Unit 3 – Meaning of a Derivative

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Derivative at a Point
- ❖ Definition of a Derivative
- ❖ Tangent & Normal Lines
- ❖ Graphing f′ from f
- ❖ Graphing f′′ from f and f′
- ❖ Graphing f from f′ and f′′

Test is ______________________

Name: ______________________

Rates of Change and The Derivative

Find an equation for the tangent line and the normal line to the graph of each function at the indicated value.

1. $f(x) = x^2 + 2, x = -1$ $f(-1)=3$

$\lim_{x\to -1} \frac{x^2+2-3}{x-(-1)}$

$\lim_{x\to -1} \frac{x^2-1}{x+1}$

$\lim_{x\to -1} x-1 = -2$

tan line: $y-3=-2(x+1)$

norm. line $y-3=\frac{1}{2}(x+1)$

2. $f(x) = x^3 + 1, x = 1$ $f(1)=2$

$\lim_{x\to 1} \frac{x^3+1-2}{x-1}$

$\lim_{x\to 1} \frac{x^3-1}{x-1}$

$\lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$\lim_{x\to 1} x^2+x+1 = 1+1+1=3$

tan. line: $y-2=3(x-1)$

norm. line: $y-2=-\frac{1}{3}(x-1)$

3. $f(x) = \frac{2-5x}{1+x}$ at 0 $f(0)=2$

$\lim_{x\to 0} \frac{\frac{2-5x}{1+x}-2}{x-0}$

$\lim_{x\to 0} \frac{2-5x-2-2x}{1+x}\cdot\frac{1}{x}$

$\lim_{x\to 0} \frac{-7\cancel{x}}{\cancel{x}(1+x)} = \frac{-7}{1+0} = -7$

tan. line: $y-2=-7x$

norm. line: $y-2=\frac{1}{7}x$

4. $f(x) = \sqrt{x+3}, x = 6$ $f(6)=3$

$\lim_{x\to 6} \frac{\sqrt{x+3}-3}{x-6}$

$\lim_{x\to 6} \frac{(\sqrt{x+3}-3)(\sqrt{x+3}+3)}{(x-6)(\sqrt{x+3}+3)}$

$\lim_{x\to 6} \frac{\cancel{x+3-9}}{\cancel{(x-6)}(\sqrt{x+3}+3)}$

$\lim_{x\to 6} \frac{1}{\sqrt{x+3}+3} = \frac{1}{\sqrt{6+3}+3} = \frac{1}{6}$

tan line: $y-3=\frac{1}{6}(x-6)$

norm. line: $y-3=-6(x-6)$

5. $f(x) = \frac{1}{\sqrt{x}}, x = 4$ $f(4)=1/2$

$\lim_{x\to 4} \frac{\frac{1}{\sqrt{x}}-\frac{1}{2}}{x-4}$

$\lim_{x\to 4} \frac{2-\sqrt{x}}{2\sqrt{x}}\cdot\frac{1}{x-4}$

$\lim_{x\to 4} \frac{2-\sqrt{x}}{2\sqrt{x}(x-4)}\frac{(2+\sqrt{x})}{(2+\sqrt{x})}$

$\lim_{x\to 4} \frac{\cancel{4-x}\ -1}{2\sqrt{x}\,\cancel{(x-4)}(2+\sqrt{x})} = \frac{-1}{2\sqrt{4}(2+\sqrt{4})} = \frac{-1}{16}$

Tan Line: $y-\frac{1}{2}=-\frac{1}{16}(x-4)$

norm line: $y-\frac{1}{2}=16(x-4)$

6. $f(x) = \frac{1}{x^2}, x = 2$ $f(2)=1/4$

$\lim_{x\to 2} \frac{\frac{1}{x^2}-\frac{1}{4}}{x-2}$

$\lim_{x\to 2} \frac{4-x^2}{4x^2}\cdot\frac{1}{x-2}$

$\lim_{x\to 2} \frac{\cancel{(2-x)}^{-1}(2+x)}{4x^2\cancel{(x-2)}} = \frac{-1(2+2)}{4(2)^2} = \frac{-4}{16} = \frac{-1}{4}$

tan. line: $y-\frac{1}{4}=-\frac{1}{4}(x-2)$

norm. line: $y-\frac{1}{4}=4(x-2)$

Find the rate of change of f at the indicated number.

7. $f(x) = 5x - 2, c = 0$ $f(0) = -2$

$IROC = \lim_{x \to 0} \frac{5x-2-(-2)}{x-0}$

$\lim_{x \to 0} 5$

$= 5$

8. $f(x) = x^2 - 1, c = -1$ $f(-1) = 0$

$\lim_{x \to -1} \frac{x^2-1-0}{x-(-1)}$

$\lim_{x \to -1} \frac{(x+1)(x-1)}{x+1}$

$\lim_{x \to -1} x - 1$

$-1 - 1 = -2$

9. $f(x) = \frac{x^2}{x+3}, c = 0$ $f(0) = 0$

$\lim_{x \to 0} \frac{\frac{x^2}{x+3} - 0}{x-0}$

$\lim_{x \to 0} \frac{x^{\cancel{2}}}{x+3} \cdot \frac{1}{\cancel{x}}$

$\lim_{x \to 0} \frac{x}{x+3} = 0$

10. $f(x) = \frac{x}{x^2-1}, c = 2$ $f(2) = \frac{2}{3}$

$\lim_{x \to 2} \frac{\frac{x}{x^2-1} - \frac{2}{3}}{x-2}$

$\lim_{x \to 2} \frac{3x - 2x^2 + 2}{3(x^2-1)} \cdot \frac{1}{x-2}$

$\lim_{x \to 2} \frac{-1(2x^2-3x-2)}{3(x^2-1)(x-2)}$

$\lim_{x \to 2} \frac{-1(2x+1)\cancel{(x-2)}}{3(x^2-1)\cancel{(x-2)}} = \frac{-1(2 \cdot 2+1)}{3(2^2-1)}$

$= \frac{-5}{9}$

Find the derivative of each function at the given number.

11. $f(x) = 2x + 3$ at 1 $f(1) = 5$

$f'(x) = \lim_{x \to 1} \frac{2x+3-5}{x-1}$

$\lim_{x \to 1} \frac{2\cancel{(x-1)}}{\cancel{x-1}}$

$\lim_{x \to 1} 2$

$f'(1) = 2$

12. $f(x) = 3x^2 + x + 5$ at -1 $f(-1) = 7$

$f'(x) = \lim_{x \to -1} \frac{3x^2+x+5-7}{x+1}$

$\lim_{x \to -1} \frac{3x^2+x-2}{x+1}$

$\lim_{x \to -1} \frac{(3x-2)\cancel{(x+1)}}{\cancel{x+1}}$

$\lim_{x \to -1} 3x - 2 = 3(-1) - 2$

$f'(-1) = -5$

Using the Definition of a Derivative

Use the definition of the derivative to find the derivative of each function with respect to x.

