Unit 4 – Derivative Rules

- Homework will be assigned daily
- Daily skills checks will accumulate to form a quiz grade

Topics Covered:

- ❖ Power Rule
- ❖ Product & Quotient Rule
- ❖ Chain Rule
- ❖ Derivatives from Charts & Graphs
- ❖ Derivatives of Trig Functions
- ❖ Derivatives of Exponential & Log Functions
- ❖ L'Hopital's Rule

Quiz is ____________________

Test is ____________________

Name: Key

Constant and Power Rule Practice

Find the derivative of each function. Make sure your answers are simplified completely. If a point is given, find the value of the derivative at that point.

1. $y = 3$

$y' = 0$

2. $f(x) = x + 1$

$f'(x) = 1$

3. $f(t) = -3t^2 + 2t - 4$

$f'(t) = -6t + 2$

4. $s(t) = t^3 - 2t + 4$

$s'(t) = 3t^2 - 2$

5. $y = 4t^{\frac{4}{3}}$

$y' = \frac{16}{3} t^{1/3}$

6. $f(x) = 4\sqrt{x} = 4x^{1/2}$

$f'(x) = 2x^{-1/2}$

$f'(x) = \frac{2}{\sqrt{x}}$

7. $y = 4x^{-2} + 2x^2$

$y' = -8x^{-3} + 4x$

$y' = \frac{-8}{x^3} + 4x$

8. $y = \frac{1}{4x^3} = \frac{1}{4}x^{-3}$

$y' = \frac{-3}{4}x^{-4}$

$y' = \frac{-3}{4x^4}$

9. $y = \frac{1}{(4x)^3} = \frac{1}{64}x^{-3}$

$y' = \frac{-3}{64x^4}$

10. $y = \frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x} = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2}$

$y' = \frac{-1}{2x^{3/2}}$

11. $f(x) = x^2 - \frac{4}{x}$

$f'(x) = 2x + \frac{4}{x^2}$

12. $f(x) = x^2 - 2x - \frac{2}{x^4}$

$f'(x) = 2x - 2 + \frac{8}{x^5}$

13.

13. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2}$

$f(x) = 2x - 4 + 3x^{-2}$

$f'(x) = 2 - \dfrac{6}{x^3}$

14. $y = x(x^2 + 1)$

$y = x^3 + x$

$y' = 3x^2 + 1$

15. $f(x) = x^{4/5}$

$f'(x) = \frac{4}{5}x^{-1/5}$

$f'(x) = \dfrac{4}{5x^{1/5}}$

16. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$

$f(x) = x^{1/3} + x^{1/5}$

$f'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5}$

$f'(x) = \dfrac{1}{3x^{2/3}} + \dfrac{1}{5x^{4/5}}$

17. $f(x) = \dfrac{4}{x^{-3}} = 4x^3$

$f'(x) = 12x^2$

18. $f(x) = \dfrac{\pi}{(3x)^2} = \frac{\pi}{9}x^{-2}$

$f'(x) = -\frac{2\pi}{9}x^{-3}$

$f'(x) = \dfrac{-2\pi}{9x^3}$

19. $f(x) = \dfrac{1}{\sqrt[9]{x^7}} = x^{-7/9}$

$f'(x) = -\frac{7}{9}x^{-16/9}$

$f'(x) = \dfrac{-7}{9x^{16/9}}$

20. $f(x) = (x^2 + 2x)(x + 1)$

$f(x) = x^3 + 3x^2 + 2x$

$f'(x) = 3x^2 + 6x + 2$

21. $f(x) = \dfrac{5x^7 + 9x^4 + 2x - 9}{10}$

$f(x) = \frac{1}{2}x^7 + \frac{9}{10}x^4 + \frac{1}{5}x - \frac{9}{10}$

$f'(x) = \frac{7}{2}x^6 + \frac{18}{5}x^3 + \frac{1}{5}$

22. $f(x) = (2x + 1)^2 \quad (0, 1)$

$f(x) = 4x^2 + 4x + 1$

$f'(x) = 8x + 4$

$f'(0) = 8(0) + 4$

$f'(0) = 4$

23. $f(x) = \dfrac{1}{x}$ at $\left(2, \dfrac{1}{2}\right)$

$f'(x) = \dfrac{-1}{x^2}$

$f'(2) = -\dfrac{1}{(2)^2}$

$f'(2) = -1/4$

24. $f(x) = x(x^2 + 1) \quad (7, 350)$

$f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$

$f'(7) = 3(7)^2 + 1$

$f'(7) = 148$

PRODUCT & QUOTIENT RULE

PRODUCT RULE: Find the derivative 2 different ways: 1) using the Product Rule and 2) distribute first & then derive.

1. $f(x) = x^2(x^3 - 1)$

① $f'(x) = x^2(3x^2) + (x^3-1)(2x)$
$= 3x^4 + 2x^4 - 2x$
$= 5x^4 - 2x$

② $f(x) = x^5 - x^2$
$f'(x) = 5x^4 - 2x$

2. $f(x) = (x^3 - 1)(x^2 - 2x + 1)$

① $f'(x) = (x^3-1)(2x-2) + (x^2-2x+1)(3x^2)$
$= 2x^4 - 2x^3 - 2x + 2 + 3x^4 - 6x^3 + 3x^2$
$= 5x^4 - 8x^3 + 3x^2 - 2x + 2$

② $f(x) = x^5 - 2x^4 + x^3 - x^2 + 2x - 1$
$f'(x) = 5x^4 - 8x^3 + 3x^2 - 2x + 2$

Differentiate with respect to x using the Product Rule.

3. $f(x) = (4x + 1)(3x - 2)$

$f'(x) = (4x+1)(3) + (3x-2)(4)$
$= 12x + 3 + 12x - 8$
$= 24x - 5$

4. $f(x) = (x + 3)^2$

$f(x) = (x+3)(x+3)$
$f'(x) = (x+3)(1) + (x+3)(1)$
$f'(x) = 2x + 6$

5. $f(x) = (2x^{\frac{5}{3}} - 3)(-x^5 + 3)$

$f'(x) = (2x^{5/3} - 3)(-5x^4) + (-x^5+3)(\frac{10}{3}x^{2/3})$
$f'(x) = -10x^{17/3} + 15x^4 - \frac{10}{3}x^{17/3} + 10x^{2/3}$
$f'(x) = -\frac{40}{3}x^{17/3} + 15x^4 + 10x^{2/3}$

6. $f(x) = (1 + \sqrt{x})(x^3)$

$f'(x) = (1+\sqrt{x})(3x^2) + (x^3)\left(\frac{1}{2\sqrt{x}}\right)$
$f'(x) = 3x^2 + 3x^{5/2} + \frac{1}{2}x^{5/2}$
$f'(x) = 3x^2 + \frac{7}{2}x^{5/2}$

When dividing by a monomial, you don't need to use the Quotient Rule. You can simplify first and then derive. Find the derivative <u>without</u> using the Quotient Rule.

