AP Calc AB - Derivative Rules – Fall 2020

EQ: What are some Short Cut Derivative Rules?

Day	Date	Topic	Assignment
1	Friday, September 11th	**4.1 The Power Rule** *EQ: How do you find the derivative of a polynomial using the power rule?*	Constant and Power Rule Practice (Packet p. 1 - 2)
2	Monday, September 14th	**4.2 The Product and Quotient Rules** *EQ: How do you find the derivative when functions are multiplied or divided?*	**Skills Check 4.1** Power and Quotient Rule (Packet p. 3 - 4) Mixed Derivatives (Packet p. 5 - 6)
3	Tuesday, September 15th	**4.3 Particle Motion** EQ: How are Derivatives used in the real world?	**Quick Derivatives Quiz 1** Particle Motion (Packet p. 7 – 8)
4	Wednesday, September 16th	**Optional Q&A Session at 10:00 AM**	**Get caught up on all Keeper Notes & HW**
5	Thursday, September 17th	**4.4 Derivative of Trigonometric Functions** *EQ: How can you take the derivative of trig functions?* **4.5 The Chain Rule** *EQ: How do you take the derivative of a composition of functions?*	**Quick Derivatives Quiz 2** Derivatives of Trig Functions (Packet p. 9) Chain Rule (Packet p. 10 – 12)
6	Friday, September 18th	**4.6 Derivatives of Logarithmic Functions and Exponential Functions** *EQ: How do you differentiate a logarithmic or exponential function?*	**Skills Check 4.1-4.5** Derivative of ln(x) (Packet p. 13) Derivative of e^x and a^x (Packet p. 14)
7	Monday, September 21st	**4.7 Implicit Differentiation** **4.8 Logarithmic Differentiation** *EQ: How can you take a derivative if y is in the problem more than once?*	**Quick Derivatives Quiz 3** Implicit Differentiation (Packet p. 15 – 16) Logarithmic Differentiation (Packet p. 17)
8	Tuesday, September 22nd	**4.9 Derivatives of Inverse Functions** *EQ: How do you find the derivative of an inverse function?*	**Skills Check 4.7/4.8** Derivatives of Inverse Functions (packet p. 20 – 21) Derivatives of Inverse Trig Functions (Packet p. 22 – 23)
9	Wednesday, September 23rd	**Optional Q&A Session at 10:00 AM**	Packet p. 18 - 19 **Get caught up on all Keeper Notes & HW**
10	Thursday, September 24th	**4.10 L'Hopital's Rule** *EQ: What is the quick way to evaluate limits in indeterminate form?* **Review**	**Quick Derivatives Quiz 4** Limits and L'Hopital's Rule (Packet p. 24 – 25) AP Calculus Multiple Choice and Free Response Practice (Packet p. 26 – 30)
11	Friday, September 25th	**Test**	Good Luck!

Constant and Power Rule Practice

Find the derivative of each function. Make sure your answers are factored completely. If a point is given, find the value of the derivative at that point.

