} \text { Condense } \\ \log _{5} 6+\log _{5} 7 \end{gathered}$ |
| QUOTIENT RULE | $\log _{b}\left(\frac{m}{n}\right)=$ | Expand $\log \left(\frac{3}{10}\right)$ | $\begin{gathered} \text { Condense } \\ \log _{4} 48-\log _{4} 12 \end{gathered}$ |
| POWER RULE | $\log _{b} m^{n}=$ | Expand $\log _{3} 5^{2}$ | Condense $(x-1) \log _{5} 7$ |
| CHANGE OF BASE FORMULA | $\log _{b} a=\square$ | Use a common base to evaluate $\log _{7} 3$ |  |
| Remember: | Common Logs are base10 and are often written with no base shown. Natural Logs are base $\boldsymbol{e}$ and are denoted using $\ln$ |  |  |

## Unit 5 Practice Test

Rewrite each equation in exponential form.

1) $\log _{9} 81=2$
2) $\log _{256} 16=\frac{1}{2}$

Rewrite each equation in logarithmic form.
3) $18^{2}=324$
4) $125^{\frac{1}{3}}=5$

Evaluate each expression.
5) $\log _{16} 4$
6) $\log _{3} \frac{1}{27}$

Condense each expression to a single logarithm.
7) $8 \log _{6} x-2 \log _{6} y$
8) $\log _{7} a+\log _{7} b+5 \log _{7} c$

Expand each logarithm.
9) $\log _{2}\left(z^{2} \sqrt{x}\right)$
10) $\log _{7}\left(\frac{x}{y^{4}}\right)^{5}$

Solve each equation.
11) $\log _{3}(x+5)-\log _{3} 7=\log _{3} 18$
12) $\log _{9} x+\log _{9}(x+24)=2$
13) $625^{-3 x}=25$
14) $2^{2 v}=2^{3 v-2}$

Solve each equation. Round your answers to the nearest ten-thousandth.
15) $7^{x-10}=80.9$
16) $10 \cdot 9^{5.8 p+5}-2=48$

The wind speed $s$ (in miles per hour) near the center of a tornado can be modeled by $s=93 \log d+65$
where $d$ is the distance (in miles) that the tornado travels.
A. In April 1947, a tornado traveled over 125 miles, from Texas to Oklahoma. Estimate the wind speed of the tornado.
B. In May 2011, a tornado stuck Joplin, Missouri, with wind speeds up to200 miles per hour. Determine the distance that the tornado traveled.

$$
A=P\left(1+\frac{r}{n}\right)^{n \cdot t}
$$

In 1998, Jana invested $\$ 1500$ in an account with a $4.5 \%$ interest rate compounded monthly. What will be her account balance in 2022?

Exponential Growth: $y=a(1+r)^{t}$ Exponential Decay: $y=a(1-r)^{t}$
A. Jorge purchased an investment property for $\$ 55,000$. Its value is expected to increase by $1.7 \%$ each year. What will be the value of the property in 10 years?
B. Julie bought a car for $\$ 29,500$ in 2019 . It depreciates at a rate of $15 \%$ per year. What will be the value of the car in 2025 ?
C. When will the depreciated value of Julie's car be $\$ 9,000$ ?

