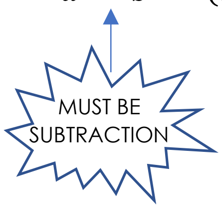
Factoring Polynomials

Always look for a GCF First!

2 TERMS

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$



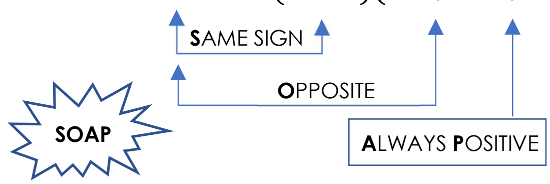
*Remember that a perfect square is the product of two identical factors!

Sum or Difference of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

or

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$



*A perfect cube is the product of __________ identical factors!

Difference of Squares	Sum or Difference of Cubes
a. $3x^2 - 12$	d. $x^3 + 8$
b. $4x^6 - 9$	e. $x^3 - 27$
c. $x^4 - 16$	f. $64x^3 + 1$

You Try: Factor completely

a. $2x^4 - 50$ b. $x^3 - 125$ c. $8x^3 + 27$

3 TERMS

Sum and Product: when $a = 1$	Slide and Divide: when $a > 1$
a. $x^2 - 10x + 21$	c. $2x^4 + 9x^2 - 5$
b. $x^4 + 2x^2 - 24$	

You Try: Factor completely

a. $x^4 + 4x^2 - 5$

b. $3x^4 + 11x^2 + 6$

4 TERMS	
Factor by Grouping	
a. $x^3 + 5x^2 + 2x + 10$	c. $x^3 - 2x^2 - 9x + 18$
b. $2x^3 - 8x^2 + 3x - 12$	d. $4x^3 + 16x^2 - x - 4$

You Try: Factor Completely

a. $5x^3 - 15x^2 + 3x - 9$

b. $x^3 + 8x^2 - 4x - 32$

c. $5x^3 + 40$

c. $3x^4 - 24x^2 - 27$

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Factoring Polynomials Practice

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Factor each completely.

1) $15a^4 - 45a^2$

2) $10x^4 + 30x^2$

3) $a^2 - 3a - 18$

4) $3p^2 - 14p + 16$

5) $4b^4 - 1$

6) $2n^4 - 18$

7) $16p^6 - 25$

8) $25n^6 - 9$

9) $x^3 - 216$

10) $27u^3 - 8$

11) $64x^3 + 27$

12) $x^3 + 125$

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13) $x^4 - 7x^2 - 30$

14) $x^4 + 3x^2 - 18$

15) $x^4 - 9x^2 + 18$

16) $x^4 + 4x^2 - 12$

17) $7a^4 - 13a^2 + 6$

18) $2x^4 + x^2 - 6$

19) $u^6 + 3u^3 - 70$

20) $5x^6 - 10x^3 - 15$

21) $5m^4 + 27m^2 + 10$

22) $4x^4 + 2x^2 - 30$

23) $3x^3 + 6x^2 + 8x + 16$

24) $v^3 - 8v^2 + v - 8$

25) $6p^3 + 7p^2 - 18p - 21$

26) $7x^3 + 5x^2 - 35x - 25$

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Solving Polynomial Equations by Factoring

1. Make the equation equal 0.
2. Factor the polynomial completely using the strategies we've learned.
3. Set each factor equal to zero and solve. You may have to use square root or quadratic formula to solve factors in quadratic form.
4. Check the solutions by substituting each one into the original equation.

> The number of solutions is equal to the________________, *n*, of the polynomial.
>
> This includes counting double roots or triple roots according to the multiplicity.

Factor completely, then solve.

