Unit 3 – Meaning of a Derivative

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Derivative at a Point
- ❖ Definition of a Derivative
- ❖ Tangent & Normal Lines
- ❖ Graphing f’ from f
- ❖ Graphing f’’ from f and f’
- ❖ Graphing f from f’ and f’’

Test is _____

Name: _____

Rates of Change and The Derivative

Find an equation for the tangent line and the normal line to the graph of each function at the indicated value.

1. $f(x) = x^2 + 2, x = -1$

2. $f(x) = x^3 + 1, x = 1$

3. $f(x) = \frac{2-5x}{1+x}$ at 0

4. $f(x) = \sqrt{x+3}, x = 6$

5. $f(x) = \frac{1}{\sqrt{x}}, x = 4$

6. $f(x) = \frac{1}{x^2}, x = 2$

Find the rate of change of f at the indicated number.

7. $f(x) = 5x - 2, c = 0$

8. $f(x) = x^2 - 1, c = -1$

9. $f(x) = \frac{x^2}{x+3}, c = 0$

10. $f(x) = \frac{x}{x^2-1}, c = 2$

Find the derivative of each function at the given number.

11. $f(x) = 2x + 3$ at 1

12. $f(x) = 3x^2 + x + 5$ at -1

Using the Definition of a Derivative

Use the definition of the derivative to find the derivative of each function with respect to x.

1. $y = -5x^2 - 2x + 5$

2. $y = 2x - 1$

3. $y = -\frac{2}{x+4}$

4. $f(x) = 2\sqrt{x+3}$

5. $f(x) = \sqrt{2x-5}$

6. $y = x^3$

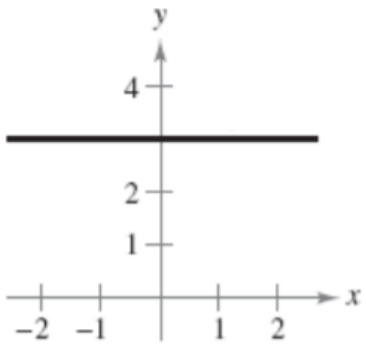
7. $f(x) = (3x-5)^2$

8. $f(x) = \frac{1}{3x}$

Curve Sketching - Graphing f' from f

The graph of f is given below. Sketch a possible graph of f' and f''

1.



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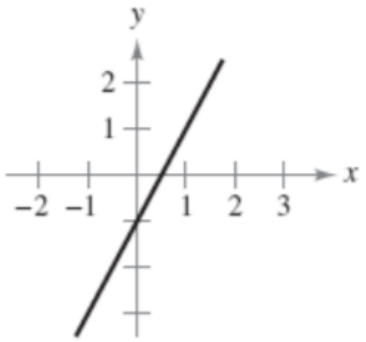

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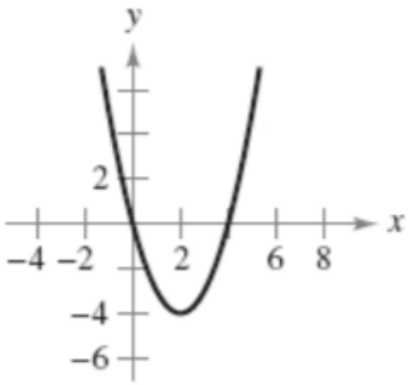

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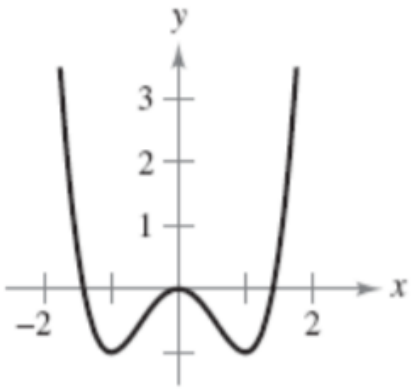

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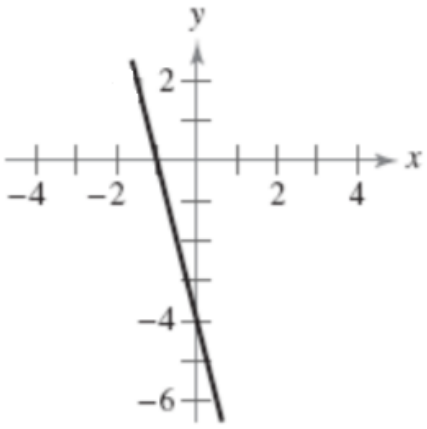

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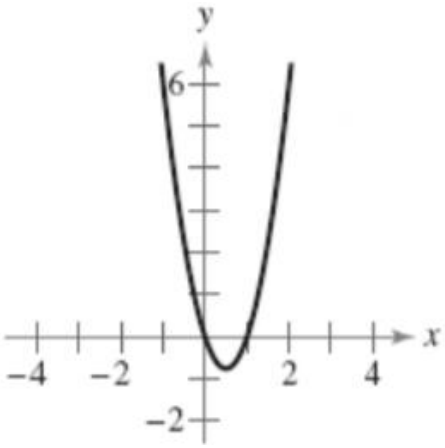