7. $f(x) = \frac{x^2+3x}{6}$

$f(x) = \frac{1}{6}x^2 + \frac{1}{2}x$
$f'(x) = \frac{1}{3}x + \frac{1}{2}$

8. $f(x) = \frac{5x^4-16x^2+4}{x^2}$

$f(x) = 5x^2 - 16 + 4x^{-2}$
$f'(x) = 10x - \frac{8}{x^3}$

9. $f(x) = \frac{-3(3x-2x^2)}{7x}$

$f(x) = -\frac{9x}{7x} + \frac{6x^2}{7x}$
$f(x) = -\frac{9}{7} + \frac{6}{7}x$

$f'(x) = \frac{6}{7}$

Differentiate with respect to x using the Quotient Rule.

10. $f(x) = \frac{x^2}{1-x^3}$

$f'(x) = \frac{(1-x^3)(2x) - x^2(-3x^2)}{(1-x^3)^2}$

$f'(x) = \frac{2x - 2x^4 + 3x^4}{(1+x^3)^2}$

$f'(x) = \frac{x^4 + 2x}{(1-x^3)^2}$

11. $f(x) = \frac{x^2-2}{x^2+2}$

$f'(x) = \frac{(x^2+2)(2x) - (x^2-2)(2x)}{(x^2+2)^2}$

$f'(x) = \frac{2x^3 + 4x - 2x^3 + 4x}{(x^2+2)^2}$

$f'(x) = \frac{8x}{(x^2+2)^2}$

12. $f(x) = \frac{x^3+3x+2}{x^2-1}$

$f'(x) = \frac{(x^2-1)(3x^2+3) - (x^3+3x+2)(2x)}{(x^2-1)^2}$

$f'(x) = \frac{3x^4 + 3x^2 - 3x^2 - 3 - 2x^4 - 6x^2 - 4x}{(x^2-1)^2}$

$f'(x) = \frac{x^4 - 6x^2 - 4x - 3}{(x^2-1)^2}$

13. $f(x) = (2x-5)(4-x)^{-1}$ $\quad f(x) = \frac{(2x-5)}{(4-x)}$

$f'(x) = \frac{(4-x)(2) - (2x-5)(-1)}{(4-x)^2}$

$f'(x) = \frac{8 - 2x + 2x - 5}{(4-x)^2}$

$f'(x) = \frac{3}{(4-x)^2}$

Differentiate with respect to x using the Product or Quotient Rule.

14. $g(x) = \frac{3x^4+5x^2}{x^5+2}$

$g'(x) = \frac{(x^5+2)(12x^3+10x) - (3x^4+5x^2)(5x^4)}{(x^5+2)^2}$

$g'(x) = \frac{12x^8 + 10x^6 + 24x^3 + 20x - 15x^8 - 25x^6}{(x^5+2)^2}$

$g'(x) = \frac{-3x^8 - 15x^6 + 24x^3 + 20x}{(x^5+2)^2}$

15. $y = \frac{\sqrt{x}}{x^3+1}$

$y' = \frac{(x^3+1)\left(\frac{1}{2x^{1/2}}\right) - x^{1/2}(3x^2)}{(x^3+1)^2}$

$y' = \frac{\frac{1}{2}x^{5/2} + \frac{1}{2x^{1/2}} - 3x^{5/2}}{(x^3+1)^2}$

16. $m(x) = (-4 + \frac{4}{x})(3x^3 + 4)$

$m'(x) = (-4 + \frac{4}{x})(9x^2) + (3x^3+4)\left(\frac{-4}{x^2}\right)$

$m'(x) = -36x^2 + 36x - 12x - \frac{16}{x^2}$

$m'(x) = -36x^2 + 24x - \frac{16}{x^2}$

17. $h(x) = \frac{2x^4-5x^2+6x}{2x^2}$

$h(x) = x^2 - \frac{5}{2} + 3x^{-1}$

$h'(x) = 2x - 3x^{-2}$

$h'(x) = 2x - \frac{3}{x^2}$

18. $f(x) = (2x^3 - 5)(3x^2 + 4x - 7)$

$f'(x) = (2x^3-5)(6x+4) + (3x^2+4x-7)(6x^2)$

$f'(x) = 12x^4 + 8x^3 - 30x - 20 + 18x^4 + 24x^3 - 42x^2$

$f'(x) = 30x^4 + 32x^3 - 42x^2 - 30x - 20$

19. $y = 5\sqrt{x}(3x^2 + 4x - \sqrt{x})$

$y' = 5\sqrt{x}\left(6x + 4 - \frac{1}{2\sqrt{x}}\right) + (3x^2+4x-\sqrt{x})\left(\frac{5}{2\sqrt{x}}\right)$

$y' = 30x^{3/2} + 20x^{1/2} - \frac{5}{2} + \frac{15}{2}x^{3/2} + 10x^{1/2} - \frac{5}{2}$

$y' = \frac{75}{2}x^{3/2} + 30x^{1/2} - 5$

Mixed Derivatives #1

Find the derivative using the power, product, or quotient rule. If necessary, rewrite first.

1. $y = 6x^3 + 4x^2 - 2x + 5$

$y' = 18x^2 + 8x - 2$

2. $y = \sqrt[4]{x^3}$

$y = x^{3/4}$

$y' = \frac{3}{4}x^{-1/4}$ $\quad y' = \frac{3}{4\sqrt[4]{x}}$

3. $y = 3x^2 + \frac{12}{\sqrt{x}} - \frac{1}{x^2}$

$y = 3x^2 + 12x^{-1/2} - x^{-2}$

$y' = 6x - 6x^{-3/2} + 2x^{-3}$

$y' = 6x - \frac{6}{x^{3/2}} + \frac{2}{x^3}$

4. $y = 3 - 7x^3 + 3x^7$

$y' = -21x^2 + 21x^6$

5. $y = 3x^{-\frac{2}{3}} + x^{\frac{3}{4}}$

$y' = -2x^{-5/3} + 3/4x^{-1/4}$

$y' = \frac{-2}{x^{5/3}} + \frac{3}{4x^{1/4}}$

6. $y = \frac{3x^3 - 5}{7}$ $\quad y = \frac{3}{7}x^3 - \frac{5}{7}$

$y' = \frac{9}{7}x^2$

7. $y = \frac{4x^{\frac{3}{2}}}{x}$ $\quad y = 4x^{1/2}$

$y' = 2x^{-1/2}$ $\quad y' = \frac{2}{\sqrt{x}}$

8. $y = \frac{x^2 + 1}{x}$ $\quad y = x + x^{-1}$

$y' = 1 - \frac{1}{x^2}$

9. $y = \frac{x^7 + 5x^6 - x^3}{x^2}$ $\quad y = x^5 + 5x^4 - x$

$y' = 5x^4 + 20x^3 - 1$

10. $y = \frac{x + 1}{\sqrt{x}}$ $\quad = \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}$

$y = x^{1/2} + x^{-1/2}$

$y' = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$ or $\frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$