1. $y = -5x^2 - 2x + 5$

$\lim_{h\to 0} \frac{-5(x+h)^2 - 2(x+h) + 5 - (-5x^2 - 2x + 5)}{h}$

$\lim_{h\to 0} \frac{-5x^2 - 10xh - 5h^2 - 2x - 2h + 5 + 5x^2 + 2x - 5}{h}$

$\lim_{h\to 0} \frac{-10xh - 5h^2 - 2h}{h}$

$\lim_{h\to 0} -10x - 5h - 2 = -10x - 5(0) - 2$

$y' = -10x - 2$

2. $y = 2x - 1$

$\lim_{h\to 0} \frac{2(x+h) - 1 - (2x-1)}{h}$

$\lim_{h\to 0} \frac{2x + 2h - 1 - 2x + 1}{h}$

$\lim_{h\to 2} \frac{2h}{h}$

$\lim_{h\to 2} 2$

$y' = 2$

3. $y = -\frac{2}{x+4}$

$y' = \lim_{h\to 0} \frac{\frac{-2}{x+h+4} + \frac{2}{x+4}}{h}$

$\lim_{h\to 0} \frac{-2(x+4) + 2(x+h+4)}{(x+h+4)(x+4)h}$

$\lim_{h\to 0} \frac{-\cancel{2x} - \cancel{8} + \cancel{2x} + 2\cancel{h} + \cancel{8}}{(x+h+4)(x+4)\cancel{h}}$

$\lim_{h\to 0} \frac{2}{(x+h+4)(x+4)} = \frac{2}{(x+4)^2}$

4. $f(x) = 2\sqrt{x+3}$

$\lim_{h\to 0} \frac{2\sqrt{x+h+3} - 2\sqrt{x+3}}{h} \cdot \frac{(2\sqrt{x+h+3} + 2\sqrt{x+3})}{(2\sqrt{x+h+3} + 2\sqrt{x+3})}$

$2x + 2h + 6 - 2x - 6$

$\lim_{h\to 0} \frac{2(x+h+3) - 2(x+3)}{h(2\sqrt{x+h+3} + 2\sqrt{x+3})}$

$\lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(2\sqrt{x+h+3} + 2\sqrt{x+3})}$

$\lim_{h\to 2} \frac{\cancel{2}\ 1}{\cancel{2}(\sqrt{x+0+3} + \sqrt{x+3})} = \frac{1}{2\sqrt{x+3}}$

5. $f(x) = \sqrt{2x-5}$

$\lim_{h\to 0} \frac{\sqrt{2(x+h)-5} - \sqrt{2x-5}}{h} \cdot \frac{(\sqrt{2(x+h)-5} + \sqrt{2x-5})}{\sqrt{2(x+h)-5} + \sqrt{2x-5}}$

$\lim_{h\to 0} \frac{2x + 2h - 5 - 2x + 5}{h(\sqrt{2x+2h-5} + \sqrt{2x-5})}$

$\lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2x+2h-5} + \sqrt{2x-5})} = \frac{2}{2\sqrt{2x-5}}$

$f'(x) = \frac{1}{\sqrt{2x-5}}$

6. $y = x^3$

$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

$\lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$\lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$

$\lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2 + 3x(0) + 0^2$

$y' = 3x^2$

7. $f(x) = (3x-5)^2$

$f(x) = 9x^2 - 30x + 25$

$\lim_{h\to 0} \frac{9(x+h)^2 - 30(x+h) + 25 - (9x^2 - 30x + 25)}{h}$

$\lim_{h\to 0} \frac{\cancel{9x^2} + 18xh + 9h^2 - \cancel{30x} - 30h + \cancel{25} - \cancel{9x^2} + \cancel{30x} - \cancel{25}}{h}$

$\lim_{h\to 0} 18x + 9h - 30$

$18x + 9(0) - 30$

$f'(x) = 18x - 30$

8. $f(x) = \frac{1}{3x}$

$\lim_{h\to 0} \frac{\frac{1}{3(x+h)} - \frac{1}{3x}}{h}$

$\lim_{h\to 0} \frac{x - (x+h)}{3x(x+h)} \cdot \frac{1}{h}$

$\lim_{h\to 0} \frac{\cancel{x} - \cancel{x} - \overset{-1}{\cancel{h}}}{3x(x+h)\cancel{h}}$

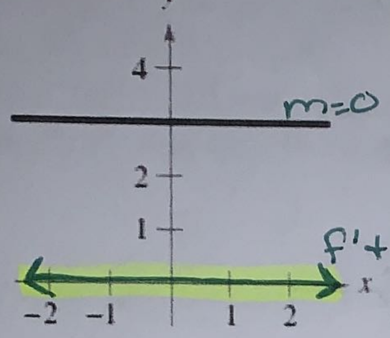
$= \frac{-1}{3x(x+0)}$

$f'(x) = \frac{-1}{3x^2}$

Curve Sketching - Graphing f' from f

The graph of f is given below. Sketch a possible graph of f' and f''

1.



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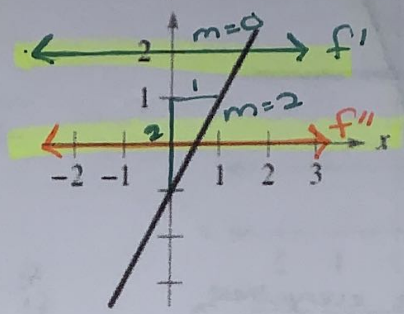

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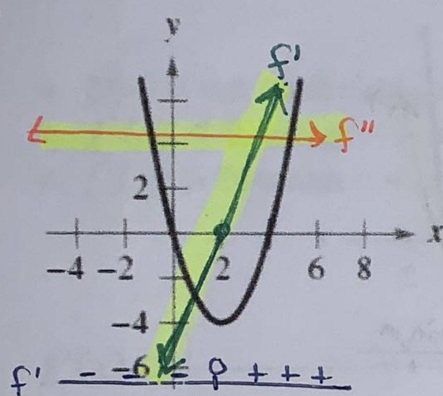

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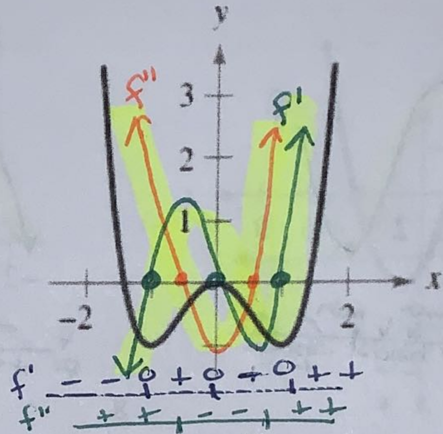

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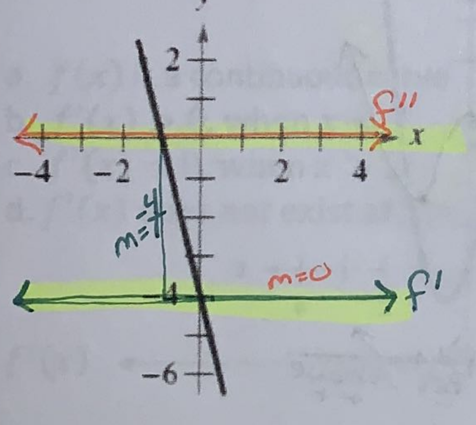

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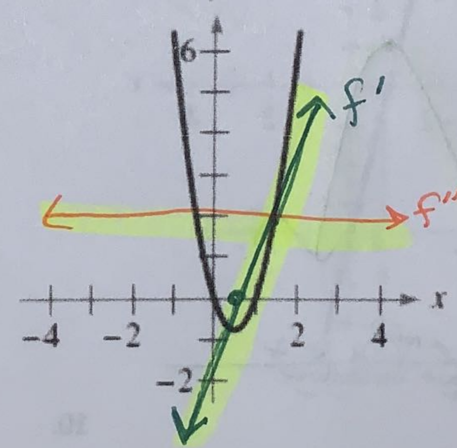

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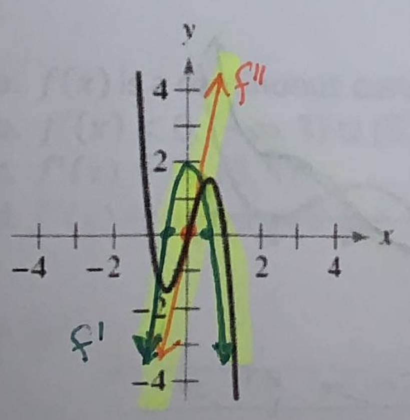

