Unit 4 Additional Review – Derivative Rules

1. Find the equation of the tangent line to the curve $f(x) = \cos 2x$ at $x = \frac{2\pi}{3}$

$f'(x) = -2\sin 2x$

$f'(\frac{2\pi}{3}) = -2\sin(2 \cdot \frac{2\pi}{3})$

$f'(\frac{2\pi}{3}) = -2\sin(\frac{4\pi}{3}) = -2(\frac{-\sqrt{3}}{2}) = \sqrt{3}$

$f(\frac{2\pi}{3}) = \cos(\frac{4\pi}{3}) = -\frac{1}{2}$

$y + \frac{1}{2} = \sqrt{3}(x - \frac{2\pi}{3})$

b. Find the equation of the normal line to $f(x) = \cos 2x$ at $x = \frac{2\pi}{3}$

$y + \frac{1}{2} = -\frac{1}{\sqrt{3}}(x - \frac{2\pi}{3})$

2. Find $f'(-1)$ if $f(x) = \frac{5x^2+3}{x}$

$f(x) = 5x + 3x^{-1}$

$f'(x) = 5 - \frac{3}{x^2}$

$f'(-1) = 5 - \frac{3}{(-1)^2} = 5 - 3 = 2$

Find the derivative.

3. $f(x) = 3x^4 + e^{2x} + \tan(x^3)$

$f'(x) = 12x^3 + 2e^{2x} + \sec^2(x^3) \cdot 3x^2$

$f'(x) = 12x^3 + 2e^{2x} + 3x^2\sec^2(x^3)$

4. $y = e^{\frac{x^2+1}{5x-2}}$

$y' = e^{\frac{x^2+1}{5x-2}} \cdot \frac{(5x-2)2x - (x^2+1)5}{(5x-2)^2}$

$y' = e^{\frac{x^2+1}{5x-2}} \cdot \frac{5x^2-4x-5}{(5x-2)^2}$

5. $y = \frac{e^{x^3}}{\sin x}$

$y' = \frac{\sin x \cdot e^{x^3} \cdot 3x^2 - e^{x^3}\cos x}{\sin^2 x}$

$y' = \frac{e^{x^3}(3x^2\sin x - \cos x)}{\sin^2 x}$

6. $y = \sin^4(6x+1) = (\sin(6x+1))^4$

$y' = 4\sin^3(6x+1)\cos(6x+1) \cdot 6$

$y' = 24\sin^3(6x+1)\cos(6x+1)$

7. $f(x) = \cos\sqrt[3]{3x-4}$ $= \cos(3x-4)^{1/3}$

$f'(x) = -\sin\sqrt[3]{3x-4} \cdot \frac{1}{\cancel{3}}(3x-4)^{-2/3} \cdot \cancel{3}$

$f'(x) = \dfrac{-\sin\sqrt[3]{3x-4}}{(3x-4)^{2/3}}$

8. $y = \ln(6x^2 - 4x) + 7\csc(x^2) - \dfrac{10}{x}$ $-10x^{-1}$

$y' = \frac{1}{6x^2-4x} \cdot (12x-4) - 7\csc(x^2)\cot(x^2) \cdot 2x + \frac{10}{x^2}$

$y' = \dfrac{6x-2}{3x^2-2x} - 14x\csc(x^2)\cot(x^2) + \dfrac{10}{x^2}$

9. $f(x) = \sin^{-1}(5e^x)$

$f'(x) = \dfrac{1}{\sqrt{1-(5e^x)^2}} \cdot 5e^x$

$f'(x) = \dfrac{5e^x}{\sqrt{1-25e^{2x}}}$

Use logarithmic differentiation to find the derivative.

10. $y = (2x-3)^{x^2}$

$\ln y = x^2 \ln(2x-3)$

$\frac{1}{y}\frac{dy}{dx} = x^2 \cdot \frac{2}{2x-3} + 2x\ln(2x-3)$

$\frac{dy}{dx} = y\left(\frac{2x^2}{2x-3} + 2x\ln(2x-3)\right)$

$\frac{dy}{dx} = (2x-3)^{x^2}\left(\frac{2x^2}{2x-3} + 2x\ln(2x-3)\right)$

11. $y = x^4(2x-5)^3$

$\ln y = \ln\left[x^4(2x-5)^3\right]$

$\ln y = 4\ln x + 3\ln(2x-5)$

$\frac{1}{y}\frac{dy}{dx} = \frac{4}{x} + \frac{3 \cdot 2}{2x-5}$

$\frac{dy}{dx} = y\left(\frac{4}{x} + \frac{6}{2x-5}\right)$

$\frac{dy}{dx} = x^4(2x-5)^3\left(\frac{4}{x} + \frac{6}{2x-5}\right)$

In #12 and 13, use implicit differentiation to find the derivative.

12. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ when $y^{-2} = 2x^3$

$-2y^{-3}\frac{dy}{dx} = 6x^2$

$-\frac{2}{y^3}\frac{dy}{dx} = 6x^2$

$\frac{dy}{dx} = \frac{6x^2y^3}{-2}$

$\frac{dy}{dx} = -3x^2y^3$

$\frac{d^2y}{dx^2} = -3x^2 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot -6x$

$\frac{d^2y}{dx^2} = -9x^2y^2(-3x^2y^3) - 6xy^3$

$\frac{d^2y}{dx^2} = 27x^4y^5 - 6xy^3$

13. $3xy^2 - \sin y = 4x + 3$

$3x \cdot 2y\frac{dy}{dx} + y^2 \cdot 3 - \cos y\frac{dy}{dx} = 4$

$(6xy - \cos y)\frac{dy}{dx} = 4 - 3y^2$

$\frac{dy}{dx} = \frac{4-3y^2}{6xy - \cos y}$

Entire page 3 points each
In #14-16, use the chart information to find the following.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
−4	−2	−3	5	3
−2	0	3	7	1
0	2	4	4	−2
1	-2	-3	-1	3
2	4	1	3	−3
4	5	−4	2	−4

14. Find $\frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$ when $x = -4$.

$\frac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}$

$\frac{f(-4)g'(-4) - g(-4)f'(-4)}{[f(-4)]^2} = \frac{-2(3) - (-3)(5)}{(-2)^2} = \frac{-6+15}{4} = \frac{9}{4}$

15. Find $\frac{d}{dx}\left[x - f\left(\sqrt{x}\right)\right]$ when $x = 4$.

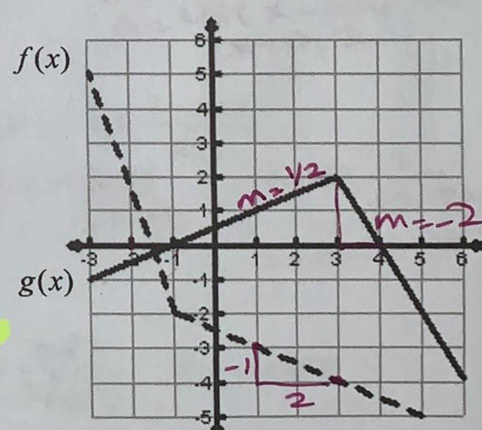
$y' = 1 - f'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$y' = 1 - f'(2) \cdot \frac{1}{4} = 1 - 3\left(\frac{1}{4}\right) = \frac{1}{4}$

16. Find $\frac{d}{dx}\left[g^{-1}(x)\right]$ when $x = 1$.

$\frac{1}{g'(g^{-1}(x))} = \frac{1}{g'(g^{-1}(1))} = \frac{1}{g'(2)} = \frac{1}{-3}$

In 17- 19, f and g are functions whose graphs are given to the right. Find the following.



17. $p(x) = 3f(x) \cdot g(x)$, find $p'(2)$.

$p'(x) = 3f(x)g'(x) + g(x) \cdot 3f'(x)$

$p'(2) = 3f(2)g'(2) + 3g(2)f'(2)$

$p'(2) = 3(-\frac{7}{2})(\frac{1}{2}) + 3(\frac{3}{2})(-\frac{1}{2})$

$-\frac{21}{4} - \frac{9}{4} = -\frac{30}{4} = -\frac{15}{2}$

18. $m(x) = \frac{g(x)}{x}$, find $m'(-2)$.

$m'(x) = \frac{x \cdot g'(x) - g(x)}{x^2}$

$m'(-2) = \frac{-2 \cdot g'(-2) - g(-2)}{4} = \frac{-2(\frac{1}{2}) - (-\frac{1}{2})}{4} = \frac{-1 + 1/2}{4} = \frac{-1}{2} \cdot \frac{1}{4} = \frac{-1}{8}$

19. h(x) = f(g(x)), find h'(4).

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(4) = f'(g(4)) \cdot g'(4)$

$= f'(0) \cdot g'(4)$

$= -\frac{1}{2} \cdot -2$

$h'(4) = 1$

Multiple Choice: Show all work leading to your answer.

c 20. $\lim_{h\to 0}\dfrac{\sin\left(\dfrac{\pi}{2}+h\right)-1}{h}=$

(a) 1 (b) –1 (c) 0 (d) ∞ (e) none of these

$f(x)=\sin x \quad a=\frac{\pi}{2}$
$f'(x)=\cos x$
$f'(\frac{\pi}{2})=\cos\frac{\pi}{2}=0$

c 21. If $f(x)=x\ln x$, which of the following is $f'''(e)$?