Example:	Practice:
A. $x^3 - 10x^2 = 0$	1. $2x^3 + 12x^2 = 0$ 2. $x^3 - 7x^2 = 0$ 3. $8x^3 + 12x^2 = 0$
B. $3x^3 - 27x = 0$	4. $5x^3 - 80x = 0$ 5. $x^3 - 25x = 0$ 6. $6x^3 - 6x = 0$
C. $x^3 + 8 = 0$	7. $x^3 - 27 = 0$ 8. $27x^3 + 125 = 0$ 9. $64x^3 - 1 = 0$

D. $x^3 + 5x^2 - 24x = 0$	10. $x^3 - 3x^2 - 54x = 0$ 11. $x^3 + 15x^2 + 56x = 0$ 12. $x^3 - x^2 - 72x = 0$
E. $3x^3 + 14x^2 - 5x = 0$	13. $2x^3 + x^2 - x = 0$ 14. $4x^3 - 14x^2 + 12x = 0$ 15. $5x^3 - 21x^2 - 20x = 0$
F. $x^4 + 4x^2 - 5 = 0$	16. $x^4 - 10x^2 + 24 = 0$ 17. $x^4 + 9x^2 + 14 = 0$ 18. $x^4 - 18x^2 + 81 = 0$
G. $4x^3 + 12x^2 - 3x - 9 = 0$	19. $5x^3 + 3x^2 + 20x + 12 = 0$ 20. $2x^3 + 5x^2 - 4x - 10 = 0$ 21. $4x^3 + 4x^2 + 5x + 5 = 0$

Polynomial Long Division

When dividing a polynomial by a monomial, we divide each term by the monomial, ______________________ the coefficients and _________________________ the exponents.

Ex 1: Divide

a. $(14x^3 + 28x^2 - 70x) \div (7x)$

b. $\frac{12x^5 - 3x^3 + 9x^2}{3x^2}$

However, when we divide a polynomial by another polynomial, we must use long division.

Recall: What is the process for long division? $3728 \div 5$

The process for polynomial long division is very similar:

Ex 2: Divide

a. $(x^3 + 5x^2 + 5x - 2) \div (x + 2)$

b. $(x^4 + 2x^3 - 5x^2 + 3x - 1) \div (x - 1)$

Notice that the remainder for each of these quotients is ______. This means that the divisor is a factor of the dividend. This is just like division with integers:

$15 \div 3 = 5$ and the remainder is ______. This means that 3 is a factor of 15.

Powers must count down!

Insert a place holder for any missing power, including the constant!

Ex 3: Divide $(4x^3 - 5x + 3) \div (2x + 1)$

Ex 4: Divide $(4a^4 + 2a^2 - 4a + 12) \div (a + 2)$

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Dividing Polynomials Practice

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Divide.

1) $(20x^3 + 20x^2 + 4x) \div 4x$

2) $(12n^6 + 4n^5 + 5n^4) \div 4n^2$

3) $(6x^3 + 24x^2 + 18x) \div 6x$

4) $(18x^3 + 24x^2 + 18x) \div 6x$

Divide using long division.

5) $(2v^3 + 19v^2 - 10v) \div (v + 10)$

6) $(b^3 + b^2 - 20b - 50) \div (b - 5)$

7) $(2n^3 - 6n^2 - 12n + 22) \div (n - 4)$

8) $(8x^4 - 45x^3 + 23x^2 + 17x - 36) \div (x - 5)$

9) $(k^4 - 88k^2 + 70k - 69) \div (k - 9)$

10) $(m^5 + 13m^4 + 40m^3 - 2m - 17) \div (m + 5)$

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Synthetic Division

When dividing a polynomial by a linear (degree _____) factor, we can use a shorthand method called synthetic division.

Compare the two methods, dividing $(2x^3 + 9x^2 + 5x - 1) \div (x + 3)$	
Long Division	Synthetic Division

Ex 2: $(5a^5 + 2a^3 + a^2 - 7a + 1) \div (a - 1)$

Don't forget!

Ex 3: $(2x^3 - 45x - 7) \div (x - 5)$

Ex 4: $(x^3 - 4x^2 - 9x + 36) \div (x - 3)$

Recall that if the remainder is ____, this means that the divisor is a factor of the dividend.

Can you factor $x^3 - 4x^2 - 9x + 36$ completely?

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Synthetic Division Practice

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Divide using synthetic division.

1) $(n^3 - 3n^2 - 42n + 16) \div (n - 8)$

2) $(10a^4 + 62a^3 - 53a^2 + 22a + 7) \div (a + 7)$

3) $(m^3 + 10m^2 + 8m - 9) \div (m + 9)$

4) $(x^4 + 2x^3 - 34x^2 + 45x - 21) \div (x - 4)$

5) $(7x^4 + 14x^3 - 6x - 17) \div (x + 2)$

6) $(r^5 - 2r^4 - 48r^3 - 8r - 52) \div (r + 6)$

Factor each. One factor has been given.