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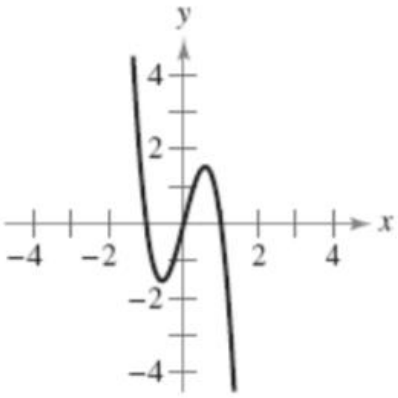

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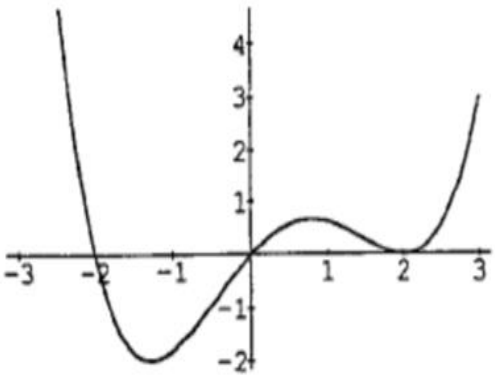
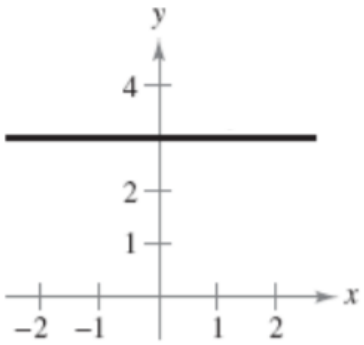


Curve Sketching – Graphing f from f'

The graph of f' is given below. Sketch a possible graph of f

1.



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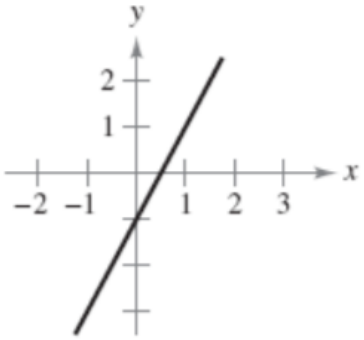

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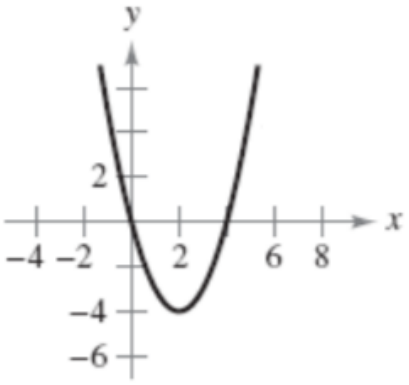

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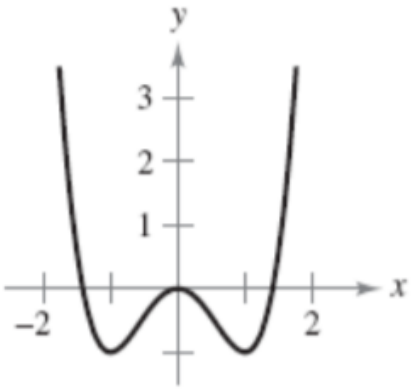

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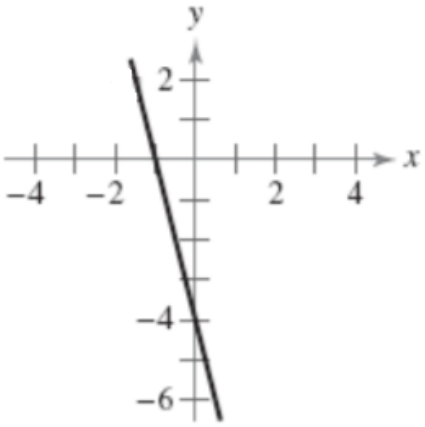

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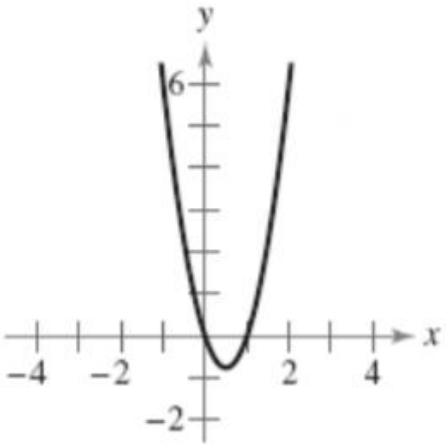