1. $y = 3$

2. $f(x) = x + 1$

3. $f(t) = -3t^2 + 2t - 4$

4. $s(t) = t^3 - 2t + 4$

5. $y = 4t^{\frac{4}{3}}$

6. $f(x) = 4\sqrt{x}$

7. $y = 4x^{-2} + 2x^2$

8. $y = \dfrac{1}{4x^3}$

9. $y = \dfrac{1}{(4x)^3}$

10. $y = \dfrac{\sqrt{x}}{x}$

11. $f(x) = x^2 - \frac{4}{x}$

12. $f(x) = x^2 - 2x - \frac{2}{x^4}$

13. $f(x) = \frac{2x^3 - 4x^2 + 3}{x^2}$

14. $y = x\left(x^2 + 1\right)$

15. $f(x) = x^{4/5}$

16. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$

17. $f(x) = \frac{4}{x^{-3}}$

18. $f(x) = \frac{\pi}{(3x)^2}$

19. $f(x) = \frac{1}{\sqrt[9]{x^7}}$

20. $f(x) = \frac{5x^7 + 9x^4 + 2x - 9}{10}$

Product and Quotient Rule

1. $f(x) = (1+\sqrt{x})(x^3)$

2. $g(t) = \left(\frac{2}{t} + t^5\right)(t^3+1)$

3. $h(y) = \frac{1}{y^3+2y+1}$

4. $y = \frac{1}{x+\sqrt{x}}$

5. $y = 2^x e^x$

6. $g(z) = \frac{z^2+1}{z^3-5}$

7. $y = \frac{\sqrt{x}}{x^3+1}$

8. $z = \frac{t^2}{(t-4)(2-t^3)}$

9. $h(x) = \frac{\left((x^3+1)\sqrt{x}\right)}{x^2}$

10. $y(m) = \frac{(e^m)(\sqrt[3]{m})}{m^2+3}$

11. $g(x) = \left(x + \sqrt{x}\right)(3^x)$

12. Let $f(x) = g(x)h(x), g(10) = -4, h(10) = 560, g'(10) = 0$, and $h'(10) = 35$. find $f'(10)$.

13. Let $y(x) = \frac{z(x)}{1+x^2}, z(-3) = 6$, and $z'(-3) = 15$. Find $y'(-3)$

Mixed Derivatives

Find the derivative using the <u>power</u>, <u>product</u>, or <u>quotient rule</u>. If necessary, rewrite first.

1. $y = 6x^3 + 4x^2 - 2x + 5$

2. $y = \sqrt[4]{x^3}$

3. $y = 3x^2 + \dfrac{12}{\sqrt{x}} - \dfrac{1}{x^2}$

4. $y = 3 - 7x^3 + 3x^7$

5. $y = 3x^{-\frac{2}{3}} + x^{\frac{3}{4}}$

6. $y = \dfrac{3x^3 - 5}{7}$

7. $y = \dfrac{4x^{\frac{3}{2}}}{x}$

8. $y = \dfrac{x^2 + 1}{x}$

9. $y = \dfrac{x^7 + 5x^6 - x^3}{x^2}$

10. $y = \dfrac{x+1}{\sqrt{x}}$

11. $y = \left(x^3 - 2\right)^2$

12. $y = \dfrac{x^2 - 4}{x+3}$

13. $y = \dfrac{2x+1}{2x-1}$

14. $y = \dfrac{x^2+1}{x^2-1}$

15. $y = \dfrac{1}{1+\sqrt{x}}$

16. $y = \dfrac{(x+1)(2x-5)}{(x+2)}$

17. $y = \left(3x^3 + 4x\right)(x-5)(x+1)$

Particle Motion

Answer the following questions for each position function $s(t)$ in meters where t is in seconds if a particle is moving along the x-axis.

$s(t) = t^3 - 3t + 3 \quad [0,6]$

a. What is the velocity function?

b. What is the velocity at $t = 3$ seconds?

c. When is the particle at rest?

d. When is the particle moving right? Moving left?

e. What is the acceleration function?

f. What is the acceleration at $t = 1$ second?

g. What is the displacement?

h. What is the total distance traveled?

i. When is the particle speeding up? Slowing Down?

j. Find the velocity when the acceleration is 0.

$s(t) = t^3 - 6t^2 \quad [0,7]$

a. What is the velocity function?

b. What is the velocity at $t = 3$ seconds?

c. When is the particle at rest?

d. When is the particle moving right? Moving left?

e. What is the acceleration function?

f. What is the acceleration at $t = 1$ second?

g. What is the displacement?

h. What is the total distance traveled?

i. When is the particle speeding up? Slowing Down?

j. Find the velocity when the acceleration is 0.

$s(t) = 2t^3 - 21t^2 + 60t + 3$ [0,8]

a. What is the velocity function?

b. What is the velocity at $t = 3$ seconds?

c. When is the particle at rest?

d. When is the particle moving right? Moving left?

e. What is the acceleration function?

f. What is the acceleration at $t = 1$ second?

g. What is the displacement?

h. What is the total distance traveled?

i. When is the particle speeding up? Slowing Down?

j. Find the velocity when the acceleration is 0.