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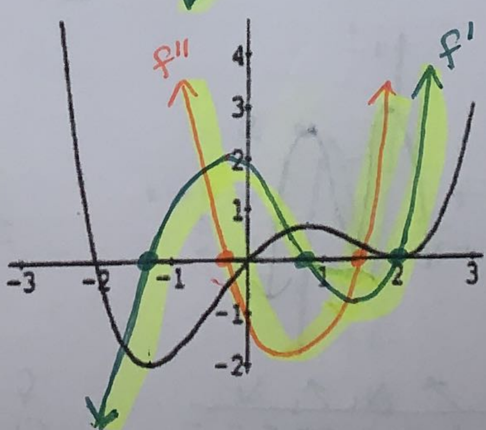
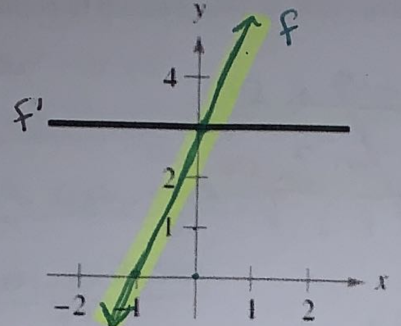


Curve Sketching – Graphing f from f'

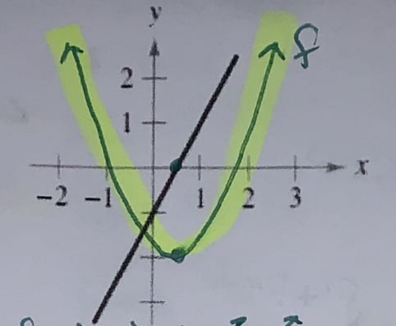
The graph of f' is given below. Sketch a possible graph of f

1.



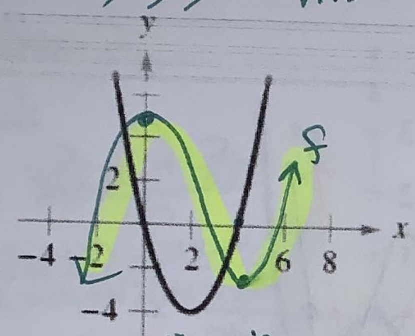
f above everywhere
↗ ↗ ↗ m=3

2.



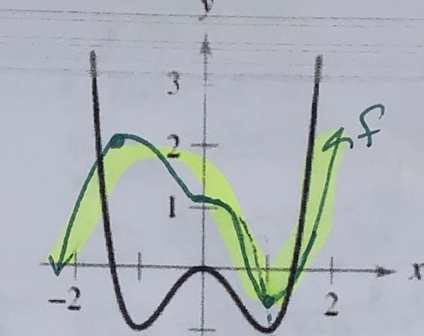
f ↘ ↘ ↗ ↗
f' below above
– – + +

3.



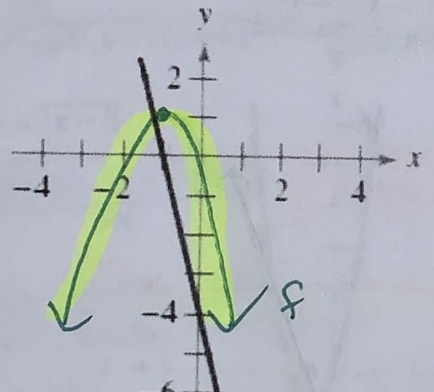
f ↗ max ↘ ↘ min ↗ ↗
f' above below above
+ + + – – + +

4.



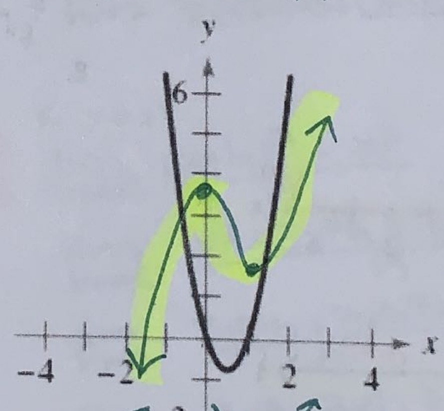
f ↗ max ↘ HT ↘ min ↗
f' + + + – – – + +

7.



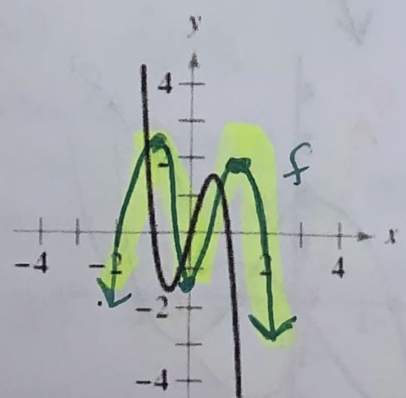
f ↗ ↗ max ↘ ↘ ↘
f' Above on Below
+ + – –

8.



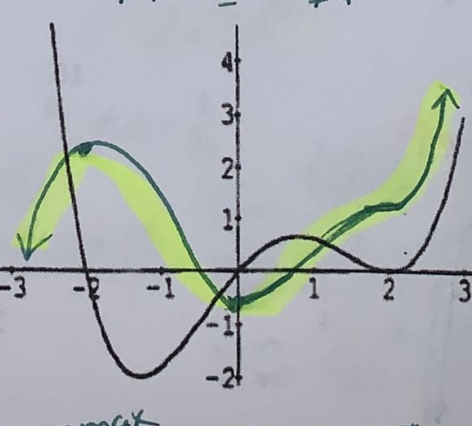
f ↗ ↘ ↗
Above Bel Above
+ + – + +

9.



f ↗ ↘ ↗ ↘
f' + + – + + – – –

10.

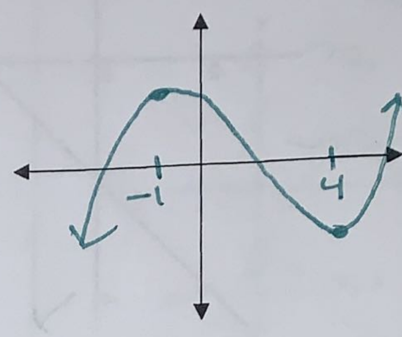


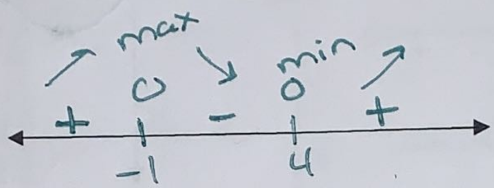
f ↗ max ↘ min ↗ HT ↗
f' + – – + + + +

First Derivative Test & Critical Points

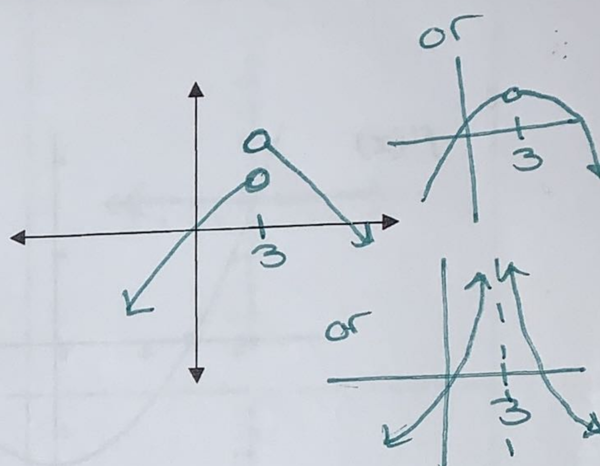
Draw a possible graph of f(x) given the information below.