(a) $\dfrac{1}{e}$ (b) 0 (c) $-\dfrac{1}{e^2}$ (d) $\dfrac{1}{e^2}$ (e) $\dfrac{2}{e^3}$

$f'(x)=\frac{x}{x}+\ln x = 1+\ln x$
$f''(x)=0+\frac{1}{x}=\frac{1}{x}$
$f'''(x)=\frac{-1}{x^2}$
$f'''(e)=-\frac{1}{e^2}$

e 22. If $f(x)=16\sqrt{x}$, then $\left.\dfrac{d^2y}{dx^2}\right|_{x=4}$ is

(a) –32 (b) –16 (c) –4 (d) –2 (e) $-\dfrac{1}{2}$

$f'(x)=\frac{16}{2\sqrt{x}}$
$f'(x)=\frac{8}{\sqrt{x}}=8x^{-1/2}$
$f''(x)=-4x^{-3/2}$
$f''(4)=\frac{-4}{\sqrt{4}^3}=\frac{-4}{8}$

e 23. If $f(x)=x^4-4x^3+4x^2-1$, then the values of x for which f(x) has a horizontal tangent line are:

(a) 1, 2 (b) 0, -1, -2 (c) -1, 2 (d) 0 (e) 0, 1, 2

$f'(x)=4x^3-12x^2+8x$
$0=4x^3-12x^2+8x$
$0=4x(x^2-3x+2)$
$0=4x(x-2)(x-1)$
$x=0,2,1$

d 24. If $\sin x-\cos y-2=0$, then $\dfrac{dy}{dx}=$

(a) $-\cot x$ (b) $-\cot y$ (c) $\dfrac{\cos x}{\sin y}$ (d) $-\csc y\cos x$ (e) $\dfrac{2-\cos x}{\sin y}$

$\cos x+\sin y\frac{dy}{dx}-0=0$ $\quad \frac{dy}{dx}=\frac{-\cos x}{\sin y}=-\cos x\cdot\frac{1}{\sin y}$
$=-\cos x\csc y$

a 25. If $f(x)=\dfrac{1}{2\sin 2x}$, then $f'(x)=$

(a) $-\csc(2x)\cot(2x)$ (b) $\dfrac{1}{4\cos 2x}$ (c) $-4\csc(2x)\cot(2x)$ (d) $\dfrac{\cos 2x}{2\sqrt{\sin 2x}}$ (e) $-\csc^2(2x)$

$f(x)=\frac{1}{2}\csc(2x)$
$f'(x)=\frac{1}{2}\cdot-\csc(2x)\cot(2x)\cdot 2$
$f'(x)=-\csc(2x)\cot(2x)$

b 26. $\dfrac{d}{dx}\left(\dfrac{2}{(5x+1)^3}\right)=$

a. $\dfrac{-30}{(5x+1)^2}$ b. $\dfrac{-30}{(5x+1)^4}$ c. $\dfrac{-6}{(5x+1)^4}$ d. $\dfrac{-10}{3(5x+1)^{-4/3}}$ e. $\dfrac{30}{(5x+1)^4}$

$\frac{(5x+1)^3\cdot 0-2\cdot 3(5x+1)^2\cdot 5}{(5x+1)^6}=\frac{-30}{(5x+1)^4}$

Calculator Allowed

27. The equation of motion of a particle is: $s(t)=t^3-12t^2+36t+4$, where s is in meters and t is in seconds. Find the following when 0<t<7. Be sure to include units where appropriate.

A. Find the velocity function in terms of t.

$v(t)=3t^2-24t+36$

B. When is the particle at rest?

$0=3(t^2-8t+12)$ $\quad t=2,6$ sec.

$0=(t-6)(t-2)$

C. When is the particle moving forward?

$(0,2)\cup(6,7)$ sec.

+ − +

0 2 6 7

D. What is the acceleration function in terms of t?

$a(t)=6t-24$

F. What is the velocity when the acceleration is zero?

$0=6t-24$ $\quad v(4)=3(4)^2-24(4)+36$

$t=4$ sec $\quad v(4)=-12$ m/s²

G. When is the particle slowing down?

a(t)
$0=6t-24$
$t=4$

v(t) + − − + (0, 2, 6, 7) $\quad (0,2)\cup(4,6)$ sec

a(t) − − − + + + + (0, 4, 7)

H. What is the displacement from t = 0 to t = 7?

$s(7)-s(0)=7$ m

I. What is the total distance traveled from t = 0 to t = 7?

$|s(2)-s(0)|+|s(6)-s(2)|+|s(7)-s(6)|$

$32 + 32 + 7 = 71$ m

23. If $f(x)=\ln(x+1)-\sin^2 x$, find the following:

(a) f(3) ≈ 1.366

(b) $f'(3)$ ≈ 0.529

(c) Find the smallest positive value of x at which the tangent line to f(x) is horizontal. $x\approx 0.398$

Solve $(\frac{d}{dx}(f(x))=0, x) | x>0$

(d) Find the smallest positive value of x at which the tangent line to the graph of f has a slope of $\frac{1}{2}$.

Solve $(\frac{d}{dx}(f(x))=\frac{1}{2}, x) | x>0$ $\quad x\approx 0.178$

(e) $f''(0.22)$ ≈ -2.481