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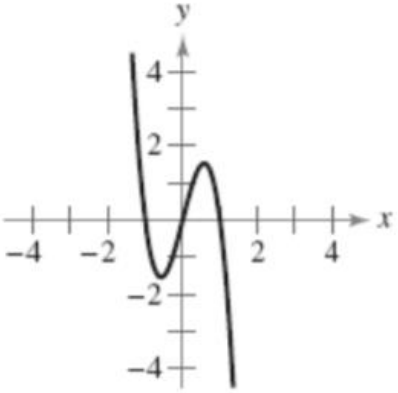

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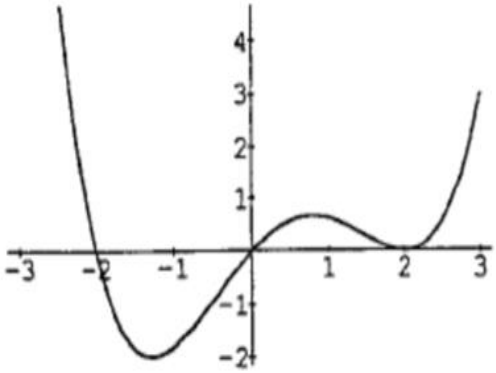
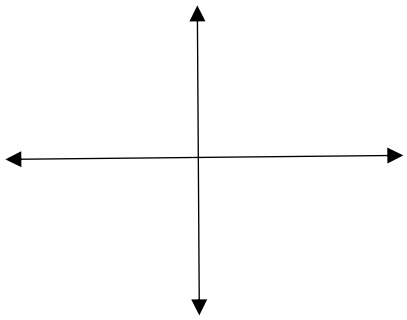


First Derivative Test & Critical Points

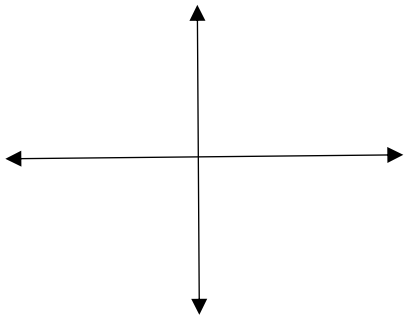
Draw a possible graph of f(x) given the information below.

1. a. $f(x)$ is a continuous curve
   b. $f'(x) < 0, (-1, 4)$
   c. $f'(x) > 0, (-\infty, -1) \cup (4, \infty)$
   d. $f'(x) = 0$, at $x = -1$ and $x = 4$



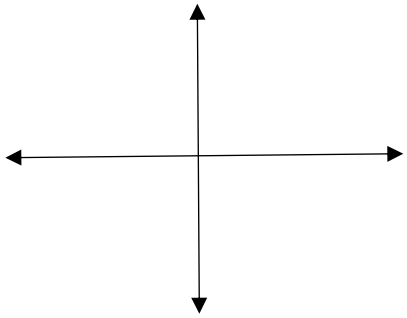
$f'(x)$

2. a. $f(x)$ is not continuous at x = 3
   b. $f'(x) < 0$, when $x > 3$
   c. $f'(x) > 0$, when $x < 3$



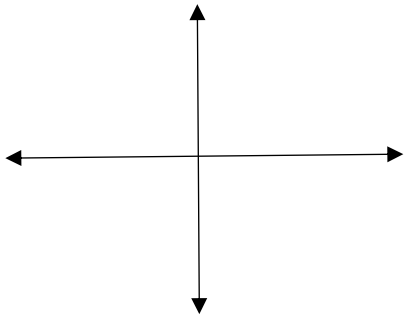
$f'(x)$

3. a. $f(x)$ is a continuous curve
   b. $f'(x) > 0$, when $x < 2$
   c. $f'(x) < 0$, when $x > 2$
   d. $f'(x)$ does not exist at $x = 2$



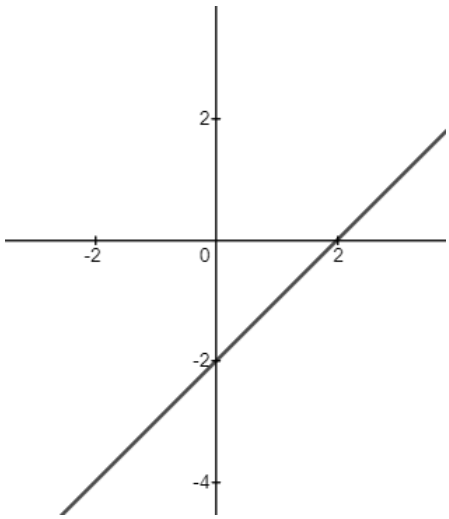
$f'(x)$

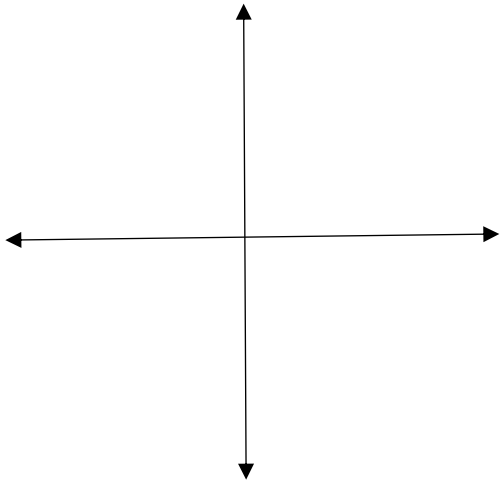
4. a. $f(x)$ is a continuous curve
   b. $f'(x) < 0, (-\infty, 3) \cup (5, \infty)$
   c. $f'(x) > 0, \ (3, 5)$
   d. $f'(x) = 0$, at $x = 3$ and $x = 5$