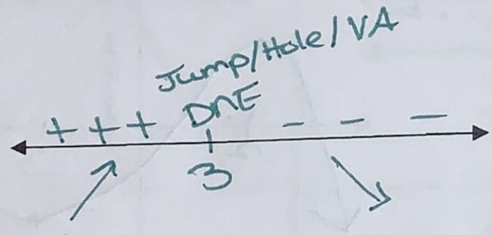
1. a. $f(x)$ is a continuous curve
   b. $f'(x) < 0, (-1, 4)$
   c. $f'(x) > 0, (-\infty, -1) \cup (4, \infty)$
   d. $f'(x) = 0$, at $x = -1$ and $x = 4$



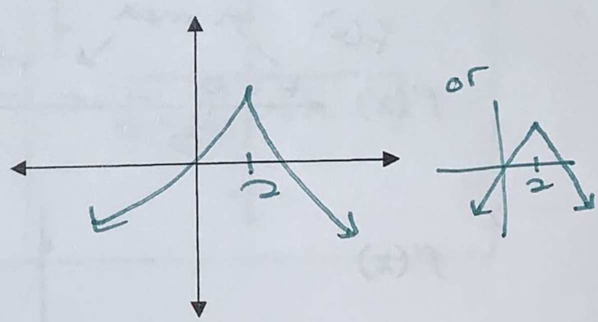
$f'(x)$ 

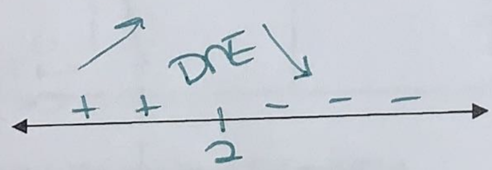
2. a. $f(x)$ is not continuous at x = 3
   b. $f'(x) < 0$, when $x > 3$
   c. $f'(x) > 0$, when $x < 3$



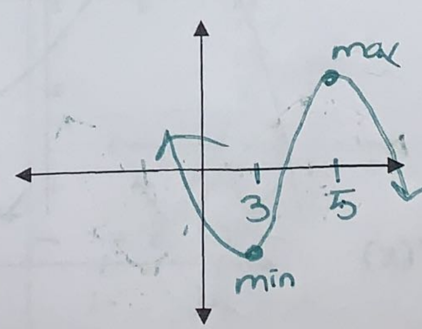
$f'(x)$ 

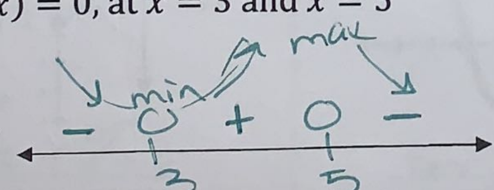
3. a. $f(x)$ is a continuous curve
   b. $f'(x) > 0$, when $x < 2$
   c. $f'(x) < 0$, when $x > 2$
   d. $f'(x)$ does not exist at $x = 2$



$f'(x)$ 

4. a. $f(x)$ is a continuous curve
   b. $f'(x) < 0, (-\infty, 3) \cup (5, \infty)$
   c. $f'(x) > 0, \ (3, 5)$
   d. $f'(x) = 0$, at $x = 3$ and $x = 5$