7) $3x^3 - 8x^2 - 13x + 30 = 0$; $x - 3$

8) $3x^3 - 16x^2 + x + 20 = 0$; $x - 5$

9) $2x^3 + 5x^2 + x - 2 = 0$; $x + 2$

10) $2x^3 - 9x^2 - 2x + 24 = 0$; $x - 2$

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Factoring and Solving Given one Factor or Root

We can use synthetic division to factor and solve polynomial equations.

$(x-4)$ is a factor of the polynomial equation $x^3+x^2-14x-24=0$	
FACTOR We can **factor** completely: 1. Use synthetic division to depress the polynomial 2. Factor the remaining polynomial using one of the strategies we've learned 3. Write the polynomial in factored form.	**SOLVE** We can find the **roots**, or solutions, of the polynomial equation: 1. Factor completely. 2. Set each factor equal to zero and solve. 3. Write the solutions using set notation.

Ex 1: Factor $x^3-3x^2-40x+84=0$ given that one factor is $(x-2)$.

What are the roots of the equation?

Ex 2: Factor $x^3-4x^2+5x-20=0$ given that one factor is $(x-4)$.

Ex 3: Factor $2x^3-3x^2-17x-12=0$ given that one factor is $(x+1)$.

Ex 4: Find all roots of $x^3 - 11x^2 + 38x - 40 = 0$,

given that one root is 2.

What are the factors of the equation?

Ex 5: Find all roots of $x^3 - 9x^2 + 22x - 12 = 0$,

given that one root is 3.

Ex 6: Find all roots of $x^4 + 6x^3 + 3x^2 - 12x - 10 = 0$,

given the roots -5 and -1.

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Factoring & Solving Given one Factor or Root

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Factor each. One factor has been given.

1) $x^3 - 7x^2 + 16x - 12 = 0;\ x - 3$

2) $x^3 + x^2 - 9x - 9 = 0;\ x + 3$

3) $x^3 + 2x^2 - 9x - 18 = 0;\ x + 3$

4) $x^3 - 21x - 20 = 0;\ x - 5$

5) $2x^3 - 7x^2 + 9 = 0;\ x - 3$

6) $3x^3 + 26x^2 + 61x + 30 = 0;\ x + 5$

Find all roots.

7) $x(x + 4)(x + 2) = 0$

8) $x(x - 5)(x - 3) = 0$

Find all roots. One root has been given.

9) $x^3 + 6x^2 + 5x - 12 = 0;\ -3$

10) $x^3 - 3x + 2 = 0;\ -2$

11) $x^3 - 2x^2 - x + 2 = 0;\ 2$

12) $x^3 - 5x^2 - x + 5 = 0;\ 5$

13) $5x^3 + 39x^2 + 78x + 40 = 0;\ -5$

14) $5x^3 - 7x^2 - 46x - 24 = 0;\ -2$

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Finding the Zeros of a Polynomial Function

Recall that the zeros, or x-intercepts, of a function are found by substituting ______ for y.

Equation	Factors	Zeros
$f(x) = x^3 + 4x^2 + 3x$		

We can locate the zeros of a function on the x-axis, or in the table where y= ______.

Ex 1: Find all zeros of the function $f(x) = x^3 - x^2 - 21x + 45$.

# of zeros:	Table:	Synthetic Division:	Zeros:

What if the zeros are not evident from the table or graph?

We can find all the possible rational zeros: $\pm\frac{p}{q} = \frac{\text{all factors of the \_\_\_\_\_\_\_\_\_\_}}{\text{all factors of the \_\_\_\_\_\_\_\_\_\_}}$

Ex 2: List the possible rational zeros of the function $f(x) = 5x^4 + 12x^3 - 16x^2 + 10$.

Ex 3: List the possible rational zeros of the function $f(x) = 20x^3 - 4x^2 - 5x + 1$.

Ex 4: Find all zeros of the function $f(x) = 20x^3 - 4x^2 - 5x + 1$.