$s(t) = 2t^3 - 14t^2 + 22t - 5$ [0,6]

a. What is the velocity function?

b. What is the velocity at $t = 3$ seconds?

c. When is the particle at rest?

d. When is the particle moving right? Moving left?

e. What is the acceleration function?

f. What is the acceleration at $t = 1$ second?

g. What is the displacement?

h. What is the total distance traveled?

i. When is the particle speeding up? Slowing Down?

j. Find the velocity when the acceleration is 0.

Derivatives of Trigonometric Functions

1. $y = 4sin^2x + 5cos^2x$

2. $f(x) = sin^3x \cdot cos\ x$

3. $y = 2sec\ x + tan\ x$

4. $y = \frac{1+tan^2x}{sec\ x}$

5. $f(x) = sin^4 3x - cos^4 3x$

6. $f(x) = \frac{sec\ 4x}{tan\ 4x}$

7. $f(x) = csc^4x - 21cot^2x$

8. $f(x) = (1 + cos\ 3x)^2$

9. $f(x) = cot\left(\frac{x}{2}\right)\ sin\left(\frac{x}{2}\right)$

10. $y = \frac{1}{cos\ 6x}$

Chain Rule

1. $y = (x^3 - 4)^4$

2. $y = (2x^2 + 5)^7$

3. $f(x) = (x^2 + 2x + 5)^6$

4. $f(x) = \sqrt[3]{x^2 + x}$

5. $y = \sqrt{(3x + 1)^3}$

6. $y = \left(\sqrt{x} + 1\right)^2$

7. $f(x) = \frac{1}{\sqrt{2x^3 - 7x^2}}$

8. $y = (5x^2 - 3x)^{-\frac{2}{3}}$

9. $f(x) = \sin^2 x$

10. $f(x) = \sin(x^2)$

11. $f(x) = \tan(3x)$

12. $f(x) = (3x - \cos x)^4$

13. $f(x) = e^{x^2+2x}$

14. $f(x) = \sec^2(4x)$

15. $y = (1 - x)(3x^2 - 5)^5$

16. $y = (x^2 - 5x)^6(2x - 5)^{-1}$

17. $y = \left(\frac{7-2x^5}{5x^2-8}\right)^2$

18. $f(x) = \sqrt{\frac{x^2+9}{x+3}}$

19. $f(x) = (x^2 - 3)^4(5x - 1)^6$

20. $f(x) = \sqrt{4 - \sqrt{x^2 - 5}}$

21. $f(x) = \cos^2(\sin 5x)$

22. $f(x) = \frac{(4x^2-6)^3}{(6x-7)^5}$

23. $f(x) = \frac{e^{3x}-5}{e^{2x}+7}$

24. $f(x) = \cot^3\left(e^{x^2}\right)$

25. $f(x) = x^2 \sin\left(\frac{1}{x}\right)$

26. $f(x) = \sin^3\left(\sqrt{e^{3x} - 5x}\right)$

Derivatives of $\ln x$

Find each Derivative

1. $y = \ln(x^3 + 1)$
2. $y = \ln\sqrt{x}$
3. $y = \sqrt{\ln(x)}$
4. $y = \ln\ |\sin x|$
5. $y = \ln(\sec x)$
6. $y = x \cdot \ln x$
7. $y = \frac{\ln x}{x^2}$
8. $y = \ln(\ln x)$
9. $y = (\sin x)(\ln x)$
10. $y = \frac{x^2}{\ln x}$
11. $y = \ln\left(\frac{5}{5-x}\right)$
12. $y = \ln\sqrt{x^2 + 4}$
13. $y = \ln(2 - cos\ x)$
14. $y = \ln(5 - x)^6$
15. $y = e^{\ln x^2}$
16. $y = \ln(3x^2 + 2)^3$
17. $y = \ln x^3 + (\ln x)^3$
18. $y = \ln\sqrt{\ln(x)}$

Derivatives of e^x and a^x

Find the derivative of each.