$f'(x)$

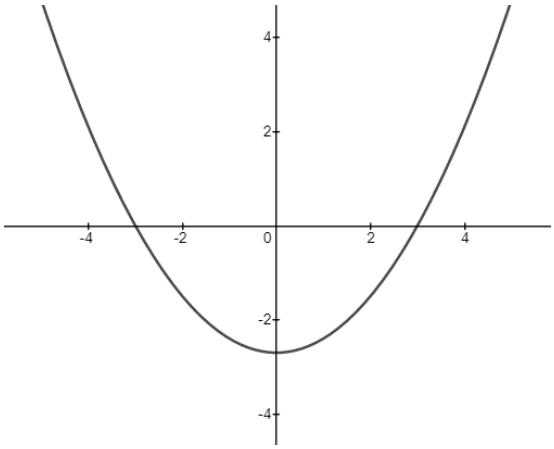
5. $f'(x)$

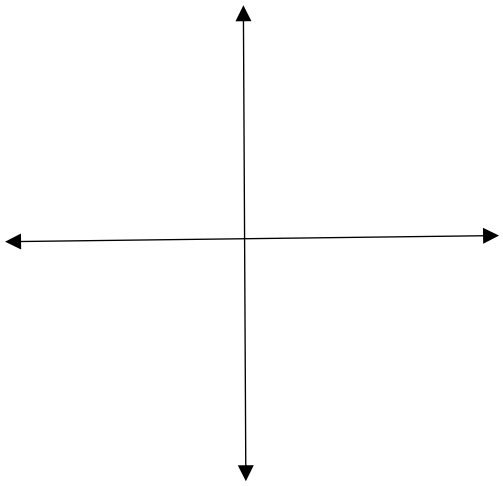




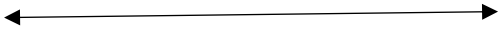
$f'(x)$

6. $f'(x)$

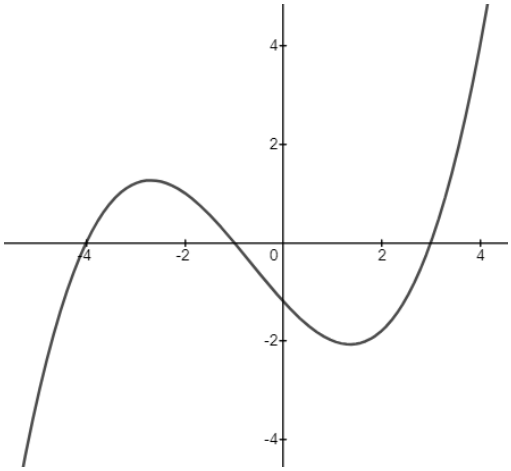


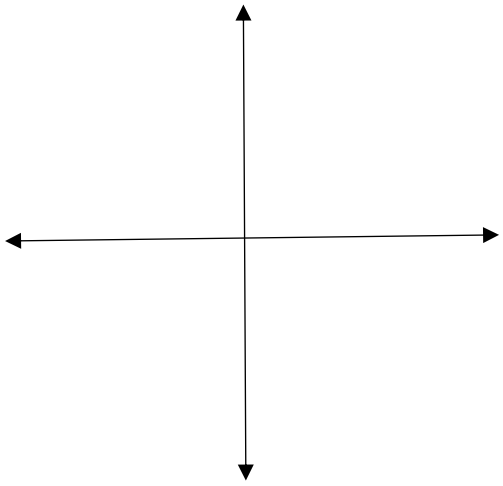


$f'(x)$



7. $f'(x)$



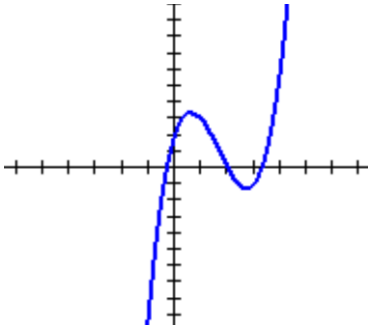


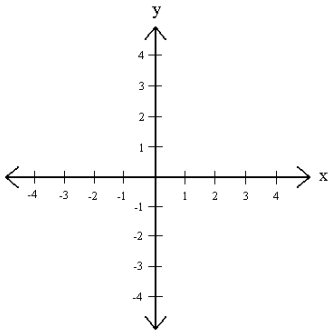
$f'(x)$

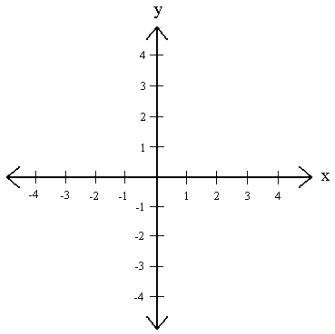
Curve Sketching Review

Given f(x), sketch the graphs of f'(x) and f"(x)

1.