Ex 5: Find all zeros of the function $f(x) = 2x^4 + 7x^3 + 11x^2 + 28x + 12$

1. List possible rational zeros $\pm\frac{p}{q}$	
2. Check the table or graph and perform synthetic division with the known integer zeros.	
3. If there are no integer zeros, or if it's not yet quadratic, use synthetic division to test rational zeros. (Remember, you will have a remainder of zero if it is a solution.)	
4. Once the polynomial is depressed to quadratic, solve using factoring, square root property, completing the square or quadratic formula.	

Ex 6: Find all zeros

a. $f(x) = x^3 - 7x^2 + 10x + 6$

b. $f(x) = 4x^3 - 8x^2 - 15x + 9$

c. $f(x) = x^3 + 2x^2 - 34x + 7$

d. $f(x) = x^4 + 4x^3 - 14x^2 - 20x - 3$

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Solving Polynomial Functions Practice

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State the possible rational zeros for each function.

1) $f(x) = 3x^3 + 13x^2 + 13x + 3$

2) $f(x) = 4x^3 + 7x^2 + 2x - 1$

3) $f(x) = 3x^3 + 11x^2 + 5x - 3$

4) $f(x) = 3x^3 + x^2 - 3x - 1$

State the possible rational zeros for each function. Then find all zeros.

5) $f(x) = 5x^3 - 16x^2 + 8x - 1$

6) $f(x) = 2x^3 + x^2 - 2x - 1$

7) $f(x) = x^3 + 4x + 16$

8) $f(x) = 2x^3 - 5x^2 + 4x - 1$

9) $f(x) = 2x^4 - 3x^3 + x$

10) $f(x) = x^4 + 15x^3 + 49x^2 - 5x$

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Finding the Zeros of Polynomial Function

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State the possible rational zeros for each function. Then find all zeros.

1) $f(x) = 5x^3 + x^2 - 5x - 1$

2) $f(x) = 3x^3 + 10x^2 + 18x + 5$

3) $f(x) = 3x^3 + 8x^2 - 2x + 3$

4) $f(x) = 3x^3 + 5x^2 - 11x + 3$

5) $f(x) = x^3 + 2x^2 - x - 2$

6) $f(x) = 9x^4 - 24x^3 + 15x^2 - 2x$

7) $f(x) = 3x^4 + 7x^3 + 5x^2 + x$

8) $f(x) = 2x^4 - x^3 - 2x^2 + x$

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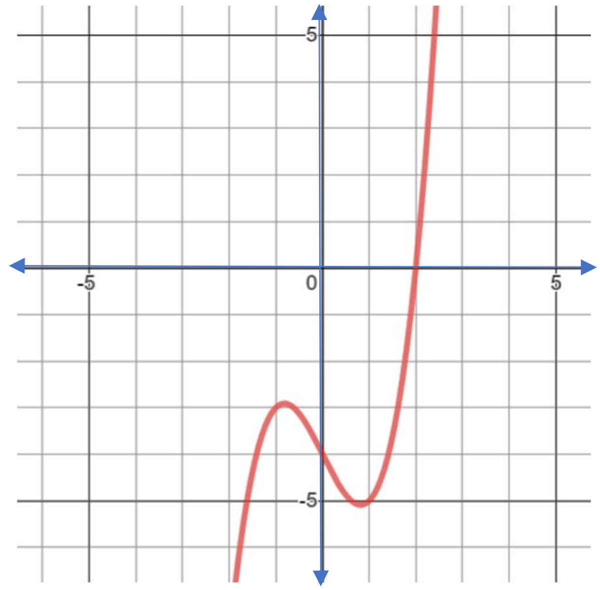
Writing Equations of Polynomial Functions

The graph of the function $f(x) = x^3 - 2x - 4$ is shown at the right.

There are _____ zeros, and ______ real solution(s).

This means that the other _____ are imaginary.

Find all the zeros!



Complex roots and irrational roots come in pairs.

Ex 1: A polynomial function has the given zeros. Find all additional zeros and state the least possible degree of the function.