1. $y = e^{2x}$
2. $y = e^{5x^2}$
3. $y = e^{\sin x}$
4. $y = e^{\tan x}$
5. $y = e^{x^2+2x}$
6. $y = e^{\sqrt{x}}$
7. $y = 5^x$
8. $y = e^{e^x}$
9. $y = 7^{x^2+2x^3}$
10. $y = \sin e^{3x}$
11. $y = xe^x$
12. $y = (\sin x\,)e^x$
13. $y = x^2e^x$
14. $y = \frac{e^x}{x^2}$
15. $y = 2^x(x^2+1)$
16. $y = 3^{\ln x}$
17. $y = x^2 + 4^x$
18. $y = \ln e^{x^2}$
19. $y = e^{\ln x^3}$
20. $y = e^{3x} \cdot 4^{5x}$
21. $y = e^{\csc x}$
22. $y = 10^{\sin x}$
23. $y = x^2e^x - xe^x$
24. $y = xe^2 - e^x$

Implicit Differentiation

Find the derivative:

1. $(3x+7)^2 = 2y^3$

2. $x^2 = \dfrac{x-y}{x+y}$

3. $y^2 = \dfrac{x-1}{x+1}$

4. $x^3 - xy + y^3 = 1$

5. $x = \tan y$

6. $x + \sin y = xy$

7. $y^2 \cos\left(\frac{1}{y}\right) = 2x + 2y$

8. $e^{xy} + \ln(y) = 2x$

Find y' and y''.

9. $x^2 + y^2 = 1$

10. $y^2 = x^2 + 2x$

11. $x^{2/3} + y^{2/3} = 1$

12. $xy + y^2 = 1$

Find the equation of the lines that are tangent and normal to the curve at the given point.

13. $x^2 y^2 = 9 \quad at\ (-1,3)$

14. $2xy + \pi \sin y = 2\pi$ at $\left(1,\ \pi/2\right)$

Logarithmic Differentiation

1. $y = x^x$

2. $y = x^{2x+1}$

3. $y = x^{\sin x}$

4. $y = x^{\frac{1}{x}}$

5. $y = (4x+3)^{x+2}$

6. $y = (\ln x)^x$

7. $y = (x^2+5x+1)^{x+2}$

8. $y = x^{\ln x}$

9. $y = (3x-7)^4(8x^2-1)^3$

10. $y = (2x-1)^3(4x^2+5)^5$

Derivatives from Charts and Graphs

1. If $f(3)=4,\ g(3)=2,\ f'(3)=6$ and $g'(3)=5$ find the following.

a) $(f+g)'(3)$

b) $-5g'(3)$

c) $(f\cdot g)'(3)$

d) $\left(\dfrac{f}{g}\right)'(3)$

2 – 9 Given the following chart, find the indicated derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	-4

2. If $h(x)=f(x)+g(x)$, find $h'(-1)$

3. If $h(x)=7g(x)$, find $h'(0)$

4. If $h(x)=g(x)\cdot f(x)$, find $h'(0)$

5. If $h(x)=\dfrac{f(x)}{g(x)}$, find $h'(0)$

6. If $h(x)=-4f(x)\cdot g(x)$, find $h'(-2)$

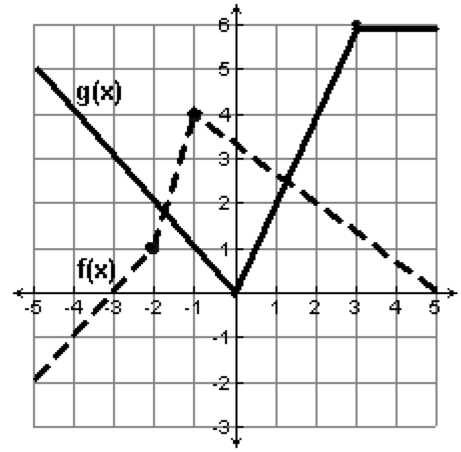
7. If $h(x)=f(g(x))$, find $h'(1)$

8. If $j(x)=g(f(x))$. Find j'(2)?

9. If $h(x)=x^2g(x)$, find $h'(-2)$

10– 15 Use the graph to find the derivative.

