

Rational Expressions: Simplifying, Multiplying and Dividing

A rational expression is a _____ of two polynomials.

To **simplify** a rational expression:

- _____ the numerator and denominator completely.
- Simplify the monomial factor by _____ coefficients & _____ exponents.
- Eliminate all factors common to numerator and denominator.

Ex 1: Simplify

a. $\frac{20x^6}{14x^2}$	b. $\frac{5x^4-20x^3}{15x-60}$
c. $\frac{y+8}{y^2+2y-48}$	d. $\frac{6n^2+8n+2}{2n^2-2}$
e. $\frac{p^2-49}{7-p}$	f. $\frac{45-5w}{3w^2-28w+9}$
<div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: fit-content; margin: 0 auto;"> <p>CAREFUL!</p> $\frac{a-b}{b-a} = -1$ </div>	

To **multiply** rational expressions:

- Factor binomials and trinomials completely.
- Multiply monomials by _____ coefficients and _____ exponents.
- Simplify by eliminating all factors common to numerator and denominator.
- Write the product of the remaining factors in a single rational expression.
- LEAVE THE RESULT IN FACTORED FORM.

Ex 2: Multiply

d. $\frac{6x^2y^3}{2x^2y^2} \cdot \frac{10x^3y^4}{18y^2}$	b. $\frac{2x^2}{x+3} \cdot \frac{2x^2+6x}{12x^5}$
c. $\frac{4x^2-4x}{2x^2+4x-6} \cdot \frac{x^2+x-6}{4x^2+8x}$	d. $\frac{10v-5v^2}{v^2-11v+18} \cdot \frac{v^2-8v-9}{15v}$

To **divide** rational expressions:

- Multiply the first rational expression by the _____ of the second.

Ex 3: Divide

a. $\frac{5pq}{16p^3} \div \frac{3p^2q^2}{8q^5}$	b. $\frac{10}{4x-8} \div \frac{2x^2+6x}{x^2+x-6}$
c. $\frac{2x^3-12x^2}{x^2-7x+12} \div \frac{24x^2-8x^3}{x^2-8x+15}$	d. $\frac{16a^2-1}{a^2} \div \frac{4a+1}{a}$

Simplifying Rational Expressions

Simplify each expression.

1) $-\frac{20n}{90n^2}$

2) $\frac{27x^2}{21x}$

3) $\frac{42n}{48n^2}$

4) $\frac{16x^4}{10x}$

5) $-\frac{8n^4}{8n^3}$

6) $\frac{40x^2}{28x^2 - 4x}$

7) $\frac{18n^2 - 54n}{63n}$

8) $\frac{6 - v}{7v - 42}$

9) $\frac{21k + 63}{42}$

10) $\frac{16}{16a - 64}$

$$11) \frac{x^2 + 17x + 70}{10x + 70}$$

$$12) \frac{8x - 56}{-x^2 + 10x - 21}$$

$$13) \frac{n^2 + 7n - 30}{n^2 + 7n - 30}$$

$$14) \frac{63k + 18}{18k + 81}$$

$$15) \frac{n^2 + 11n + 24}{n^2 + 5n - 24}$$

$$16) \frac{3m^3 - 12m^2 - 180m}{m^2 - m - 90}$$

$$17) \frac{x^3 - x^2 - 6x}{6x - 18}$$

$$18) \frac{2n^2 + 10n + 8}{3n^2 + 12n}$$

$$19) \frac{70p^3 + 170p^2 + 60p}{70p^2 - 30p}$$

$$20) \frac{9x^2 - 36x + 36}{3x^3 - 33x^2 + 54x}$$

Mult & Div Rational Expressions

Simplify each expression.