11. $y = (x^3 - 2)^2$ $\quad y = x^6 - 4x^3 + 4$

$y' = 6x^5 - 12x^2$

12. $y = \frac{x^2 - 4}{x + 3}$

$y' = \frac{(x+3)(2x) - (x^2-4)(1)}{(x+3)^2}$

$y' = \frac{2x^2 + 6x - x^2 + 4}{(x+3)^2}$ $\quad y' = \frac{x^2 + 6x + 4}{(x+3)^2}$

13. $y = \frac{2x + 1}{2x - 1}$

$y' = \frac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2}$

$y' = \frac{4x - 2 - 4x - 2}{(2x-1)^2}$ $\quad y' = \frac{-4}{(2x-1)^2}$

14. $y = \frac{x^2 + 1}{x^2 - 1}$

$y' = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$

$y' = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$ $\quad y' = \frac{-4x}{(x^2-1)^2}$

15. $y = \frac{1}{1 + \sqrt{x}}$

$y' = \frac{(1+\sqrt{x})(0) - 1\left(\frac{1}{2\sqrt{x}}\right)}{(1+\sqrt{x})^2}$ $\quad y' = \frac{\frac{-1}{2\sqrt{x}}}{(1+\sqrt{x})^2}$

$y' = \frac{-1}{2\sqrt{x}(1+\sqrt{x})^2}$

16. $y = \frac{(x+1)(2x-5)}{(x+2)}$

$y = \frac{2x^2 - 3x - 5}{x + 2}$

$y' = \frac{(x+2)(4x-3) - (2x^2-3x-5)(1)}{(x+2)^2}$

$y' = \frac{4x^2 + 5x - 6 - 2x^2 + 3x + 5}{(x+2)^2}$

$y' = \frac{2x^2 + 8x - 1}{(x+2)^2}$

17. $y = (3x^3 + 4x)(x - 5)(x + 1)$

$y = (3x^4 - 15x^3 + 4x^2 - 20x)(x + 1)$

$y = 3x^5 - 15x^4 + 4x^3 - 20x^2 + 3x^4 - 15x^3 + 4x^2 - 20x$

$y = 3x^5 - 12x^4 - 11x^3 - 16x^2 - 20x$

$y' = 15x^4 - 48x^3 - 33x^2 - 32x - 20$

Mixed Derivatives #2

Find the derivative using the power, product, or quotient rule. Remember to rewrite if necessary.

1. $y=\sqrt{x}+\frac{1}{\sqrt{x}}$

$y = x^{1/2} + x^{-1/2}$

$y' = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$

2. $y=\sqrt{x}\left(\sqrt{x}+3\right)$

$y = x + 3\sqrt{x}$

$y' = 1 + \frac{3}{2\sqrt{x}}$

3. $y=x\left(x^2+\sqrt{x}\right)$

$y = x^3 + x^{3/2}$

$y' = 3x^2 + \frac{3}{2}x^{1/2}$

4. $y=\frac{x^4-x^3}{x^{\frac{1}{2}}}$

$y = x^{7/2} - x^{5/2}$

$y' = \frac{7}{2}x^{5/2} - \frac{5}{2}x^{3/2}$

5. $y=\frac{4x}{3x+1}$

$y' = \frac{(3x+1)(4) - 4x(3)}{(3x+1)^2}$

$y' = \frac{12x+4-12}{(3x+1)^2}$ $\quad y' = \frac{4}{(3x+1)^2}$

6. $y=\sqrt[3]{x^2}-\sqrt[3]{x}$

$y = x^{2/3} - x^{1/3}$

$y' = \frac{2}{3}x^{-1/3} - \frac{1}{3}x^{-2/3}$

$y' = \frac{2}{3x^{1/3}} - \frac{1}{3x^{2/3}}$

7. $y=(3x-2)\left(x^3+1\right)$

$y = 3x^4 + 3x - 2x^3 - 2$

$y' = 12x^3 - 6x^2 + 3$

8. $y=x^3\left(2x^4-x\right)$

$y = 2x^7 - x^4$

$y' = 14x^6 - 4x^3$

9. $y=\frac{3x+4}{2x-3}$

$y' = \frac{(2x-3)(3) - (3x+4)(2)}{(2x-3)^2}$

$y' = \frac{6x-9-6x-8}{(2x-3)^2}$ $\quad y' = \frac{-17}{(2x-3)^2}$

10. $y=\frac{x^3-x^2+2}{x^2}$

$y = x - 1 + 2x^{-2}$

$y' = 1 - 4x^{-3}$

$y' = 1 - \frac{4}{x^3}$

11. Find all derivatives:

$y=x^5+\frac{1}{6}x^2-\frac{1}{3}x$

$y' = 5x^4 + \frac{1}{3}x - \frac{1}{3}$

$y'' = 20x^3 + \frac{1}{3}$

$y''' = 60x^2$

$y'''' = 120x$

$y^{V} = 120$

$y^{VI} = 0$

12. Given: $y=x^3-5x^2+3x-1$.

Write the equation of the tangent line to the curve at $x=2$. (Hint: find y' first.) $f(2) = -7$