10. If $K(x) = f(x) + g(x)$, find $K'(-3)$



11. If $K(x) = f(x) \cdot g(x)$, find $K'(2)$

12. If $K(x) = \dfrac{f(x)}{g(x)}$, find $K'(2)$

13. If $K(x) = f(g(x))$, find $K'(1)$

14. If $K(x) = (g \circ f)(x)$, find $K'(-4)$

15. If $K(x) = f(x) - g(x)$, find $K'(0)$

Derivative of the Inverse

1. Which inverse trigonometric function $g(x)$ has the derivative $g'(x) = \frac{1}{x^2+1}$?

2. If $g(x) = \sqrt[3]{x-1}$ and f is the inverse function of g, then $f'(x) =$

3. Let $f(x) = x^2 - 3x,\ x > 0$ Find: $f^{-1}(4) =$ $(f^{-1})'(4) =$

4. Let $f(x) = x^2 - 13,\ x > 0$ Find: $f^{-1}(3) =$ $(f^{-1})'(3) =$

5. Let $g(x)$ be the inverse of $f(x) = x^3 + 2x + 4$. Calculate $g(7) =$ $g'(7) =$

6. Find $g'\left(-\frac{1}{2}\right)$ where $g(x)$ is the inverse of $f(x) = \frac{x^3}{x^2+1}$

7. Let $f(x) = \frac{\ln e^{2x}}{x-1}$ for $x > 1$. If g is the inverse of f, then $g'(3) =$

For questions 8-10, calculate $g(b)$ and $g'(b)$, where g is the inverse of f.

8. $f(x) = x + \cos x, \qquad b = 1$

9. $f(x) = 4x^3 - 2x, \qquad b = -2$

10. $f(x) = \sqrt{x^2 + 6x}$ for $x \geq 0 \qquad\qquad b = 4$

Derivatives of Inverse Trig Functions

Find the derivative of the following.

1. $y = \tan^{-1}(2x)$

2. $y = \sin^{-1}(x^2)$

3. $y = \sec^{-1}(x^3)$

4. $y = \arctan(x^2 + 1)$

5. $y = \arcsin(5x)$

6. $y = arc\sec(5x)$

7. $y = \arctan\left(\sqrt{x}\right)$

8. $y = \sin^{-1}\left(\sqrt{x}\right)$

9. $y = \tan^{-1}(x^2 + 2x)$

10. $y = \tan^{-1}(e^x)$

11. $y = x\tan^{-1}(x)$

12. $y = x^2 \sin^{-1}(x)$

13. $y = e^x \sin^{-1}(x)$

14. $y = \ln\left(x\right)\arctan(x)$

Limits and L'Hopitals Rule

Evaluate each Limit. Use L'Hopitals Rule when possible.

1. $\lim_{x\to 2} \frac{x^3-x-2}{x-2}$

2. $\lim_{x\to\infty} \frac{(\ln x)^3}{x}$

3. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x}$

4. $\lim_{x\to 0} \frac{e^x-(1-x)}{x}$

5. $\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$

6. $\lim_{x\to 0} \frac{x+\sin 3x}{x-\sin 3x}$

7. $\lim_{x\to\infty} \frac{x^2+2x+1}{x-1}$

8. $\lim_{x\to 16} \frac{\sqrt[4]{x}-2}{x-16}$

9. $\lim_{x\to\infty} \frac{x^2}{e^{5x}}$

10. $\lim_{x\to\infty} \frac{\ln x}{x}$

11. $\lim_{x\to 0} \frac{e^{2x}-1}{e^x}$

12. $\lim_{x\to 3} \frac{2x-6}{x^2-9}$

AP Calculus Multiple Choice and Free Response Practice

Non-Calculator Section

1. If $y = x \cdot \sin x$, then $\frac{dy}{dx} =$
   a. $\sin x + \cos x$
   b. $\sin x + x \cdot \cos x$
   c. $\sin x - x \cdot \cos x$
   d. $x(\sin x + \cos x)$
   e. $x(\sin x - \cos x)$

2. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$
   a. 4
   b. 5
   c. 6
   d. 7
   e. 8

3. If y= $(x^3 - \cos x)^5$, then $y' =$
   a. $5(x^3 - \cos x)^4$
   b. $5(3x^2 + \sin x)^4$
   c. $5(3x^2 + \sin x)$
   d. $5(3x^2 + \sin x)^4 \cdot (6x + \cos x)$
   e. $5(x^3 - \cos x)^4 \cdot (3x^2 + \sin x)$

4. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is
   a. $\frac{7}{\sqrt{5}}$
   b. $\frac{14}{\sqrt{5}}$
   c. $\frac{18}{\sqrt{5}}$
   d. $\frac{15}{\sqrt{21}}$
   e. $\frac{30}{\sqrt{21}}$

5. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?
   a. (0,0) only
   b. $\left(\frac{1}{2}, \frac{1}{5}\right)$ only
   c. (0,0) and $(-4,2)$
   d. (0,0) and $\left(4, \frac{2}{3}\right)$
   e. There are no such points

6. Let $f(x) = (2x + 1)^3$ and let g be the inverse of f. Given that $f(0) = 1$, what is the value of $g'(1)$?
   a. $-\frac{2}{27}$
   b. $\frac{1}{54}$
   c. $\frac{1}{27}$
   d. $\frac{1}{6}$
   e. 6

7. The $\lim_{h\to 0}\frac{\ln[\sin(x+h)]-\ln(\sin x)}{h}$ is …

a. $\sin x$
b. x
c. $\frac{1}{x}$
d. $\cot x$
e. $\tan x$

8. The $\lim_{x\to\frac{\pi}{2}}\frac{\sin(x)-\sin\left(\frac{\pi}{2}\right)}{x-\frac{\pi}{2}}$ has a value of …

a. 0
b. 1
c. $\frac{\sqrt{2}}{2}$
d. -1
e. 2

9. The equation of the normal line to the graph of $y=e^{2x}$ when $\frac{dy}{dx}=2$ is …

a. $y=-\frac{1}{2}x+1$
b. $y=2\left(x-\frac{\ln 2}{2}\right)+2$
c. $y=2x+1$
d. $y=-\frac{1}{2}\left(x-\frac{\ln 2}{2}\right)+2$

10. If $f(x)=5\cos^2(\pi-x)$, then $f'\left(\frac{\pi}{2}\right)$ is …

a. 0
b. $-\frac{2}{3}$
c. $\frac{2}{3}$
d. $-\frac{5}{6}$
e. 1

11. For what value(s) of k does the graph of $g(x)=ke^{2x}+3x$ have a normal line whose slope is $-\frac{1}{5}$ when $x=1$?

a. e
b. $\frac{1}{e^2}$
c. $-\frac{8}{5e^2}$
d. $\frac{2}{e^2}$
e. 0

12. If $f'(x)=\tan(2x)$, then $f'\left(\frac{\pi}{6}\right)=$

a. $2\sqrt{3}$
b. 4
c. $\sqrt{3}$
d. 8
e. 6

13. If $y=3x(3^{-2x})$, then $\frac{dy}{dx}=$

a. $-\frac{6\ln 3}{3^{2x}}$
b. $\frac{3\ln 3}{3^{2x}}$
c. $\frac{3(1-2x\cdot\ln 3)}{3^{2x}}$
d. $\frac{1+x\cdot\ln 3}{9^{2x}}$
e. $\frac{1+x\cdot\ln 3}{3^{2x}}$