1) $\frac{10v}{9v} \cdot \frac{10v}{8}$

2) $\frac{7}{4n} \cdot \frac{2}{10}$

3) $\frac{8}{2x} \cdot \frac{10}{4}$

4) $\frac{(m-2)(m-6)}{m-6} \cdot \frac{m+9}{5m(m+9)}$

5) $\frac{n-1}{14(n-1)} \cdot \frac{16n(n-1)}{8n(n+7)}$

6) $\frac{n+9}{7n(n+7)} \cdot \frac{7n(n+7)}{5}$

7) $\frac{1}{a-3} \cdot \frac{7a-21}{a-7}$

8) $\frac{10x}{7} \cdot \frac{7x-70}{x^2-6x-40}$

9) $\frac{v+6}{v^2-6v-16} \cdot \frac{v+10}{v+6}$

10) $\frac{3x^2+3x}{3x} \cdot \frac{3x+24}{5x+5}$

11) $\frac{5}{15b-35} \cdot \frac{42-18b}{b^2+6b+5}$

12) $\frac{k^2+13k+40}{k^2-k-72} \cdot \frac{36+5k-k^2}{k^2-k-30}$

13) $\frac{4}{10} \div \frac{2n}{9}$

14) $\frac{7}{10} \div \frac{4x^3}{2}$

15) $\frac{9n}{3} \div \frac{3n^2}{8n}$

16) $\frac{3}{2(7n-6)} \div \frac{9}{9(7n-6)}$

17) $\frac{(3-x)(2+x)}{(x-5)(x-3)} \div \frac{x+2}{(x+5)(x-5)}$

18) $\frac{(v-2)^2}{5(v-2)} \div \frac{1}{v+9}$

19) $\frac{n^2 + 11n + 24}{n+2} \div \frac{n^2 + 11n + 24}{8}$

20) $\frac{n+1}{n^2 + 9n - 10} \div \frac{1}{n-1}$

21) $\frac{x-6}{4x-24} \div \frac{1}{x+5}$

22) $\frac{9r+24}{r^2 + 11r + 28} \div \frac{30r+80}{r^2 + 11r + 28}$

23) $\frac{3n^2}{n^2 + 7n + 10} \div \frac{5n+10}{n^2 + 7n + 10}$

24) $\frac{4n+40}{4n-20} \div \frac{n^2 + 13n + 40}{n^2 - 25}$

Adding and Subtracting Rational Expressions

Given LIKE Denominators	
<p>To ADD:</p> <ul style="list-style-type: none"> • Add the numerators • Write the sum over the common denominator • Simplify the result <p>Ex 1: Add $\frac{13a}{4a^2} + \frac{5a}{4a^2}$</p>	<p>To SUBTRACT:</p> <ul style="list-style-type: none"> • Distribute the negative • Add the numerators • Write the sum over the common denominator • Simplify the result <p>Ex 2: Subtract $\frac{9x+12}{16} - \frac{7x+2}{16}$</p>

Ex 3: Perform the indicated operation

a. $\frac{y^2+8y}{y^2-9} + \frac{2y+21}{y^2-9}$	b. $\frac{n^2+3n-7}{n^2+n-6} - \frac{n+1}{n^2+n-6}$
c. $\frac{1}{12a-3} - \frac{16a^2}{12a-3}$	d. $\frac{13x^2-9x}{6x^2-5x+1} + \frac{x^2+2x}{6x^2-5x+1}$
YOU TRY: $\frac{7x-3}{6x^2} - \frac{4x-6}{6x^2}$	YOU TRY: $\frac{v^2-8v}{5v^2+9v-2} + \frac{2v-16}{5v^2+9v-2}$

Add & Sub Rational's WITH LIKE DENOMINATORS

Date _____

Simplify each expression.