$y' = 3x^2 - 10x + 3$

$y'(2) = 3(2)^2 - 10(2) + 3$

$y'(2) = -5$

$m_{tan} = -5$

$y + 7 = -5(x-2)$

Chain Rule

1. $y = (x^3 - 4)^4$

$y' = 4(x^3-4)^3(3x^2)$

$y' = 12x^2(x^3-4)^3$

2. $y = (2x^2 + 5)^7$

$y' = 7(2x^2+5)^6(4x)$

$y' = 28x(2x^2+5)^6$

3. $f(x) = (x^2 + 2x + 5)^6$

$f'(x) = 6(x^2+2x+5)^5(2x+2)$

$f'(x) = (12x+2)(x^2+2x+5)^5$

4. $f(x) = \sqrt[3]{x^2 + x}$

$f'(x) = \frac{1}{3}(x^2+x)^{-2/3}(2x+1)$

$f'(x) = \dfrac{2x+1}{3(x^2+x)^{2/3}}$

5. $y = \sqrt{(3x + 1)^3}$

$y = (3x+1)^{3/2}$

$y' = \frac{3}{2}(3x+1)^{1/2}(3)$

$y' = \frac{9}{2}\sqrt{3x+1}$

6. $y = (\sqrt{x} + 1)^2$

$y' = 2(\sqrt{x}+1)\left(\frac{1}{2\sqrt{x}}\right)$

$y' = \dfrac{\sqrt{x}+1}{\sqrt{x}}$ or $y' = 1 + \dfrac{1}{\sqrt{x}}$

7. $g(x) = \dfrac{1}{\sqrt{2x^3 - 7x^2}}$

$g(x) = (2x^3-7x^2)^{-1/2}$

$g'(x) = -\frac{1}{2}(2x^3-7x^2)^{-3/2}(6x^2-14x)$

$g'(x) = \dfrac{-3x^2+7}{(2x^3-7x^2)^{3/2}}$

8. $y = (5x^2 - 3x)^{-\frac{2}{3}}$

$y' = -\frac{2}{3}(5x^2-3x)^{-5/3}(10x-3)$

$y' = \dfrac{-2(10x-3)}{3(5x^2-3x)^{5/3}}$

9. $y = \sqrt[3]{(x^2 + 4)^2}$

$y = (x^2+4)^{2/3}$

$y' = \frac{2}{3}(x^2+4)^{-1/3}(2x)$

$y' = \dfrac{4x}{3\sqrt[3]{x^2+4}}$

10. $f(x) = \dfrac{5}{(4x - 3)^2}$

$f(x) = 5(4x-3)^{-2}$

$f'(x) = -10(4x-3)^{-3}(4)$

$f'(x) = \dfrac{-40}{(4x-3)^3}$

11. $f(x) = (x^2 - 3)(5x - 1)^6$

$f'(x) = (x^2-3)\cdot 6(5x-1)^5\cdot 5 + (5x-1)^6(2x)$

$f'(x) = 30(x^2-3)(5x-1)^5 + 2x(5x-1)^6$

12. $y = \sqrt{3x^2 + 5}$

$y = (3x^2+5)^{1/2}$

$y' = \frac{1}{2}(3x^2+5)^{-1/2}(6x)$

$y' = \dfrac{3x}{\sqrt{3x^2+5}}$

Derivatives from Charts and Graphs

1. If $f(3)=4$, $g(3)=2$, $f'(3)=6$ and $g'(3)=5$ find the following.

a) $(f+g)'(3)$

$f'(3)+g'(3)$

$6+5 = 11$

b) $-5g'(3)$

$-5\cdot g'(3)$

$-5\cdot 5 = -25$

c) $(f\cdot g)'(3)$

$f(3)\cdot g'(3)+g(3)\cdot f'(3)$

$4\cdot 5+2\cdot 6$

$20+12$

32

d) $\left(\dfrac{f}{g}\right)'(3)$

$\dfrac{g(3)\cdot f'(3)-f(3)g'(3)}{[g(3)]^2}$

$\dfrac{2\cdot 6-4\cdot 5}{(2)^2} = \dfrac{12-20}{4} = \dfrac{-8}{4} = -2$

#2 – 7 Given the following chart, find the indicated derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	-4

2. If $h(x)=f(x)+g(x)$, find $h'(-1)$

$f'(-1)+g'(-1)$

$7+1 = 8$

3. If $h(x)=7g(x)$, find $h'(0)$

$h'(0)=7g'(0)$

$=7(-3)$

$=-21$

4. If $h(x)=g(x)\cdot f(x)$, find $h'(0)$

$h'(0)=g(0)\cdot f'(0)+f(0)\cdot g'(0)$

$=9\cdot 9+5\cdot -3$

$=81-15$

$=66$

5. If $h(x)=\dfrac{f(x)}{g(x)}$, find $h'(0)$

$h'(0)=\dfrac{g(0)\cdot f'(0)-f(0)g'(0)}{[g(0)]^2}$

$=\dfrac{9\cdot 9-5\cdot -3}{(9)^2} = \dfrac{96}{81}$

6. If $h(x)=f(g(x))$, find $h'(1)$

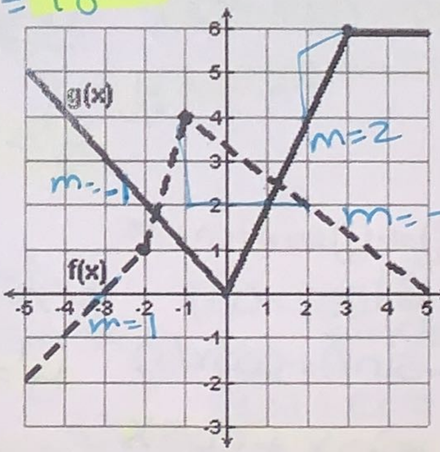
$h'(1)=f'(g(1))\cdot g'(1)$

$=f'(2)\cdot g'(1)$

$=3\cdot 6$

$=18$

#7 - 10 Use the graph to find the derivative.