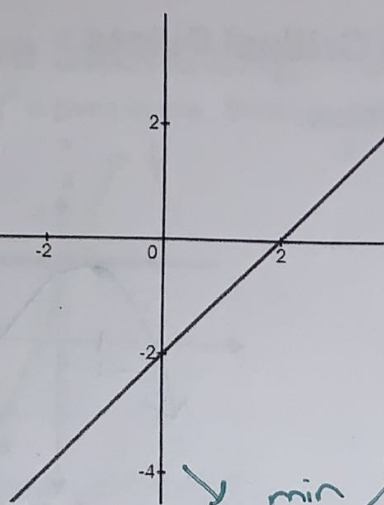
$f'(x)$ 

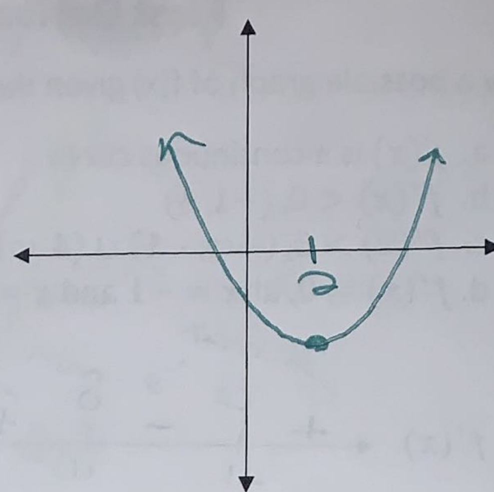
5. $f'(x)$

above

on

below



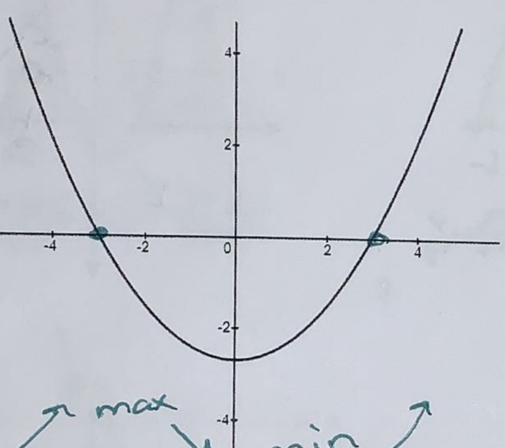


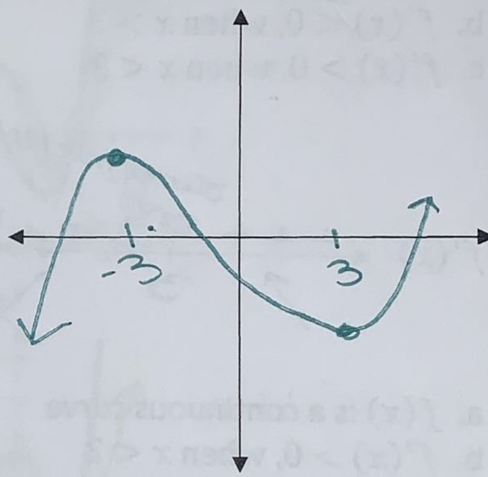
↘ min ↗

$f'(x)$ − − − 0 + +

2

6. $f'(x)$



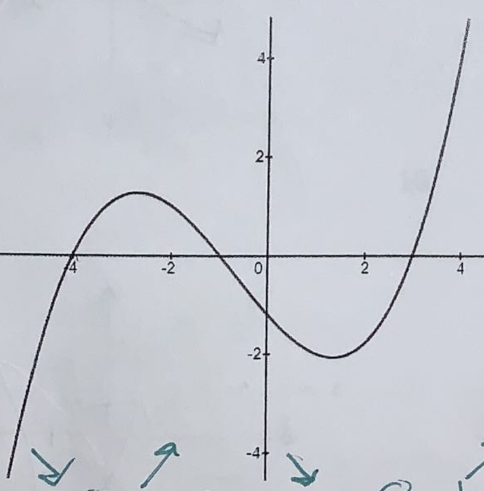


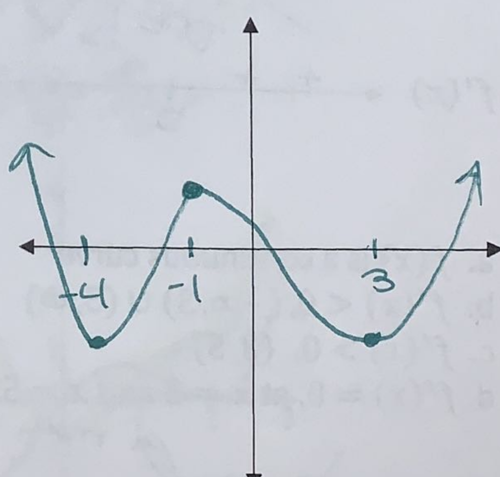
f(x) ↗ max ↘ min ↗

$f'(x)$ + + 0 − − − 0 + +

-3 3

7. $f'(x)$





↘ ↗ ↘ ↗

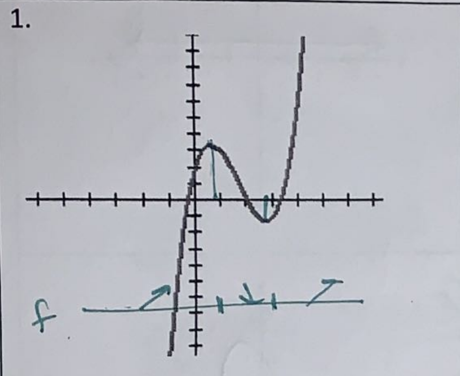
$f'(x)$ − 0 + 0 − − 0 + +

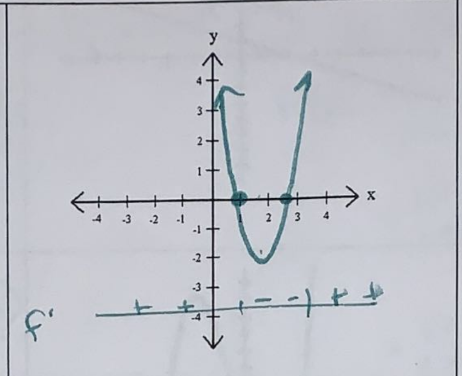
-4 -1 3

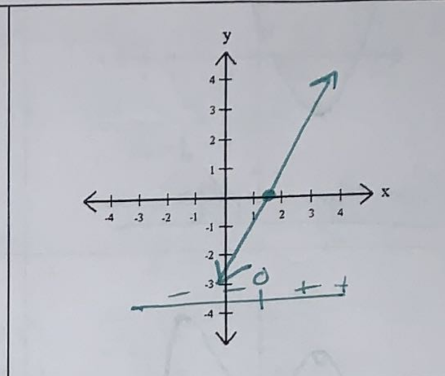
Curve Sketching Review

Given f(x), sketch the graphs of f'(x) and f''(x)

1.







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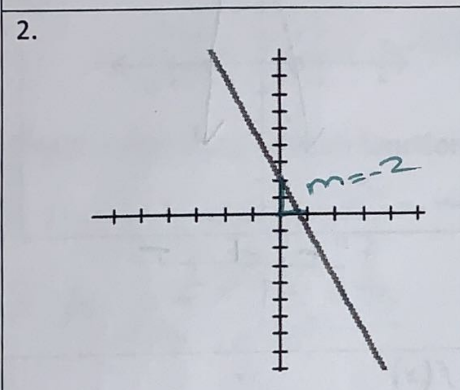
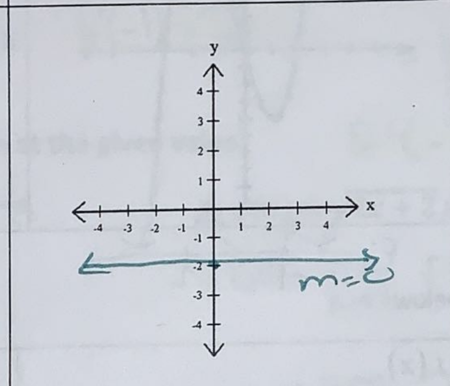
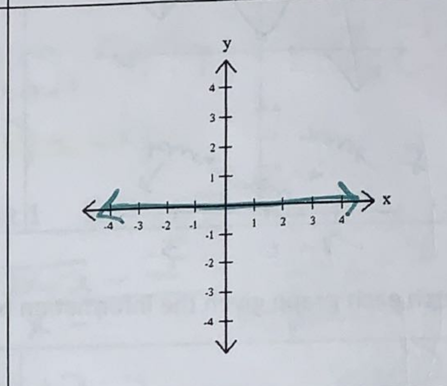
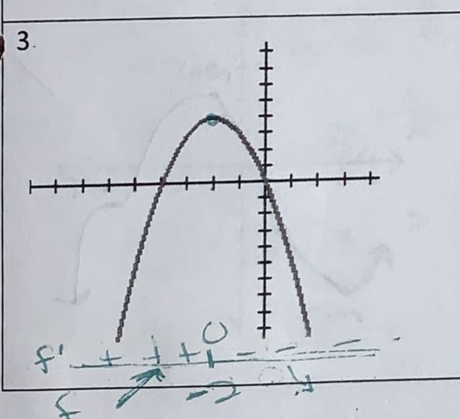
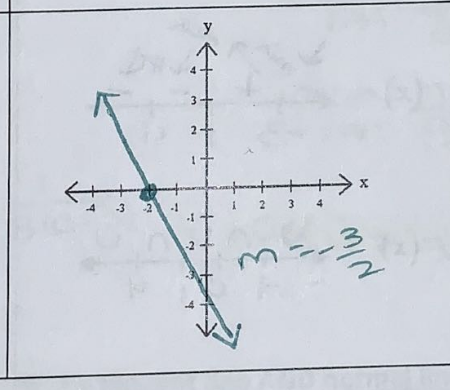


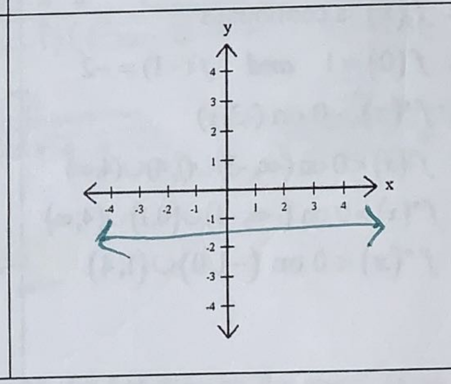




3.

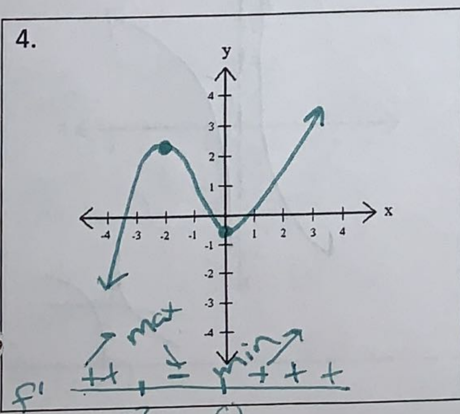


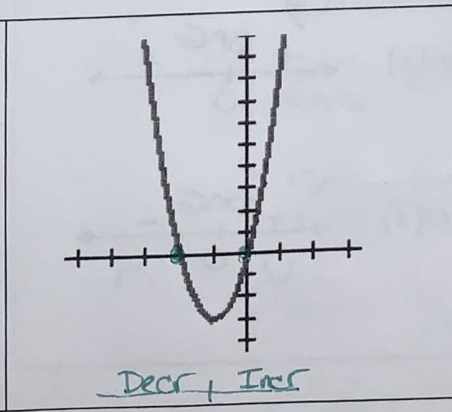


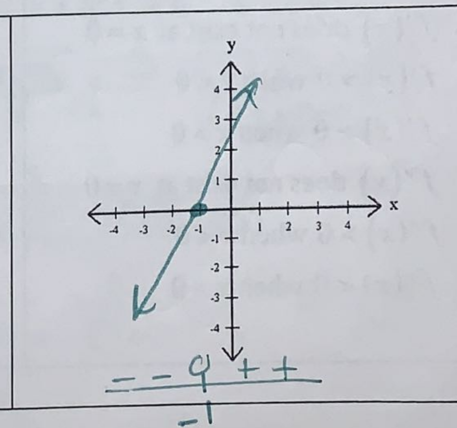


Given f'(x), sketch the graphs of f(x) and f''(x)

4.