Given Zeros	Additional Zeros	Degree of the Function
a. $-2, 3, -4i$		
b. $-1, 1 + \sqrt{3}$		
c. $-\frac{1}{2}, 0, -2 + 5i$		
d. $2 + \sqrt{7},\ -3 - \sqrt{2}$		

Ex 2: Write a polynomial function of least degree that has the given roots: $-1, 2, 6$

Ex 3: Write a polynomial function of least degree that has the given roots: $-\frac{1}{2}, 0, 3$ multi. 2

Ex 4: Write a polynomial function of least degree that has the given roots: $-3, \sqrt{2}$

Ex 5: Write a polynomial function of least degree that has the given roots: $1, 3i$

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Conjugate Roots and Writing Functions

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A polynomial function with rational coefficients has the following zeros. Find all additional zeros.

1) $\frac{3}{5}$, $2 - 2i$, $1 + \sqrt{2}$

2) $2 - i$, $-1 + \sqrt{10}$

3) $-\frac{1}{2}$, $-2 - 2i$

4) $-2 - 3i$, $-2 + \sqrt{6}$

5) $-\frac{1}{5}$, $3 + 3i$

6) 2, 1, $-1 + \sqrt{5}$

7) -1, -3, $-2i$

8) $2 + 3i$, $-3 + \sqrt{7}$

9) $-3 + \sqrt{2}$, $-2 + \sqrt{3}$

10) $-2 + \sqrt{5}$, $-1 + 2\sqrt{2}$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

11) -5, 4, -4

12) 4, $\sqrt{7}$

13) -5, 2, -3

14) -3, $\sqrt{7}$

15) $\sqrt{2}$ mult. 2, $-\sqrt{2}$ mult. 2

16) $\frac{5}{2}$, $-i$, i

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Algebra 2 Unit 3 Review Name:________________________________

Factor.			
1. $x^3 - 64$	2. 1. $x^2 - 36$	3. $2x^2 - x - 6$	4. $p^3 + 4p^2 - 9p - 36$

Find the possible rational zeros of the function.	
5. $f(x) = 5x^5 - 4x^3 + 2x - 45$	6. $f(x) = 3x^4 - 5x^3 + 2x - 8$
Divide using synthetic division.	
7. $f(x) = 2x^3 - x^2 - 7x + 6 \div (x + 2)$	8. $f(x) = x^4 - 2x^3 + 44x + 7 \div (x - 3)$

9. Is $(x - 2)$ a factor of the function $f(x) = 3x^3 - 2x + 4$? Use synthetic division to explain.

10. One factor of $x^3 - 4x^2 + x + 6$ is $(x - 3)$. Find the other factors.

11. When we say that the root $x = 7$ has a "multiplicity of 2," what do we mean?

Find all zeros of the function.	
12. $f(x) = x^3 + x^2 - 4x - 4$	13. $f(x) = 4x^3 - 3x^2 + 4x - 3$
14. $f(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$	15. $f(x) = x^4 - 4x^3 + x^2 + 16x - 20$
16. A polynomial function has the given zeros: $2 - \sqrt{3}$, $4i$. What are the missing zeros?	
Write a polynomial function of least degree that has the given zeros.	
17. -1, 3, & 5	18. -1 (multiplicity 2) & i
19. -2 & $4i$	20. $-3, 3, -2i$

Algebra 2 Polynomial Review #2 Name:___________________________________

1. Is $(x+1)$ a factor of x^3+2x^2-5x-6? How can you decide?

2. Is $(x+2)$ a factor of $x^4-x^3-11x^2+9x+18$? How can you decide?

3. Given $x=2$, find all FACTORS of the polynomial function $f(x)=4x^4-4x^3-9x^2+x+2$.

4. Given $x=4$, find all ROOTS of the polynomial function $g(x)=x^3-64$.

5. Given $x=-3$, find all ZEROS of the polynomial function $h(x)=x^4+x^3-2x^2+4x-24$.

6. Find all zeros of the polynomial function $f(x)=x^4-x^3+7x^2-9x-18$.

7. Find all roots of the polynomial function $h(x)=x^3+x^2-29x-5$.

8. Given the function $g(x)=x(x-3)(x+5)^2$

a. What does $(x+5)^2$ indicate about the root(s)?

b. Find all x-intercepts.

c. What is the degree of the polynomial function?