7. If $K(x)=f(x)+g(x)$, find $K'(-3)$

$K'(-3)=f'(-3)+g'(-3)$

$=1+-1$

$=0$

8. If $K(x)=f(x)\cdot g(x)$, find $K'(2)$

$K'(2)=f(2)\cdot g'(2)+g(2)\cdot f'(2)$

$=2\cdot 2+4(-2/3)$

$=4-8/3 = 4/3$

9. If $K(x)=f(g(x))$, find $K'(1)$

$K'(1)=f'(g(1))\cdot g'(1)$

$=f'(2)\cdot g'(1)$

$=-\frac{2}{3}\cdot 2$

$=-4/3$

10. If $K(x)=(g\circ f)(x)$, find $K'(-4)$

$K'(-4)=g'(f(-4))\cdot f'(-4)$

$=g'(-1)\cdot f'(-4)$

$=-1\cdot 1$

$=-1$

Derivatives of Trigonometric Functions

1. $y = 2\sin x$

$y' = 2\cos x$

2. $y = \frac{\sin x}{2} = \frac{1}{2}\sin x$

$y' = \frac{1}{2}\cos x$

3. $y = x + \cos x$

$y' = 1 - \sin x$

4. $y = x^2 - \frac{1}{2}\cos x$

$y' = 2x + \frac{1}{2}\sin x$

5. $y = 5 + \sin x$

$y' = \cos x$

6. $y = \frac{1}{x} - 3\sin x$

$y' = \frac{-1}{x^2} - 3\cos x$

7. $y = \pi\cos x$

$y' = -\pi\sin x$

8. $y = 1 + x - \cos x$

$y' = 1 + \sin x$

9. $y = x^{-1} + 5\sin x$

$y' = \frac{-1}{x^2} + 5\cos x$

10. $y = \csc x - 5x + 7$

$y' = -\csc x\cot x - 5$

11. $y = 4\sqrt{x} + 3\cos x$

$y' = \frac{4}{2\sqrt{x}} - 3\sin x$

$y' = \frac{2}{\sqrt{x}} - 3\sin x$

12. $y = 2\sin x + 3\cos x$

$y' = 2\cos x - 3\sin x$

13. $y = x \cdot \cos x$

$y' = x(-\sin x) + \cos x(1)$

$y' = -x\sin x + \cos x$

14. $y = 2\sin x - \tan x$

$y' = 2\cos x - \sec^2 x$

15. $y = 2x + \cot x$

$y' = 2 - \csc^2 x$

16. $y = \cot x \cdot \sec x$

$y = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$y = \frac{1}{\sin x}$

$y = \csc x$

$y' = -\csc x \cot x$

17. $y = x^2 \sin x$

$y' = x^2 \cos x + 2x \sin x$

18. $y = 5 + \frac{1}{\tan x}$

$y = 5 + \cot x$

$y' = -\csc^2 x$

19. $y = \frac{\sin x}{x}$

$y' = \frac{x\cos x - \sin x}{x^2}$

20. $y = \sin x \cdot \cos x$

$y' = \sin x \cdot \sin x + \cos x \cdot -\cos x$

$y' = \sin^2 x - \cos^2 x$

21. $y = \sin x \cdot \sec x$

$y = \sin x \cdot \frac{1}{\cos x}$

$y = \tan x$

$y' = \sec^2 x$

22. $y = \tan x \cdot \cot x$

$y = 1$

$y' = 0$

23. $y = \frac{4}{\cos x}$

$y = 4\sec x$

$y' = 4\sec x \tan x$

24. $y = \sin(3x + 1)$

$y' = \cos(3x+1) \cdot 3$

$y' = 3\cos(3x+1)$

25. $y = \tan(2x - x^3)$

$y' = \sec^2(2x - x^3) \cdot (2 - 3x^2)$

$y' = (2 - 3x^2)\sec^2(2x - x^3)$

26. $y = \cos\left(-\frac{x}{3}\right)$

$y' = -\sin\left(\frac{-x}{3}\right) \cdot -\frac{1}{3}$

$y' = \frac{1}{3}\sin\left(\frac{-x}{3}\right)$

27. $y = 4\sin^2 x + 5\cos^2 x$

$4(\sin x)^2 + 5(\cos x)^2$

$y' = 8\sin x \cdot \cos x + 10\cos x \cdot -\sin x$

$y' = 8\sin x\cos x - 10\sin x\cos x$

$y' = -2\sin x \cos x$

28. $y = \frac{1+\tan^2 x}{\sec x}$

$y = \frac{\sec^2 x}{\sec x}$

$y = \sec x$

$y' = \sec x \tan x$

29. $y = (1 + \cos 3x)^2$

$y' = 2(1 + \cos(3x)) \cdot -\sin(3x) \cdot 3$

$y' = -6(1 + \cos 3x)\sin 3x$

or

$y' = -6\sin 3x(1 + \cos 3x)$

30. $f(x) = \cot\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$

$f(x) = \frac{\cos\left(\frac{x}{2}\right)}{\cancel{\sin\left(\frac{x}{2}\right)}} \cdot \frac{\cancel{\sin\left(\frac{x}{2}\right)}}{1}$

$f(x) = \cos\left(\frac{x}{2}\right)$

$f'(x) = -\sin\left(\frac{x}{2}\right) \cdot \frac{1}{2}$

$f'(x) = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

Derivatives of e^x and a^x

Find the derivative of each.