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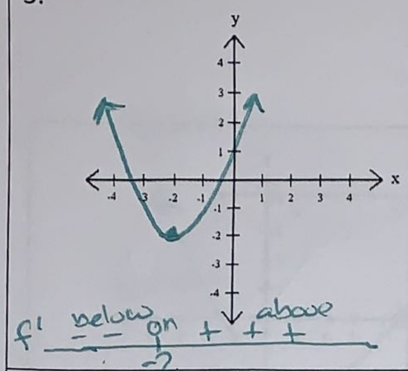

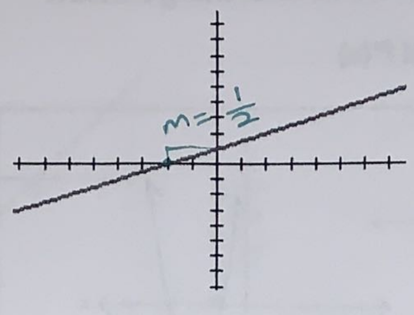

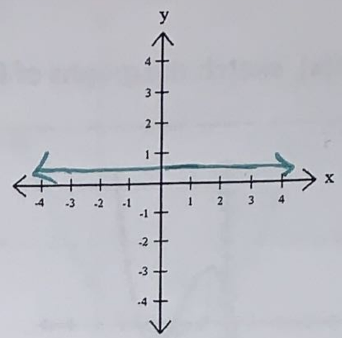

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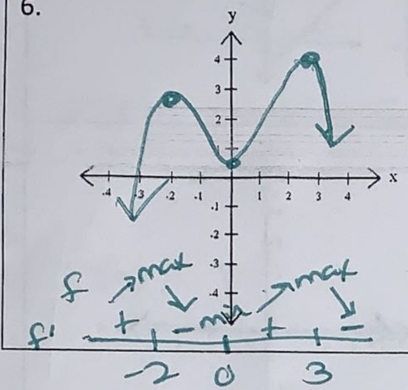
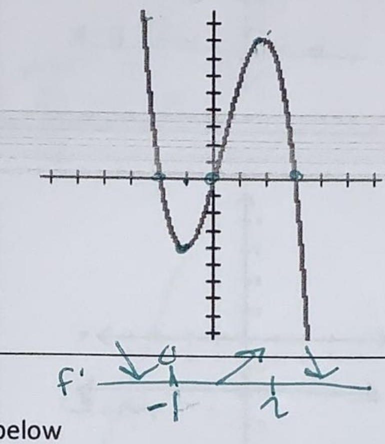
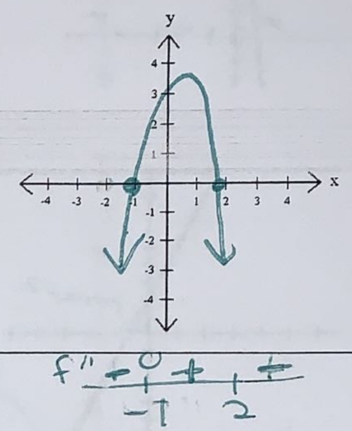






Sketch each graph given the information below

7.

a. $f(x)$ is continuous

b. $f(0)=1$ *and* $f(-1)=-2$

c. $f'(x)>0$ on (-3,1)

d. $f'(x)<0$ on $(-\infty,-3)\cup(1,4)\cup(4,\infty)$

e. $f''(x)>0$ on $(-\infty,-1)\cup(0,1)\cup(4,\infty)$

f. $f''(x)<0$ on $(-1,0)\cup(1,4)$

$f(x)$

↘ min ↗ max ↘ HT ↘

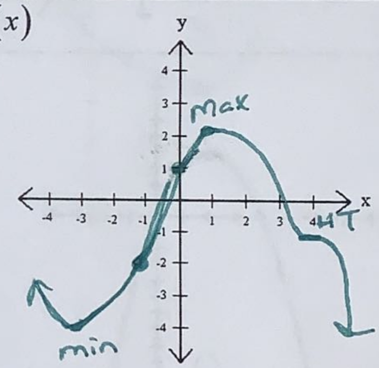
$f'(x)$ − + − −

−3 1 4

$f''(x)$ ∪ ∩ ∪ ∩ ∪

−1 0 1 4

$f(x)$



8.

a. $f(x)$ is not continuous at $x=0$

b. $f'(x)$ does not exist at $x=0$

c. $f'(x)>0$ when $x<0$

d. $f'(x)>0$ when $x>0$

e. $f''(x)$ does not exist at $x=0$

f. $f''(x)>0$ when $x<0$

g. $f''(x)<0$ when $x>0$

$f(x)$

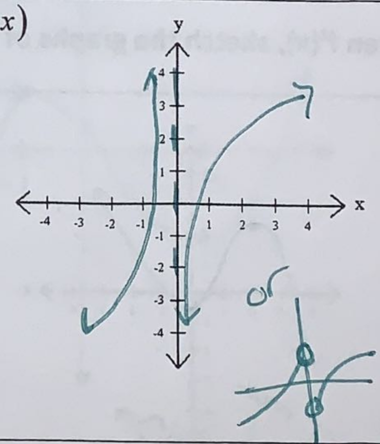
$f'(x)$ + DNE +

0

$f''(x)$ + DNE −

∪ 0 ∩

$f(x)$



Meaning of a Derivative Unit Review

Find the rate of change of the function at the indicated x-value given.

1. $f(x) = x^2 + 4x + 2$ when $x = -1$ $f(-1) = -1$

$f'(x) = \lim_{x\to -1} \frac{x^2+4x+2-(-1)}{x-(-1)}$

$= \lim_{x\to -1} \frac{x^2+4x+3}{x+1}$

$= \lim_{x\to -1} \frac{(x+1)(x+3)}{x+1}$

$= \lim_{x\to -1} x+3 = -1+3$ $f'(-1) = 2$

2. $f(x) = 2x^2 - 4$ when $x = -1$ $f(-1) = -2$

$f'(x) = \lim_{x\to -1} \frac{2x^2-4-(-2)}{x-(-1)}$

$= \lim_{x\to -1} \frac{2x^2-2}{x+1}$ $2(x^2-1)$

$= \lim_{x\to -1} \frac{2(x+1)(x-1)}{x+1}$

$= \lim_{x\to -1} 2(x-1) = 2(-1-1)$ $2(-2)$

$f'(-1) = -4$

Find the derivative of each function at the given value.

3. $f(x) = \frac{1}{x-3}$ at 0 $f(0) = -\frac{1}{3}$

$f'(x) = \lim_{x\to 0} \frac{\frac{1}{x-3}+\frac{1}{3}}{x}$

$= \lim_{x\to 0} \frac{3+x-3}{3(x-3)} \cdot \frac{1}{x}$

$= \lim_{x\to 0} \frac{\cancel{x}}{3(x-3)} \cdot \frac{1}{\cancel{x}}$

$= \lim_{x\to 0} \frac{1}{3(x-3)} = \frac{1}{3(0-3)}$

$f'(0) = -1/9$

4. $f(x) = \sqrt{2x+2}$ at 1 $f(1) = 2$

$f'(x) = \lim_{x\to 1} \frac{\sqrt{2x+2}-2}{x-1} \frac{(\sqrt{2x+2}+2)}{(\sqrt{2x+2}+2)}$

$= \lim_{x\to 1} \frac{2x+2-4}{(x-1)(\sqrt{2x+2}+2)}$ $\frac{2(\cancel{x-1})}{(\cancel{x-1})(}$

$= \lim_{x\to 1} \frac{2}{\sqrt{2x+2}+2} = \frac{2}{\sqrt{2(1)+2}+2} = \frac{2}{2+2}$

$f'(1) = \frac{1}{2}$

For each problem, find the equation of the tangent line AND <u>normal line</u> to the function at the given value or point. Write your answer in point-slope form.