14. If $f(x)=\log_5(5x+1)^4$, then what is the value of $f'(1)$?

a. $\frac{10}{3\ln 5}$
b. $\frac{4}{\ln 6}$
c. $\frac{2}{3\ln 5}$
d. $\frac{4}{\ln 5}$
e. $\frac{5}{\ln 4}$

Calculator Section

15. The graph of $y = e^{\tan x} - 2$ crosses the x −axis at one point in the interval $[0,1]$. What is the slope of the graph at this point?
    a. 0.606
    b. 2
    c. 2.242
    d. 2.961
    e. 3.747

16. Given that $f(x) = x^2 e^x$, what is an approximate value of $f(1.1)$ if you use the equation of the tangent line to the graph of f at $x = 1$?
    a. 3.534
    b. 3.635
    c. 7.055
    d. 8.155
    e. 5.263

17. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangents?
    a. -0.701
    b. -0.567
    c. -0.391
    d. -0.302
    e. -0.327

18. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$.
    a. $y = 8x - 5$
    b. $y = x + 7$
    c. $y = x + 0.763$
    d. $y = x - 0.122$
    e. $y = 2x - 3.407$

19. On the interval $-4 < x < 4$, for what value(s) of x will the graphs of $y = \log_4\left(\frac{2x}{2x+3}\right)$ and $y = x^4 + 3xe^x$ have parallel tangent lines?
    a. -0.395 only
    b. -1.568 and -0.395
    c. -0.480 only
    d. -0.817 and 0.159
    e. 0.159 only

FREE RESPONSE #1

Consider the piece-wise defined function below to answer the questions that follow.

$$f(x)=\begin{cases} ax^2+bx+2, & x\le 2 \\ ax+b, & x>2 \end{cases}$$

a. If $a = -3$ and $b = 4$, will $f(x)$ be continuous at $x = 2$? Justify your answer.

b. If $a = -3$ and $b = 4$, will $f(x)$ be differentiable at $x = 2$? Justify your answer.

c. For what value(s) of a and b will $f(x)$ be both continuous and differentiable at $x = 2$? Show your work.

FREE RESPONSE #2

A rodeo performer spins a lasso in a circle perpendicular to the ground. The height from the ground of the knot, measured in units of feet, in the lasso is modeled by the function

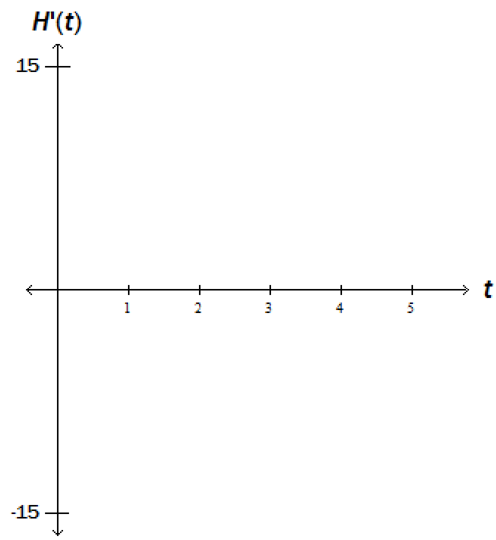
$$H(t) = -3\cos\left(\frac{5\pi}{3}t\right) + 5,$$

where t is the time measured in seconds after the lasso begins to spin.

a. Find the value of $H(0.75)$. Using correct units, explain what this value represents in the context of this problem.

b. Find the value of $H'(0.75)$. Using correct units, explain what this value represents in the context of this problem.

c. Find $H'(t)$ and sketch its graph on the axes to the right for the interval $0 < t < 5$ seconds.



d. During the first five seconds of the performer spinning the lasso, how many times is the lasso at its maximum height? Give a reason for your answer based on the graph of $H'(t)$.

e. What is the height of the lasso the first time it is at its minimum height on the interval $0 < t < 5$ seconds? Justify your answer and show your work.