1. $y = e^{2x}$

$y' = 2e^{2x}$

2. $y = e^{5x^2}$

$y' = 10x\,e^{5x^2}$

3. $y = e^{\sin x}$

$y' = e^{\sin x} \cdot \cos x$

4. $y = e^{\tan x}$

$y' = e^{\tan x} \cdot \sec^2 x$

5. $y = e^{x^2+2x}$

$y' = (2x+2)e^{x^2+2x}$

6. $y = e^{\sqrt{x}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

7. $y = 5^x$

$y' = 5^x \ln 5$

8. $y = e^{e^x}$

$y' = e^{e^x} \cdot e^x$

9. $y = 7^{x^2+2x^3}$

$y' = 7^{x^2+2x^3} \ln 7\,(2x + 6x^2)$

10. $y = \sin(e^{3x})$

$y' = \cos(e^{3x}) \cdot e^{3x} \cdot 3$

$y' = 3e^{3x} \cos e^{3x}$

11. $y = xe^x$

$y' = xe^x + e^x$

12. $y = (\sin x)e^x$

$y' = \sin x \cdot e^x + e^x \cos x$

or

$y' = e^x(\sin x + \cos x)$

13. $y = x^2 e^x$

$y' = x^2 \cdot e^x + 2x \cdot e^x$

14. $y = \frac{e^x}{x^2}$

$y' = \frac{x^2 e^x - 2x e^x}{x^4}$

$y' = \frac{xe^x - 2e^x}{x^3}$

15. $y = 2^x(x^2+1)$

$y' = 2^x(2x) + (x^2+1)2^x \ln 2$

16. $y = 3^{\ln x}$

$y' = 3^{\ln x} \ln 3 \cdot \frac{1}{x}$

$y' = \frac{3^{\ln x} \ln 3}{x}$

17. $y = x^2 + 4^x$

$y' = 2x + 4^x \ln 4$

18. $y = \ln e^{x^2}$

$y = x^2$

$y' = 2x$

19. $y = e^{\ln x^3}$

$y = x^3$

$y' = 3x^2$

20. $y = e^{3x} \cdot 4^{5x}$

$y' = e^{3x} 4^{5x} \ln 4 \cdot 5 + 4^{5x} e^{3x} 3$

$y' = e^{3x} 4^{5x}(5 \ln 4 + 3)$

21. $y = e^{\csc x}$

$y' = e^{\csc x} \cdot -\csc x \cot x$

$y' = -\csc x \cot x\, e^{\csc x}$

22. $y = 10^{\sin x}$

$y' = 10^{\sin x} \cdot \ln 10 \cdot \cos x$

23. $y = x^2 e^x - xe^x$

$y = e^x(x^2 - x)$

$y' = e^x(2x-1) + (x^2-x)e^x$

$y' = e^x(2x - 1 + x^2 - x)$

$y' = e^x(x^2 + x - 1)$

24. $y = xe^2 - e^x$

$y' = e^2 - e^x$

Derivatives of $\ln x$

Find each Derivative

1. $y = \ln(x^3 + 1)$

$y' = \frac{1}{x^3+1} \cdot 3x^2$

$y' = \frac{3x^2}{x^3+1}$

2. $y = \ln \sqrt{x}$

$y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2x}$

3. $y = \sqrt{\ln(x)} = (\ln x)^{1/2}$

$y' = \frac{1}{2} \ln(x)^{-1/2} \cdot \frac{1}{x}$

$y' = \frac{1}{2x\sqrt{\ln(x)}}$

4. $y = \ln |\sin x|$

$y' = \frac{1}{\sin x} \cdot \cos x$

$y' = \frac{\cos x}{\sin x}$

$y' = \cot x$

5. $y = \ln(\sec x)$

$y' = \frac{1}{\sec x} \cdot \sec x \tan x$

$y' = \tan x$

6. $y = x \cdot \ln x$

$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$

$y' = 1 + \ln x$

7. $y = \frac{\ln x}{x^2}$

$y' = \frac{x^2 \cdot \frac{1}{x} - \ln x \cdot 2x}{x^4}$

$y' = \frac{x - 2x \ln x}{x^4}$

$y' = \frac{1 - 2\ln x}{x^3}$

8. $y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x}$

$y' = \frac{1}{x \ln x}$

9. $y = (\sin x)(\ln x)$

$y' = (\sin x) \cdot \frac{1}{x} + (\ln x)(\cos x)$

$y' = \frac{\sin x}{x} + \ln x \cdot \cos x$

10. $y = \frac{x^2}{\ln x}$

$y' = \frac{\ln x \cdot 2x - x^2 \cdot \frac{1}{x}}{(\ln x)^2}$

$y' = \frac{2x \ln x - x}{(\ln x)^2}$

11. $y = \ln\left(\frac{5}{5-x}\right)$

$y' = \frac{5-x}{5}\left(\frac{(5-x) \cdot 0 - 5(-1)}{(5-x)^2}\right)$

$y' = \frac{5-x}{5} \cdot \frac{5}{(5-x)^2}$

$y' = \frac{1}{5-x}$

12. $y = \ln \sqrt{x^2 + 4}$

$y = \ln (x^2+4)^{1/2}$ *Prop. of Logs

$y = \frac{1}{2} \ln (x^2+4)$

$y' = \frac{1}{2} \cdot \frac{1}{x^2+4} \cdot 2x$

$y' = \frac{x}{x^2+4}$

13. $y = \ln(2 - \cos x)$

$y' = \frac{1}{2-\cos x} \cdot \sin x$

$y' = \frac{\sin x}{2-\cos x}$

14. $y = \ln(5 - x)^6$

$y = 6 \cdot \ln(5-x)$

$y' = 6 \cdot \frac{1}{5-x} \cdot -1$

$y' = \frac{-6}{5-x}$

15. $y = e^{\ln x^2}$

$y = x^2$

$y' = 2x$

16. $y = \ln(3x^2 + 2)^3$

$y = 3 \cdot \ln(3x^2+2)$

$y' = 3 \cdot \frac{1}{3x^2+2} \cdot 6x$

$y' = \frac{18x}{3x^2+2}$

17. $y = \ln x^3 + (\ln x)^3$

$y' = \frac{1}{x^3} \cdot 3x^2 + 3(\ln x)^2 \cdot \frac{1}{x}$

$y' = \frac{3}{x} + \frac{3(\ln x)^2}{x}$

or

$y' = \frac{3 + 3(\ln x)^2}{x}$

18. $y = \ln \sqrt{\ln(x)}$

$y' = \frac{1}{\sqrt{\ln x}} \cdot \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$

$y' = \frac{1}{2x \cdot \ln x}$

or

$y = \ln(\ln x)^{1/2}$

$y = \frac{1}{2} \ln(\ln x)$

$y' = \frac{1}{2} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$

$y' = \frac{1}{2x \cdot \ln x}$

Derivatives Review

1. Find the Equation of the tangent line of $f(x) = 4x^2 - 5x + 2$ at $x = 3$

$f'(x) = 8x - 5$ $\quad f(3) = 23$

$f'(3) = 24 - 5$

$f'(3) = 19$ $\quad y - 23 = 19(x - 3)$

2. Find the Equation of the normal line of $f(x) = -x^2 + 3x - 2$ at $x = 3$

$f'(x) = -2x + 3$ $\quad f(3) = -9 + 9 - 2$

$f'(3) = -6 + 3$ $\quad f(3) = -2$

$f'(3) = -3$ $\quad y + 2 = \frac{1}{3}(x - 3)$

$m_{norm} = \frac{1}{3}$

Find the Derivative.

3. $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 1} = \dfrac{(x+2)(x+1)}{(x+1)(x-1)}$

$f(x) = \dfrac{x+2}{x-1}$

$f'(x) = \dfrac{1(x+1) - 1(x+2)}{(x-1)^2}$

$f'(x) = \dfrac{-3}{(x-1)^2}$

4. $f(x) = \dfrac{x + 1}{\sqrt{x}} = \dfrac{x'}{x^{1/2}} + \dfrac{1}{x^{1/2}}$

$f(x) = x^{1/2} + x^{-1/2}$

$f'(x) = \dfrac{1}{2x^{1/2}} - \dfrac{1}{2x^{3/2}}$

5. $f(x) = \sin^2 3x = (\sin(3x))^2$

$f'(x) = 2\sin(3x)\cos(3x) \cdot 3$

$f'(x) = 6\sin(3x)\cos(3x)$

6. $f(x) = \dfrac{x(x^2 - 1)}{x + 1}$

$f(x) = \dfrac{x(x+1)(x-1)}{(x+1)}$

$f(x) = x^2 - x$

$f'(x) = 2x - 1$

or $f(x) = \dfrac{x^3 - x}{x + 1}$

$f'(x) = \dfrac{(x+1) \cdot 3x^2 - (x^3 - x) \cdot 1}{(x+1)^2}$

$f'(x) = \dfrac{3x^3 + 3x^2 - x^3 + x}{(x+1)^2}$

$f'(x) = \dfrac{2x^3 + 3x^2 + x}{(x+1)^2}$

7. $y = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

$y = (x^4 - x^3 + x^2 - x)(x^2 + x + 1)$

$y = x^6 + x^5 + x^4 - x^5 - x^4 - x^3 + x^4 + x^3 + x^2 - x^3 - x^2 - x$

$y = x^6 + x^4 - x^3 - x$

$y' = 6x^5 + 4x^3 - 3x^2 - 1$

8. $y = x \cdot \sin x + \cos x$

$y' = x\cos x + \sin x - \sin x$

$y' = x\cos x$

9. $y = 5\sec x + \tan x$

$y' = 5\sec x\tan x + \sec^2 x$

10. $f(x) = \sqrt{x} + 4\csc x$

$f'(x) = \dfrac{1}{2\sqrt{x}} - 4\csc x\cot x$

11. $f(x) = \dfrac{1}{x} - 10\sec x$

$f'(x) = \dfrac{-1}{x^2} - 10\sec x\tan x$

12. $y = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5) = \dfrac{2x^2 - 3x - 5}{x + 2}$

$y' = \dfrac{(x+2)(4x-3) - (2x^2 - 3x - 5)}{(x+2)^2}$

$y' = \dfrac{4x^2 + 5x - 6 - 2x^2 + 3x + 5}{(x+2)^2}$

$y' = \dfrac{2x^2 + 8x - 1}{(x+2)^2}$

13. $y = (2x - 7)^3$

$y' = 3(2x - 7)^2 \cdot 2$

$y' = 6(2x - 7)^2$

14. $f(x) = (9 - x^2)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(9 - x^2)^{-1/3} \cdot -2x$

$f'(x) = \dfrac{-4x}{3(9 - x^2)^{1/3}}$ or $\dfrac{-4x}{3\sqrt[3]{9 - x^2}}$

15.

$$y = \sqrt{\frac{1}{4x^2}} = \frac{1}{2x}$$

$y = \frac{1}{2}x^{-1}$ or $y = (2x)^{-1}$

$y' = -\frac{1}{2}x^{-2}$ $\quad y' = -1(2x)^{-2} \cdot 2$

$y' = \frac{-1}{2x^2}$ $\quad y' = \frac{-2}{4x^2} = \frac{-1}{2x^2}$

16.

$$f(x) = \frac{x^2}{x^2+3}$$

$f'(x) = \frac{(x^2+3)2x - x^2 \cdot 2x}{(x^2+3)^2}$

$f'(x) = \frac{2x^3 + 6x - 2x^3}{(x^2+3)^2}$

$f'(x) = \frac{6x}{(x^2+3)^2}$

17.