5. $f(x) = \frac{4}{x}$ at $(-2,-2)$

$\lim_{x\to -2} \frac{\frac{4}{x}+2}{x+2}$

$\lim_{x\to -2} \frac{4+2x}{x} \cdot \frac{1}{x+2}$

$\lim_{x\to -2} \frac{2}{x} = \frac{2}{-2}$

$f'(-2) = -1$

Tan Line: $y+2 = -1(x+2)$

Norm. Line: $y+2 = 1(x+2)$

6. $f(x) = \sqrt{x+3}$ at $x = 6$ $f(6) = 3$

$\lim_{x\to 6} \frac{\sqrt{x+3}-3}{x-6} \frac{(\sqrt{x+3}+3)}{(\sqrt{x+3}+3)}$

$\lim_{x\to 6} \frac{x+3-9}{(x-6)(\sqrt{x+3}+3)}$

$f'(x) = \lim_{x\to 6} \frac{1}{\sqrt{x+3}+3} = \frac{1}{\sqrt{6+3}+3}$

$f'(6) = 1/6$

Tan Line: $y-3 = \frac{1}{6}(x-6)$

Norm. Line: $y-3 = -6(x-6)$

Use the definition of the derivative to find the derivative of each function with respect to x.

7. $y = 3x^2 - 2x + 3$

$$y' = \lim_{h\to 0} \frac{3(x+h)^2 - 2(x+h) + 3 - (3x^2 - 2x + 3)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{3} - \cancel{3x^2} + \cancel{2x} - \cancel{3}}{h}$$

$$y' = \lim_{h\to 0} \frac{6xh + 3h^2 - 2h}{h}$$

$$y' = \lim_{h\to 0} 6x + 3h - 2 = 6x + 3(0) - 2$$

$$y' = 6x - 2$$

8. $y = -4x + 1$

$$y' = \lim_{h\to 0} \frac{-4(x+h) + 1 - (-4x + 1)}{h}$$

$$= \lim_{h\to 0} \frac{-4x - 4h + \cancel{1} + 4x - 1}{h}$$

$$= \lim_{h\to 0} -4$$

$$y' = -4$$

9. $y = \frac{3}{x-2}$

$$y' = \lim_{h\to 0} \frac{\frac{3}{x+h-2} - \frac{3}{x-2}}{h}$$

$$= \lim_{h\to 0} \frac{3x - 6 - 3x - 3h + 6}{(x+h-2)(x-2)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-3}{(x+h-2)(x-2)} = \frac{-3}{(x+0-2)(x-2)}$$

$$y' = \frac{-3}{(x-2)^2}$$

10. $f(x) = 4\sqrt{x - 6}$

$$f'(x) = \lim_{h\to 0} \frac{4\sqrt{x+h-6} - 4\sqrt{x-6}}{h} \cdot \frac{(4\sqrt{x+h-6} + 4\sqrt{x-6})}{(4\sqrt{x+h-6} + 4\sqrt{x-6})}$$

$$f'(x) = \lim_{h\to 0} \frac{16(x+h-6) - 16(x-6)}{h(4\sqrt{x+h-6} + 4\sqrt{x-6})}$$

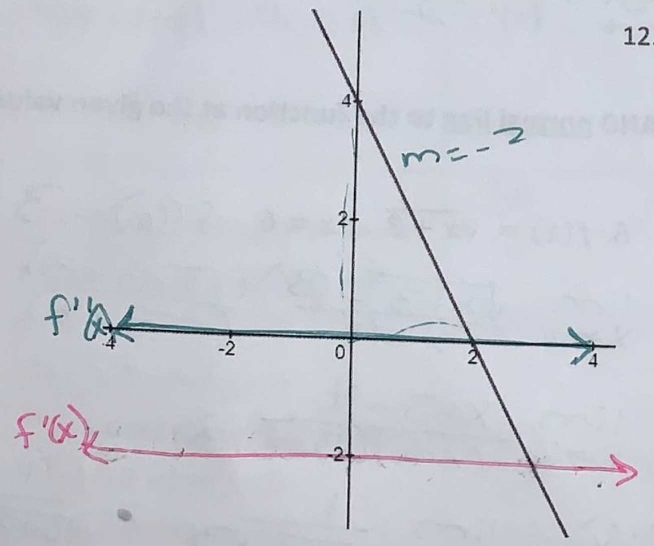
$$= \lim_{h\to 0} \frac{16x + 16h - 96 - 16x + 96}{h(4\sqrt{x+h-6} + 4\sqrt{x-6})}$$

$$= \lim_{h\to 0} \frac{16}{4\sqrt{x+h-6} + 4\sqrt{x-6}} = \frac{16}{4\sqrt{x-6} + 4\sqrt{x-6}}$$

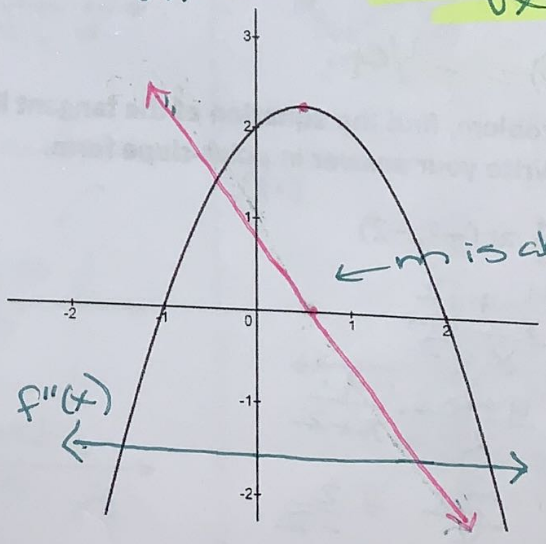
$$= \frac{16}{8\sqrt{x-6}} \qquad f'(x) = \frac{2}{\sqrt{x-6}}$$

Given the graph of f(x), sketch a graph of f'(x) and f''(x).

11.

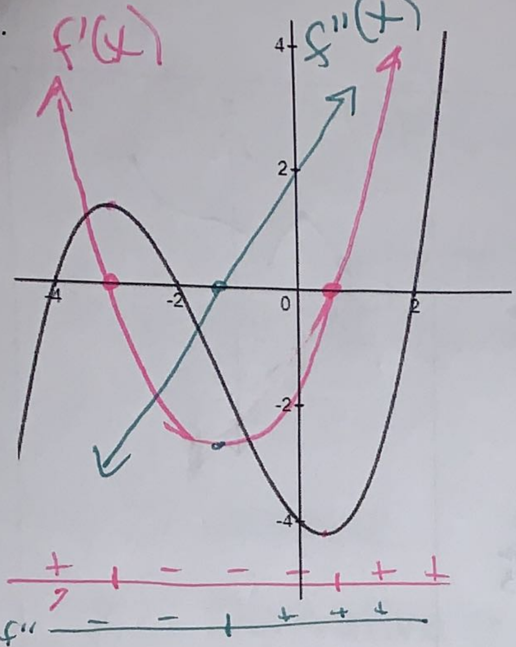


12.



f(x)	Incr	P	Decr
f'(x)	+	on	−

13.



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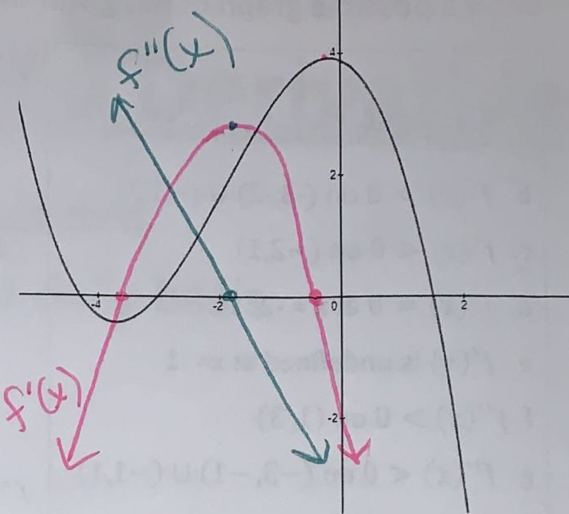