1) $\frac{a-4b}{15a} + \frac{a+4b}{15a}$

2) $\frac{6x}{30y^4} + \frac{5x}{30y^4}$

3) $\frac{5m+n}{12n} + \frac{m-5n}{12n}$

4) $\frac{a+4b}{10ab^3} + \frac{2a-5b}{10ab^3}$

5) $\frac{u+v}{6uv^2} + \frac{u+2v}{6uv^2}$

6) $\frac{a-2b}{24a^2b} + \frac{a+5b}{24a^2b}$

7) $\frac{6x-4}{3x^2+9x-12} + \frac{x-6}{3x^2+9x-12}$

8) $\frac{6p+4}{6p^2-36p} + \frac{3p}{6p^2-36p}$

9) $\frac{3v}{2v+6} - \frac{v+4}{2v+6}$

10) $\frac{x-6}{8x-16} - \frac{x+4}{8x-16}$

11) $\frac{5n+2}{6n-30} + \frac{n-5}{6n-30}$

12) $\frac{x+4}{5x^2-25x} + \frac{x+3}{5x^2-25x}$

13) $\frac{3x}{24x^2y^4} - \frac{x+4y}{24x^2y^4}$

14) $\frac{x-y}{8y^2} - \frac{x-5y}{8y^2}$

15) $\frac{x-6y}{12x} - \frac{x+6y}{12x}$

16) $\frac{x+6y}{10yx^2} - \frac{5y}{10yx^2}$

17) $\frac{x+6y}{12x^3y} - \frac{x+2y}{12x^3y}$

18) $\frac{x-y}{24xy} - \frac{2x-2y}{24xy}$

19) $\frac{x+1}{5x-30} - \frac{4}{5x-30}$

20) $\frac{x+5}{x^2+5x-6} + \frac{3}{x^2+5x-6}$

21) $\frac{v-5}{5v^2+14v+8} + \frac{6}{5v^2+14v+8}$

22) $\frac{6b+6}{3b^2-7b+2} - \frac{b+6}{3b^2-7b+2}$

23) $\frac{6v+5}{v^2-3v-10} + \frac{5v-1}{v^2-3v-10}$

24) $\frac{4v-3}{18v^3-45v^2} - \frac{v+3}{18v^3-45v^2}$

Adding and Subtracting Rational Expressions

Given **UNLIKE** Denominators

- First, you must find a common denominator!
 - Factor the denominator of each expression
 - Determine the common denominator: the product of every factor from both denominators.
 - Multiply each rational expression by the factors missing from its denominator.
- Then perform the indicated operation, combining the numerators over the common denominator.
- Simplify the result.

Ex 1: Add $\frac{15}{2x} + \frac{4}{3x}$

Ex 2: Subtract $\frac{x^2-16}{5x-10} - \frac{4}{x-2}$

Ex 3: Perform the indicated operation

a. $\frac{4a}{a^2} - \frac{7}{8a}$

b. $\frac{7}{y+8} + \frac{35}{y^2+11y+24}$

c. $\frac{x^2-5}{x^2+5x+6} + \frac{1}{x+2}$

d. $\frac{5}{w+1} + \frac{8}{w-7}$

$$e. \frac{b}{b+3} - \frac{4}{b}$$

$$f. \frac{r}{2r+8} - \frac{8}{r^2+4r}$$

$$g. \frac{m^2+5m+6}{m^2-4} + \frac{2}{m-2}$$

$$h. \frac{2}{4p-20} + \frac{p-6}{p^2-8p+15}$$

Error Analysis: Find the error

$$(x) \frac{3}{4x} - \frac{x-1}{x^2} (4)$$

$$(x) \frac{3}{4x} - \frac{x-1}{x^2} (4)$$

$$\frac{3x - 4x + 4}{4x^2} = \frac{-x + 4}{4x^2} = \frac{-x}{x^2} = \frac{-1}{x}$$

Add & Sub with UNLIKE DENOMINATORS

Simplify each expression.