$$y = \frac{\cos \pi x + 1}{x}$$

$y' = \frac{x \cdot -\sin(\pi x) \cdot \pi - (\cos \pi x + 1) \cdot 1}{x^2}$

$y' = \frac{-\pi x \sin(\pi x) - \cos(\pi x) - 1}{x^2}$

18. $f(x) = 3 \tan 4x$

$f'(x) = 3\sec^2(4x) \cdot 4$

$f'(x) = 12\sec^2(4x)$

19. $y = \sin(\cos x)$

$y' = \cos(\cos x) \cdot -\sin x$

$y' = -\sin x \cdot \cos(\cos x)$

20. $y = 3x - 5\cos^2(\pi x)$ *Double chain

$y = 3x - 5[\cos(\pi x)]^2$

$y' = 3 - 10\cos(\pi x) \cdot -\sin(\pi x) \cdot \pi$

$y' = 3 + 10\pi \cos(\pi x) \sin(\pi x)$

21. $f(x) = \sqrt{x^2 + 2x + 8}$

$f(x) = (x^2+2x+8)^{1/2}$

$f'(x) = \frac{1}{2}(x^2+2x+8)^{-1/2}(2x+2)$

$f'(x) = \frac{x+1}{\sqrt{x^2+2x+8}}$

22.

$$y = \frac{1}{x} + \sqrt{\cos x}$$

$y' = \frac{-1}{x^2} + \frac{1}{2}(\cos x)^{-1/2} \cdot -\sin x$

$y' = \frac{-1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$

23. $y = (x^2+1)e^{3x}$

$y' = (x^2+1) \cdot e^{3x} \cdot 3 + e^{3x} \cdot 2x$

$y' = 3e^{3x}(x^2+1) + 2xe^{3x}$

or $y' = e^{3x}[3(x^2+1) + 2x]$

$y' = e^{3x}(3x^2+2x+1)$

24. $y = x \cdot 5^{3x}$

$y' = x \cdot 5^{3x} \cdot \ln 5 \cdot 3 + 5^{3x}$

$y' = 3x \ln 5 \cdot 5^{3x} + 5^{3x}$

or

$y' = 5^{3x}(3x\ln 5 + 1)$

25. $y = \sin(e^{2x})$

$y' = \cos(e^{2x}) \cdot e^{2x} \cdot 2$

$y' = 2e^{2x}\cos(e^{2x})$

26. $y = e^{e^{5x}}$

$y' = e^{e^{5x}} \cdot e^{5x} \cdot 5$

$y' = 5e^{5x}e^{e^{5x}}$

27. $y = 3^{x^2+3x}$

$y' = 3^{x^2+3x} \ln 3 \, (2x+3)$

28. $y = (\ln x)^x$

$y' = (\ln x)^x \ln(\ln x)$

or Prop. of Logs 1st

$y = x \ln x$

$y' = x \cdot \frac{1}{x} + \ln x$ $\quad y' = 1 + \ln x$

29. $y = 3^{5x}$

$y' = 3^{5x} \ln 3 \cdot 5$

30. $y = e^{\ln(5x^2)}$

$y = 5x^2$

$y' = 10x$

31. Find the first TWO Derivatives:

$y = 2(x^2 - 1)^3$

$y' = 6(x^2-1)^2 \cdot 2x$

$y' = 12x(x^2-1)^2$

$y'' = 12x \cdot 2(x^2-1) \cdot 2x + 12(x^2-1)^2$

$y'' = 48x^2(x^2-1) + 12(x^2-1)^2$

32. Find the $f', f'', and\ f'''$

$f(x) = x^3 + 2x^2 - 4x + 5$

$f'(x) = 3x^2 + 4x - 4$

$f''(x) = 6x + 4$

$f'''(x) = 6$

Evaluate the Limit using L'Hopital's Rule:

33. $\lim\limits_{x \to 3} \dfrac{x^2 + 4x - 21}{x^2 - 7x + 12} = \frac{0}{0}$

$\lim\limits_{x \to 3} \dfrac{2x+4}{2x-7} = \dfrac{2(3)+4}{2(3)-7}$

$= \frac{10}{-1} = -10$

34. $\lim\limits_{x \to 0} \dfrac{\tan 3x}{\ln(1+x)} = \frac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{3\sec^2(3x)}{\frac{1}{1+x}} = \dfrac{3\sec^2(3 \cdot 0)}{\frac{1}{1+0}}$

$= \dfrac{3(1)^2}{1} = 3$

35. $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1} = \frac{0}{0}$

$\lim\limits_{x \to 1} \dfrac{2x}{1} = 2(1) = 2$

36. $\lim\limits_{x \to 3} \dfrac{\sqrt{x+6} - 3}{x - 3} = \frac{0}{0}$

$\lim\limits_{x \to 3} \dfrac{\frac{1}{2}(x+6)^{-1/2} - 0}{1}$

$\lim\limits_{x \to 3} \dfrac{1}{2\sqrt{x+6}} = \dfrac{1}{2\sqrt{3+6}} = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6}$

37. $\lim\limits_{x \to 0} \dfrac{x}{\frac{1}{3+x} - \frac{1}{3}}$ $\frac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{1}{(3+x)^{-1} - \frac{1}{3}}$

$\lim\limits_{x \to 0} \dfrac{1}{-1(3+x)^{-2} - 0}$

$\lim\limits_{x \to 0} -1(3+x)^2 = -1(3+0)^2$

$= -9$

38. $\lim\limits_{x \to \infty} \dfrac{x - 4}{x^2 - 6x + 8} = \frac{\infty}{\infty}$

$\lim\limits_{x \to \infty} \dfrac{1}{2x-6} = \frac{1}{\infty}$

$= 0$

Unit 4 Review

Given the following functions, find:

1. $f'''(-1)$ where $f(x) = \frac{5}{x^4}$ $f(x)=5x^{-4}$

$f'(x)=-20x^{-5}$

$f''(x)=100x^{-6}$

$f'''(x)=-600x^{-7}$

$f'''(-1)=\frac{-600}{(-1)^7}=\frac{-600}{-1}=600$

2. $f''(-2)$ where $f(x) = \frac{1}{x+1}$ $f(x)=(x+1)^{-1}$

$f'(x)=-1(x+1)^{-2}$

$f''(x)=2(x+1)^{-3}$

$f''(-2)=\frac{2}{(-2+1)^3}=\frac{2}{(-1)^3}=\frac{2}{-1}$

$f''(-2)=-2$

Find the following derivatives:

3. $y = \sqrt{x} + \frac{1}{x}$ or $y=x^{1/2}+x^{-1}$

$y'=\frac{1}{2\sqrt{x}}-\frac{1}{x^2}$

4. $y = \tan(3x^2 - 5)$

$y'=\sec^2(3x^2-5)\cdot 6x$

$y'=6x\sec^2(3x^2-5)$

5. $y = \frac{1}{\sqrt[3]{3-x^3}}$ $y=(3-x^3)^{-1/3}$

$y'=-\frac{1}{3}(3-x^3)^{-4/3}\cdot -3x^2$

$y'=\frac{x^2}{(3-x^3)^{4/3}}$

6. $y = x^2 \sin x$

$y'=x^2\cos x+2x\sin x$

7. $y = \sqrt{\cos^3(4x)}$

$y=([\cos(4x)]^3)^{1/2}$

$y=(\cos(4x))^{3/2}$

$y'=\frac{3}{2}(\cos 4x)^{1/2}\cdot -\sin(4x)\cdot 4$

$y'=-6\sin(4x)\sqrt{\cos(4x)}$

8. $y = \sin^2(4x)$ $y=[\sin(4x)]^2$

$y'=2\sin(4x)\cos(4x)\cdot 4$

$y'=8\sin(4x)\cos(4x)$

9. $y = 4^x x^4$

$y'=4^x\cdot 4x^3+x^4\cdot 4^x\ln 4$

or

$y'=x^3 4^x(4x+x\ln 4)$

10. $y = x(3x-9)^4$

$y'=x\cdot 4(3x-9)^3\cdot 3+(3x-9)^4$

$y'=12x(3x-9)^3+(3x-9)^4$

or

$y'=(3x-9)^3(12x+3x-9)$

$(3x-9)^3(9x-9)$

11. $y = x^3 \sin x - 5\cos x$

$y' = x^3\cos x + 3x^2\sin x + 5\sin x$

12. $y = x^3 e^x$

$y' = x^3 e^x + 3x^2 e^x$

or

$y' = x^2 e^x (x+3)$

13. $y = \frac{\ln x}{x^2}$

$y' = \frac{x^2 \cdot \frac{1}{x} - \ln x \cdot 2x}{x^4}$

$y' = \frac{x - 2x\ln x}{x^4} = \frac{1 - 2\ln x}{x^3}$

14. $y = \frac{\sin x}{x}$

$y' = \frac{x\cos x - \sin x}{x^2}$

15. $y = e^{-x^3}$

$y' = e^{-x^3} \cdot -3x^2$

$y' = \frac{-3x^2}{e^{x^3}}$

16. $y = 7^{\ln x}$

$y' = \frac{7^{\ln x}\ln 7}{x}$

17. $y = e^x - xe^x$

$y' = e^x - (xe^x + e^x)$

$y' = e^x - xe^x - e^x$

$y' = -xe^x$

18. $y = \frac{e^x}{x^2-1}$

$y' = \frac{(x^2-1)e^x - 2xe^x}{(x^2-1)^2}$

19. $y = \ln\sqrt{x^2-4}$ $\quad y = \ln(x^2-4)^{1/2}$

$y = \frac{1}{2}\ln(x^2-4)$

$y' = \frac{1}{2} \cdot \frac{1}{x^2-4} \cdot 2x$

$y' = \frac{x}{x^2-4}$

20. $y = 7^{e^x}$

$y' = 7^{e^x}\ln 7\, e^x$

21. $y = 4^x \sin x$

$y' = 4^x\cos x + \sin x\, 4^x\ln 4$

#22 – 26 Given the following chart, find the indicated derivatives.

22. If $h(x) = 3f(x) + g(x)$, find $h'(2)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-3	4	2	7
2	1	-5	9	-8
3	7	7	-2	9

$h'(2) = 3f'(2) + g'(2)$
$= 3(-5) + -8$

$h'(2) = -23$

23. If $h(x) = 7g(x)$, find $h'(3)$

$h'(3) = 7 \cdot g'(3)$
$= 7 \cdot 9$
$h'(3) = 63$

24. If $h(x) = g(x) \cdot f(x)$, find $h'(2)$

$h'(x) = g(x) \cdot f'(x) + f(x) g'(x)$
$h'(2) = g(2) f'(2) + f(2) g'(2)$
$= 9 \cdot -5 + 1 \cdot -8$
$= -45 - 8$
$h'(2) = -53$

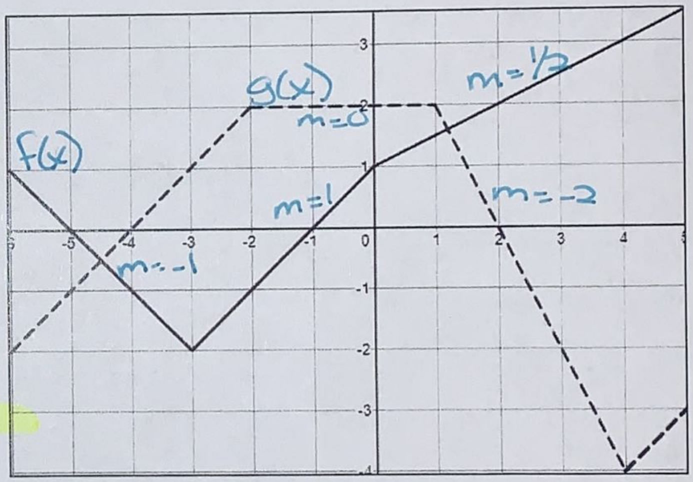
25. If $h(x) = \frac{f(x)}{g(x)}$, find $h'(1)$

$h'(1) = \dfrac{g(1) \cdot f'(1) - f(1) g'(1)}{[g(1)]^2}$
$= \dfrac{(2)(4) - (-3)(7)}{(2)^2} = \dfrac{8+21}{4} =$
$h'(1) = \frac{29}{4}$

26. If $h(x) = f(g(x))$, find $h'(1)$

$h'(1) = f'(g(1)) \cdot g'(1)$
$= f'(2) \cdot g'(1)$
$= -5 \cdot 7$
$h'(1) = -35$

#27 - 30 Use the graph to find the derivative. F(x) is the solid line and g(x) is the dashed line.



27. If $K(x) = 4f(x) - 3g(x)$, find $K'(-1)$

$K'(x) = 4f'(x) - 3g'(x)$
$K'(-1) = 4f'(-1) - 3g'(-1)$
$= 4(1) - 3(0)$
$K'(-1) = 4$

28. If $K(x) = f(x) \cdot g(x)$, find $K'(2)$

$K'(x) = f(x) g'(x) + g(x) f'(x)$
$K'(2) = f(2) \cdot g'(2) + g(2) f'(2)$
$= (2)(-2) + (0)(1/2)$
$= -4 + 0$
$K'(2) = -4$

29. If $K(x) = f(g(x))$, find $K'(0)$

$K'(x) = f'(g(x)) \cdot g'(x)$
$K'(0) = f'(g(0)) \cdot g'(0)$
$= f'(2) \cdot g'(0)$
$= \frac{1}{2} \cdot 0$

$K'(0) = 0$

30. If $K(x) = (g \circ f)(x)$, find $K'(-4)$

$K'(x) = g'(f(x)) \cdot f'(x)$
$K'(-4) = g'(f(-4)) \cdot f'(-4)$
$= g'(-1) \cdot f'(-4)$
$= 0 \cdot -1$
$K'(-4) = 0$