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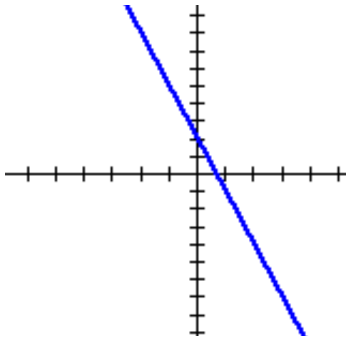

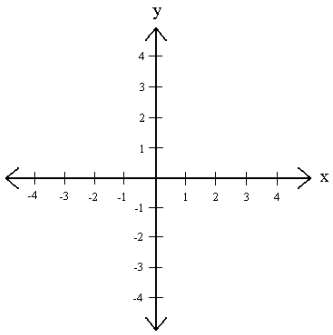

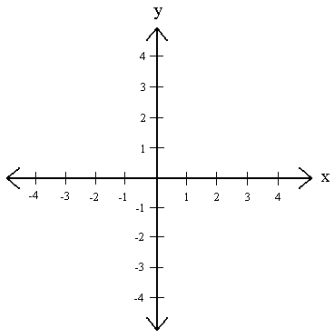

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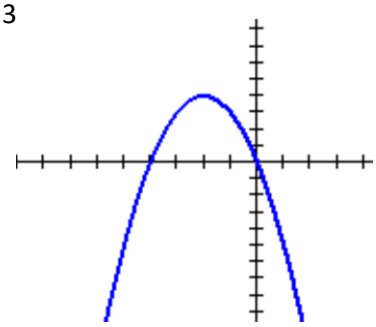

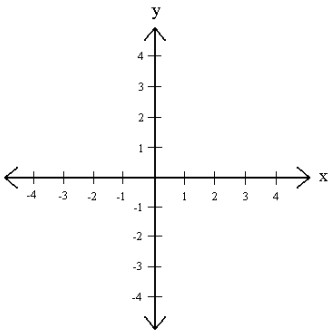

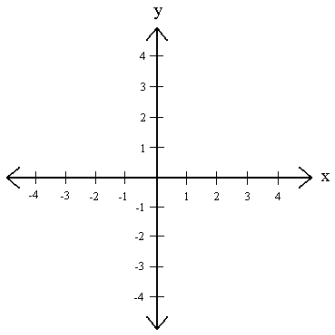

Given f'(x), sketch the graphs of f(x) and f"(x)