1) $\frac{5}{5x} + \frac{2x}{6}$

2) $\frac{4}{3} + \frac{6y}{3y}$

3) $\frac{3y}{3x^3} + \frac{6y}{4x}$

4) $\frac{2a}{5b} + \frac{3}{3b}$

5) $\frac{6}{v+4} + \frac{5v}{4}$

6) $\frac{3x}{2x(5x-3)} + \frac{3}{3x}$

7) $\frac{2}{3(3b-5)} + \frac{5b}{3b}$

8) $\frac{2x}{x+3} + \frac{6}{x-3}$

9) $\frac{3}{3k-2} + \frac{2k}{5k^2}$

10) $\frac{b-2}{5} + \frac{4}{b+3}$

11) $\frac{5}{b-6} + \frac{4b}{b+4}$

12) $\frac{3}{n-1} + \frac{2n+6}{n+6}$

$$13) \frac{3u}{4v} - \frac{5u}{4}$$

$$14) \frac{6m}{3m^2n} - \frac{6m}{4m^2}$$

$$15) \frac{3x}{4} - \frac{5y}{2y}$$

$$16) \frac{4}{5xy} - \frac{2x}{6y^2}$$

$$17) \frac{3v}{3v(v+2)} - \frac{4}{2}$$

$$18) \frac{5r}{4} - \frac{4}{r-2}$$

$$19) \frac{6}{2} - \frac{2}{6(m+3)}$$

$$20) \frac{4x}{2(x-1)} - 2$$

$$21) \frac{5n}{5n-6} - \frac{3n}{5n^2}$$

$$22) \frac{5k}{5} + \frac{2k}{2k-5}$$

$$23) \frac{4k}{k-4} - \frac{4}{k+4}$$

$$24) \frac{5x}{4x^3} - \frac{2x}{4x+6}$$

1) $\frac{5}{3x} \cdot \frac{6}{5}$

2) $\frac{6n^2}{n+8} \cdot \frac{n^2+2n-48}{n-6}$

3) $\frac{7p+2}{3p^2-23p+30} \cdot \frac{24p-40}{7p+2}$

4) $\frac{6}{8r^3} \div \frac{7}{9}$

5) $\frac{x-9}{6x^3-54x^2} + \frac{1}{x+4}$

6) $\frac{2n+3}{5n^2+16n+3} + \frac{2n+3}{10n^3+2n^2}$

7) $\frac{x-4y}{18x^5} - \frac{2x-y}{18x^5}$

8) $\frac{r-4}{15r+25} - \frac{r-5}{15r+25}$

9) $\frac{p+2}{2p^2-14p+24} + \frac{p-1}{2p^2-14p+24}$

10) $\frac{5}{4n^3} + \frac{6}{3n^3}$

11) $\frac{3}{3a} - \frac{a-6}{3a-15}$

12) $\frac{4}{n+5} + \frac{6}{6n-12}$

A	$\frac{x+4}{6x^2}$	H	$\frac{2n^2}{n+3}$
B	$6n^2$	I	$\frac{2p+1}{2p^2-14p+24}$
C	$\frac{4}{3a}$	J	$\frac{-x-3y}{18x^5}$
D	$\frac{2}{x}$	K	$\frac{5+4a^2-24a}{3a(a-6)}$
E	$\frac{13}{4n^3}$	L	$\frac{1}{15r+25}$
F	$\frac{8}{p-6}$	M	$\frac{9a-15-a^2}{3a(a-5)}$
G	$\frac{27}{28r^3}$	N	$\frac{5n-3}{(n-2)(n+5)}$

D	F	D	F	B	C	G	B	D	B	F	D
F	D	B	F	K	C	C	G	F	D	B	F
B	B	D	B	A	K	C	C	K	G	D	B
D	F	F	A	C	K	C	K	K	C	G	D
F	B	A	C	K	C	G	H	K	C	J	G
B	G	C	K	C	K	L	G	H	J	A	I
G	H	K	K	C	C	K	I	M	N	L	K
H	C	K	K	C	C	K	E	J	H	E	C
B	A	K	C	K	C	K	J	F	D	H	K
D	J	C	K	C	K	C	F	D	B	F	H
F	B	H	C	C	K	C	G	B	A	K	J
D	D	F	H	K	K	C	C	K	C	J	D

Quiz Review: Simplifying and Operating with Rational Expressions

Simplify completely. SHOW ALL WORK!!!

1.
$$\frac{xy + 3y}{3y}$$

2.
$$\frac{x^2 - 10x + 25}{x - 5}$$

3.
$$\frac{-2x - 12}{3x^2 + 18x}$$

4.
$$\frac{x^2 + 4x + 3}{18 + 3x - x^2}$$

Multiply or divide. Simplify completely. SHOW ALL WORK!!!

5.
$$\frac{\frac{r}{s} \cdot \frac{s}{r}}$$

6.
$$\frac{a+2}{15a^2} \cdot \frac{3a^3}{a^2-4}$$

7.
$$\frac{x^2-1}{x^2-x-2} \cdot \frac{x^2-4}{x^2-3x+2}$$

8.
$$\frac{15a}{3b-a} \div \frac{5a^3}{3b-a}$$

9.
$$\frac{n^2 + 8n + 16}{n^2 + 4n} \div (n^2 - 16)$$

10.
$$\frac{9 - m^2}{m^5} \div \frac{m + 3}{m}$$

Add or subtract. Simplify completely. SHOW ALL WORK!!!

$$11. \quad \frac{12x}{5} + \frac{9x}{5} - \frac{x}{5}$$

$$12. \quad \frac{4y-15}{y-2} + \frac{2y-3}{y-2}$$

$$13. \quad \frac{7x+9}{5x-1} - \frac{2x+10}{5x-1}$$

$$14. \quad \frac{r}{4y} - \frac{s}{5y}$$

$$15. \quad \frac{1}{x+1} + \frac{2x}{3(x+1)}$$

$$16. \quad \frac{m}{3m-9} - \frac{2m}{12m-36}$$

$$17. \quad \frac{3x}{x^2-1} - \frac{x}{x-1}$$

$$18. \quad \frac{x+2}{x-1} - \frac{2}{x+6} - \frac{14}{x^2+5x-6}$$

Solving Rational Equations

When the rational equation is a proportion with one rational expression on each side:

- cross multiply and set the products equal
- solve
- check for extraneous solutions!

The denominator
of a fraction
cannot = 0!

Ex 1: $\frac{7}{x-6} = \frac{4}{x}$

Ex 2: $\frac{3}{a-8} = \frac{7}{2a+1}$

Ex 3: $\frac{r}{r-1} = \frac{4}{r}$

Ex 4: $\frac{m+3}{3} = \frac{8}{m-2}$

Ex 5: $\frac{y}{3} = \frac{y+8}{y+5}$

Ex 6: $\frac{2x-3}{2} = \frac{3}{x+4}$

Homework Solve by Cross Multiplication

Date _____ Period _____

Solve each equation. Remember to check for extraneous solutions.