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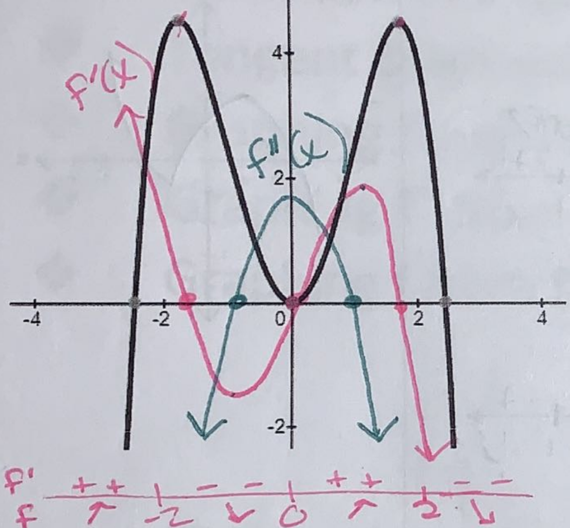

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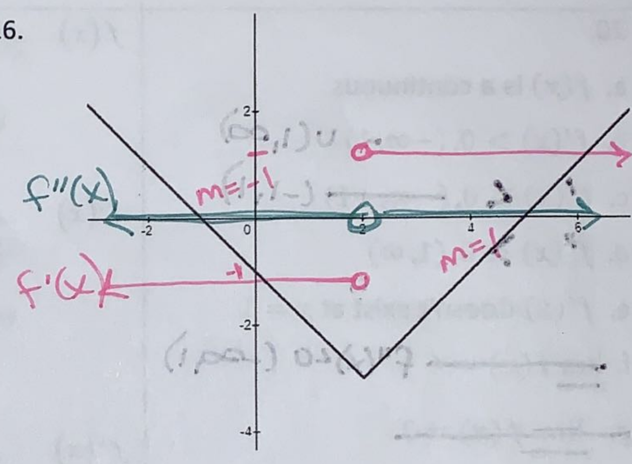
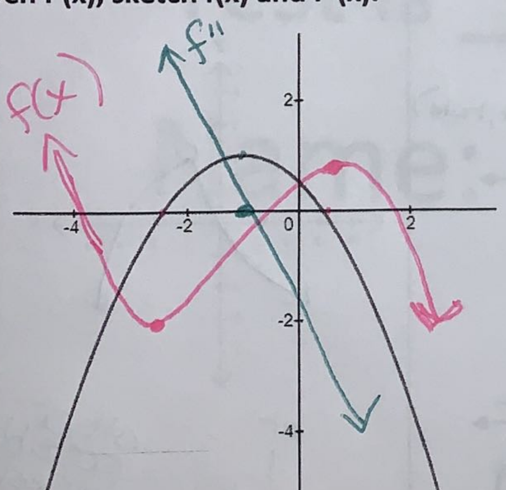


Given f'(x), sketch f(x) and f''(x).

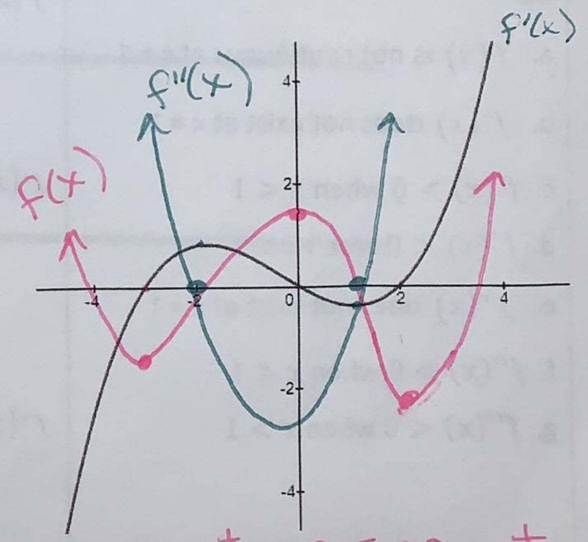
17.



f' Below on Above on Below

f min max

18.



f' Bel on Above on Bel on Above

f -3 min 0 max 2 min

Draw a possible graph of f(x) given the information below.

19.

a. $f(x)$ is continuous over [-3, 3]

b. $f'(x) > 0$ on (-3,-2) ∪ $(-1,3)$

c. $f'(x) < 0$ on $(-2,1)$

d. $f'(x) = 0$ at x = -2, 1

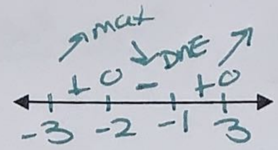
e. $f'(x)$ is undefined at x= -1

f. $f''(x) > 0$ on $(1,3)$

g. $f''(x) < 0$ on $(-3,-1) \cup (-1,1)$

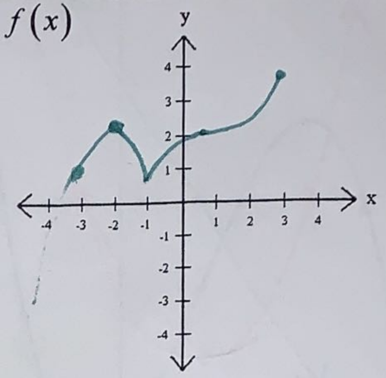
$f(x)$

$f'(x)$



$f''(x)$

$f(x)$



20.

a. $f(x)$ is a continuous

b. $f'(x) > 0, (-\infty,-1)$ ∪(1,∞)

c. $f'(x)$ < 0, ~~(−∞, −1)~~ (−1, 1)

d. $f''(x)$ > $0, (1,\infty)$

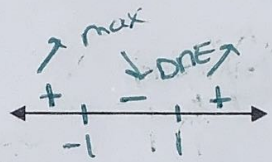
e. $f'(x)$ doesn't exist at $x = 1$

f. ~~$\lim_{x\to\infty} f(x) = 4$~~ f''(x)<0 (−∞,1)

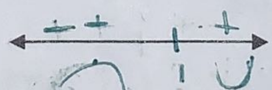
~~g. $\lim_{x\to-\infty} f(x) = 2$~~

$f(x)$

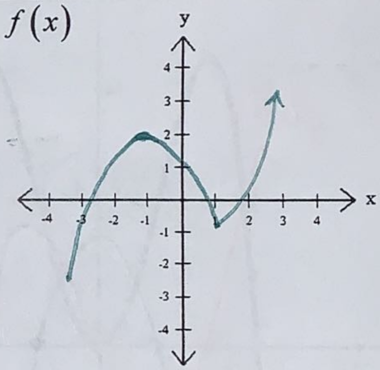
$f'(x)$



$f''(x)$



$f(x)$



21.

a. $f(x)$ is not continuous at x = 1

b. $f'(x)$ does not exist at x = 1

c. $f'(x) > 0$ when $x < 1$

d. $f'(x) < 0$ when $x > 1$

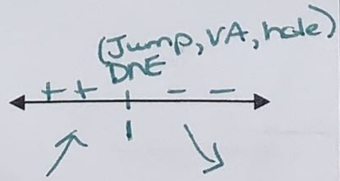
e. $f''(x)$ does not exist at x = 1

f. $f''(x) > 0$ when $x < 1$

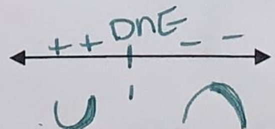
g. $f''(x) < 0$ when $x > 1$

$f(x)$

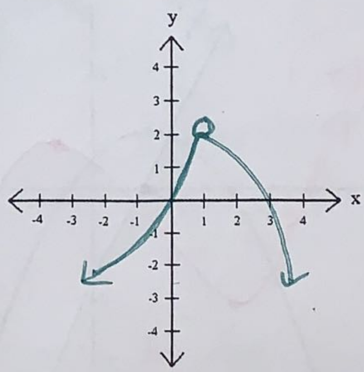
$f'(x)$



$f''(x)$



$f(x)$



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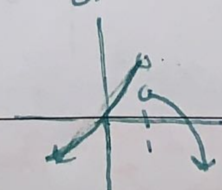

other possible answers