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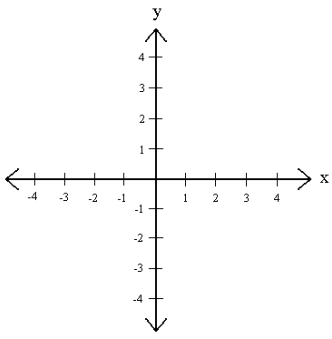
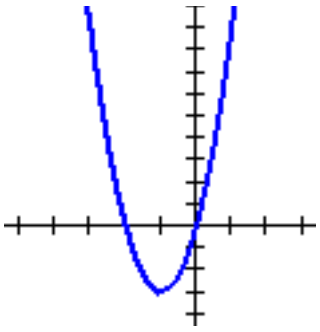
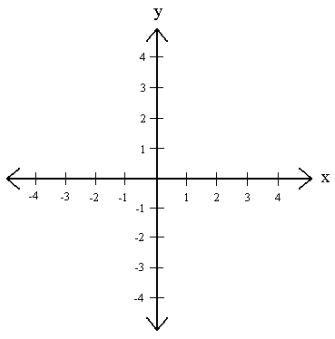
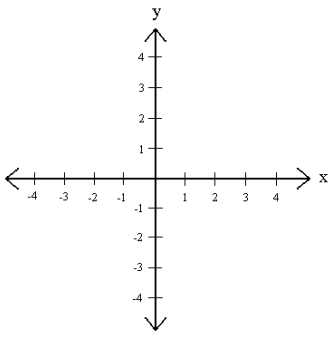
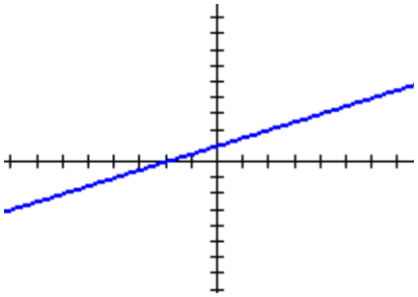
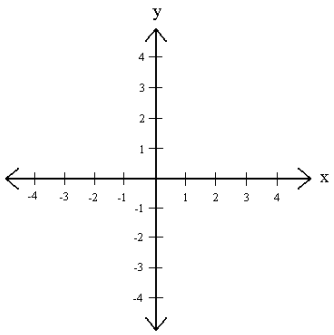
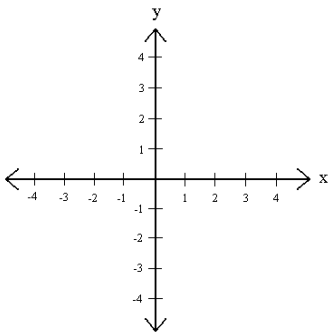
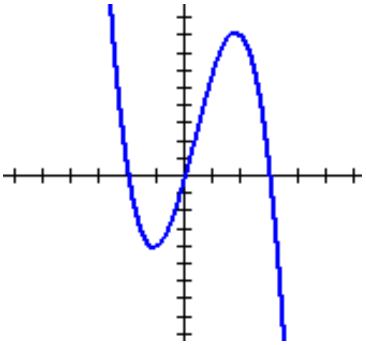
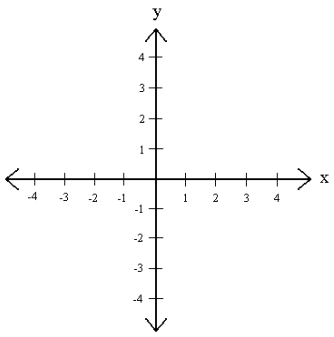
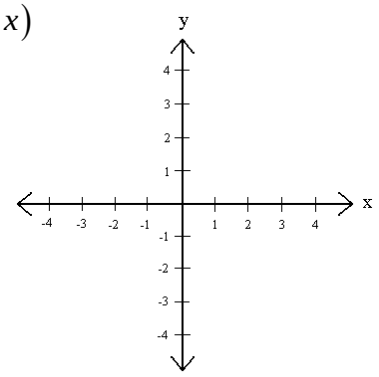
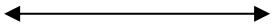
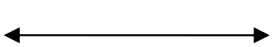
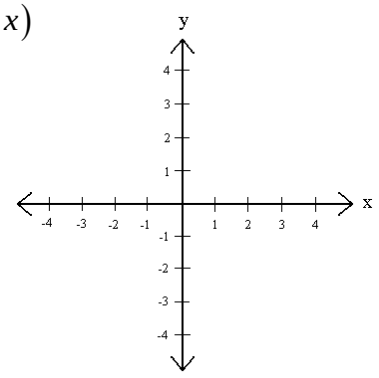






5. 		
6. 		

Sketch each graph given the information below

7. **a.** $f(x)$ is continuous **b.** $f(0)=1$ *and* $f(-1)=-2$ **c.** $f'(x)>0$ on (-3,1) **d.** $f'(x)<0$ on $(-\infty,-3)\cup(1,4)\cup(4,\infty)$ **e.** $f''(x)>0$ on $(-\infty,-1)\cup(0,1)\cup(4,\infty)$ **f.** $f''(x)<0$ on $(-1,0)\cup(1,4)$	$f(x)$ $f'(x)$  $f''(x)$	$f(x)$ 
8. **a.** $f(x)$ is not continuous at $x=0$ **b.** $f'(x)$ does not exist at $x=0$ **c.** $f'(x)>0$ when $x<0$ **d.** $f'(x)>0$ when $x>0$ **e.** $f''(x)$ does not exist at $x=0$ **f.** $f''(x)>0$ when $x<0$ **g.** $f''(x)<0$ when $x>0$	$f(x)$ $f'(x)$  $f''(x)$ 	$f(x)$ 

Meaning of a Derivative Unit Review

Find the rate of change of the function at the indicated x-value given.

1. $f(x) = x^2 + 4x + 2$ when $x = -1$

2. $f(x) = 2x^2 - 4$ when $x = -1$

Find the derivative of each function at the given value.

3. $f(x) = \frac{1}{x-3}$ at 0

4. $f(x) = \sqrt{2x+2}$ at 1

For each problem, find the equation of the tangent line AND <u>normal line</u> to the function at the given value or point. Write your answer in point-slope form.

5. $f(x) = \frac{4}{x}$ at $(-2, -2)$

6. $f(x) = \sqrt{x+3}$ at $x = 6$

Use the definition of the derivative to find the derivative of each function with respect to x.