1) $\frac{x}{9} = \frac{7}{3}$

2) $\frac{x-5}{15} = \frac{4}{5}$

3) $\frac{3x+1}{x-1} = 5$

4) $\frac{x-6}{3} = \frac{-2x-2}{15}$

5) $\frac{6}{3x-1} = \frac{x}{4}$

6) $\frac{5}{x} = \frac{x-4}{7}$

7) $\frac{x+3}{x+1} = \frac{15}{x+7}$

8) $\frac{x-1}{7} = \frac{2x-2}{3x-1}$

Solving Rational Equations

- Factor the denominators and determine the lowest common denominator.
- Multiply each side of the equation by the LCD to eliminate all denominators.
- Solve the resulting equation.
- Check for extraneous solutions!

Ex 1: $\frac{4}{3} - \frac{7}{n} = \frac{1}{6}$

Ex 2: $\frac{3}{k} - \frac{1}{2} = \frac{12}{k}$

Ex 3: $\frac{p}{p-2} + 2 = \frac{8}{p^2-4}$

Ex 4: $\frac{13}{3x-3} - \frac{1}{x-1} = \frac{x}{9}$

Ex 5: $\frac{x+6}{x+3} = 2 - \frac{5x+12}{x+3}$

Ex 6: $\frac{3y-2}{y^2-4y} = \frac{y}{y-4} - 1$

Solving by all Methods

Date _____ Period _____

Solve each equation. Remember to check for extraneous solutions.

1) $\frac{x+1}{x} = \frac{3x+3}{4x-2}$

2) $\frac{x+2}{x-2} = \frac{2x+4}{x+1}$

3) $\frac{b-5}{3b} + \frac{4}{b} = \frac{1}{b}$

4) $\frac{p-1}{4p^2} = \frac{5p+6}{4p^2} + \frac{1}{p^2}$

5) $\frac{3}{r^2} + \frac{1}{r} = \frac{2}{3r}$

6) $\frac{1}{5m^2} + \frac{1}{5m} = \frac{1}{m^2}$

7) $\frac{1}{4m^2 + 34m + 16} - \frac{1}{4m + 2} = \frac{1}{2m^2 + 17m + 8}$

8) $\frac{1}{k} = \frac{7}{4k+6} + \frac{5k+5}{4k^2+6k}$

$$9) \frac{3}{m-5} + \frac{m+2}{m^2-5m} = \frac{1}{m-5}$$

$$10) \frac{5}{m+6} - \frac{1}{m^2+4m-12} = \frac{m-7}{m^2+4m-12}$$

$$11) \frac{1}{3x+9} + \frac{1}{x^2+3x} = \frac{x-7}{3x}$$

$$12) \frac{4r-16}{r} = \frac{r-6}{6r+4} + \frac{5r-10}{r}$$

$$13) 1 = \frac{1}{x} - \frac{1}{x^2-3x}$$

$$14) 1 + \frac{1}{x^2-6x-16} = \frac{2x^2-10x-12}{x^2-6x-16}$$

Unit 4A Test Review

Date _____ Period _____

Simplify each expression.

1) $\frac{8x^3}{16x^2}$

2) $\frac{10a+8}{8}$

3) $\frac{5x^2+7x-6}{3x^2+8x+4}$

4) $\frac{1}{7r+7} \div \frac{r-3}{28r^3+28r^2}$

5) $\frac{1}{7a+8} \cdot \frac{21a^3+24a^2}{8a+8}$

6) $\frac{3n-6}{3n^2+13n-10} \cdot \frac{9n^2-15n+6}{3n-3}$

7) $\frac{6n+2}{15n^2+14n+3} \div \frac{n+2}{15n+9}$

8) $\frac{a+2}{5a^2-12a-9} - \frac{a-1}{5a^2-12a-9}$

9)
$$\frac{4n+6}{6n-12} - \frac{n+3}{6n-12}$$

10)
$$\frac{3}{v-3} + \frac{5v}{v-1}$$

11)
$$\frac{4}{5a-1} - \frac{4a}{a-4}$$

12)
$$\frac{3p}{5p-4} - \frac{2p}{p-3}$$

Solve each equation. Remember to check for extraneous solutions.

13)
$$\frac{6}{x} = 1 - \frac{1}{x}$$

14)
$$\frac{2b}{5} = \frac{b^2 - 5b}{5b}$$

15)
$$\frac{x-5}{x^2+3x} = \frac{1}{x} - \frac{2}{x+3}$$

16)
$$\frac{n-5}{n} = \frac{1}{n^2-2n} + \frac{n^2-4n-5}{n^2-2n}$$

17)
$$\frac{6}{9} = \frac{v-5}{6}$$