7. $y = 3x^2 - 2x + 3$

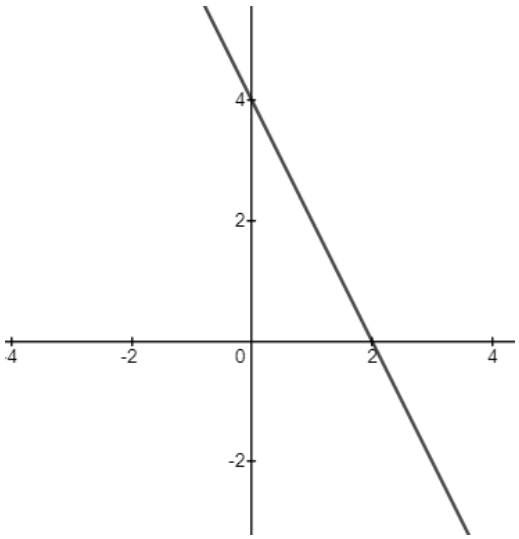
8. $y = -4x + 1$

9. $y = \frac{3}{x-2}$

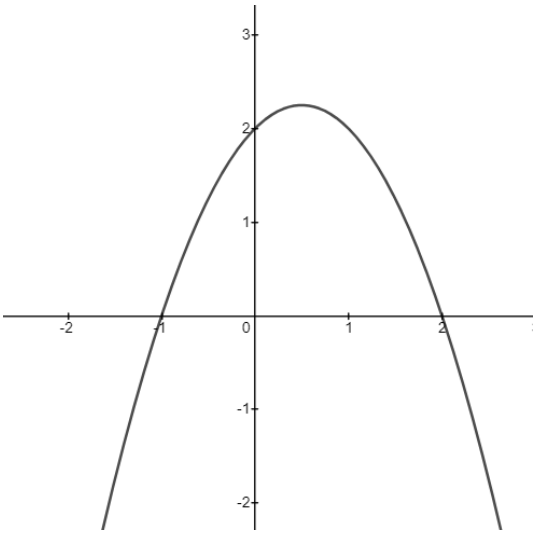
10. $f(x) = 4\sqrt{x-6}$

Given the graph of f(x), sketch a graph of f'(x) and f"(x).

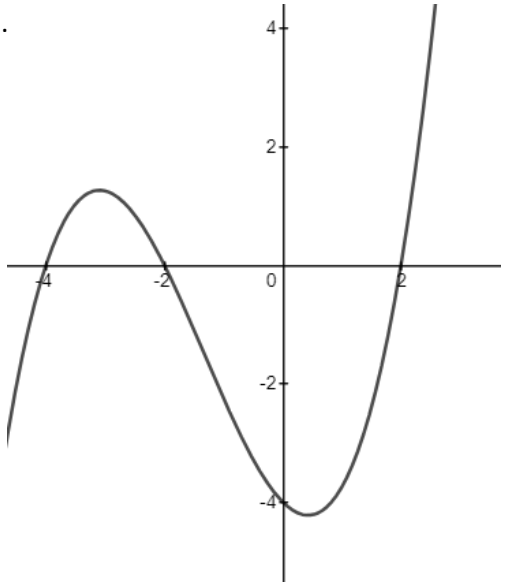
11.



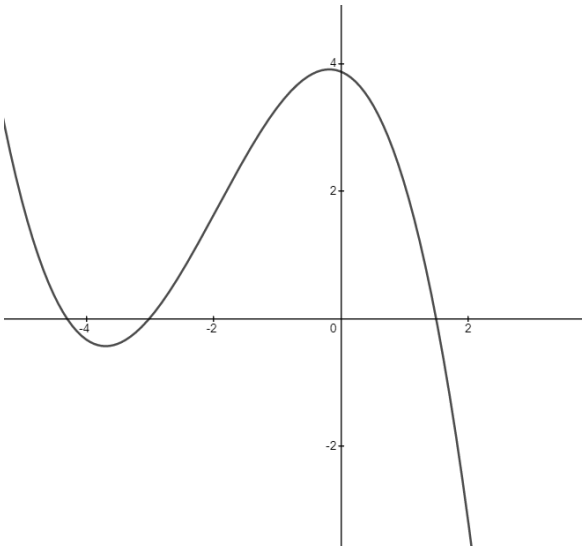
12.



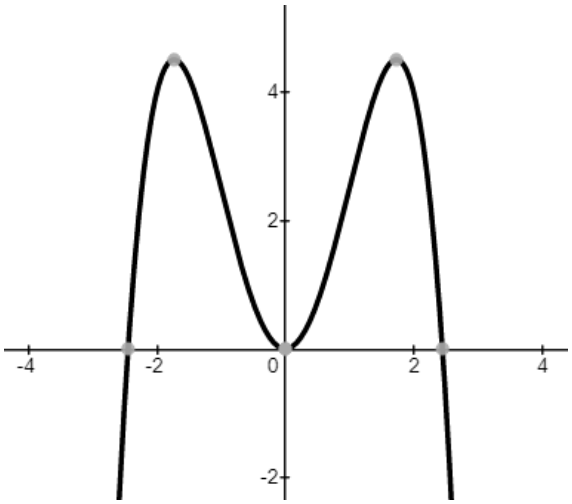
13.



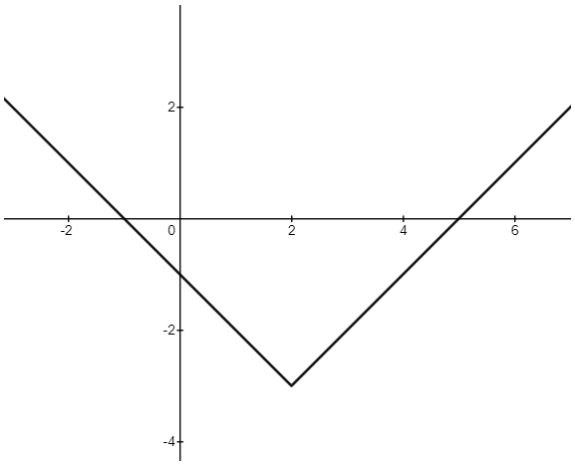
14.



15.

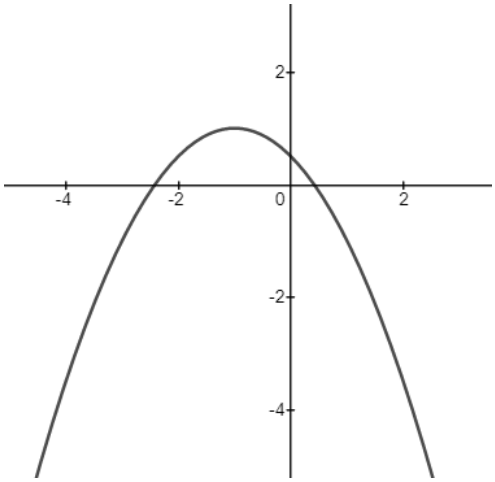


16.

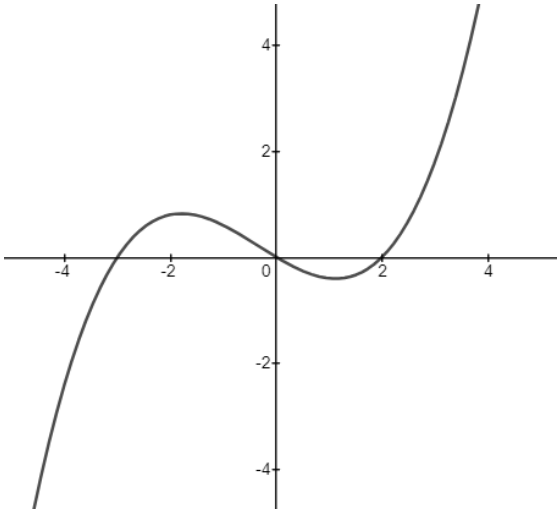


Given f'(x), sketch f(x) and f"(x).

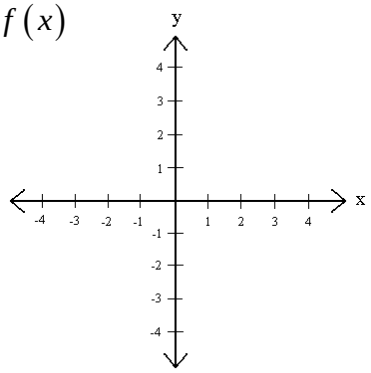
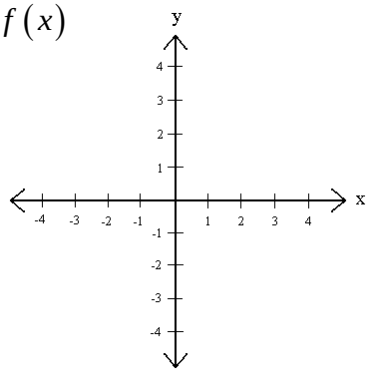
17.



18.



Draw a possible graph of f(x) given the information below.

19. a. $f(x)$ is continuous over [-3, 3] b. $f'(x) > 0$ on (-3,-2) ∪ $(-1,3)$ c. $f'(x) < 0$ on $(-2,1)$ d. $f'(x) = 0$ at x = -2, 1 e. $f'(x)$ is undefined at x= -1 f. $f''(x) > 0$ on $(1, 3)$ g. $f''(x) < 0$ on $(-3, -1) \cup (-1,1)$	$f(x)$ $f'(x)$ ⟷ $f''(x)$ ⟷	$f(x)$ 
20. a. $f(x)$ is a continuous b. $f'(x) > 0, (-\infty, -1) \cup (1, \infty)$ c. $f'(x) < 0, (-1, 1)$ d. $f''(x) > 0, (1, \infty)$ e. $f''(x) < 0, (-\infty, -1)$ f. $f'(x)$ doesn't exist at $x = 1$	$f(x)$ $f'(x)$ ⟷ $f''(x)$ ⟷	$f(x)$ 
21. a. $f(x)$ is not continuous at x = 1 b. $f'(x)$ does not exist at x = 1 c. $f'(x) > 0$ when $x < 1$ d. $f'(x) < 0$ when $x > 1$ e. $f''(x)$ does not exist at x = 1 f. $f''(x) > 0$ when $x < 1$ g. $f''(x) < 0$ when $x > 1$	$f(x)$ $f'(x)$ ⟷ $f''(x)$ ⟷	$f(x)$